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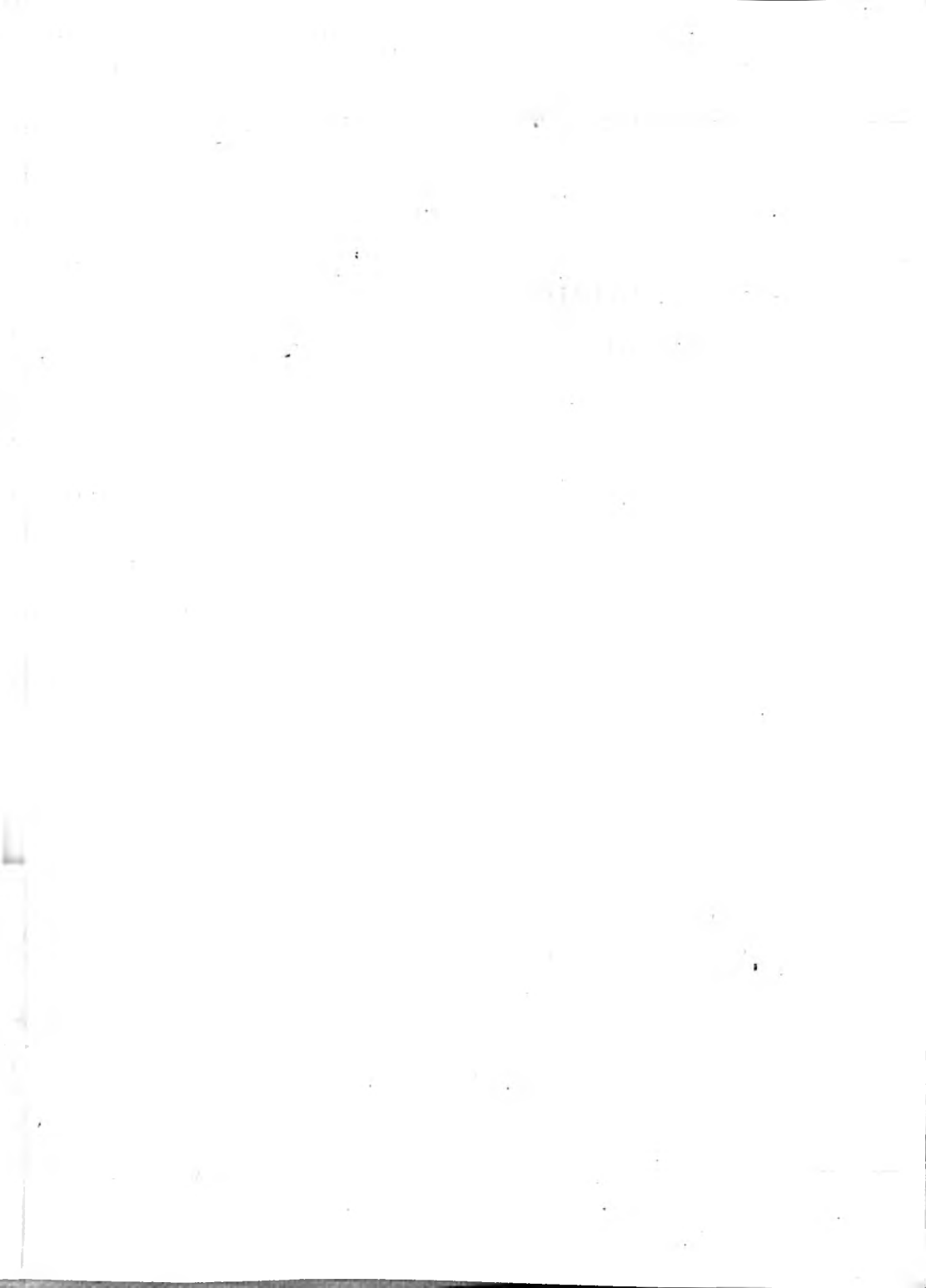
HYDROGRAPHIC AND GEODETIC
SURVEYING MANUAL

KELLAR



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UNITED STATES NAVY DEPARTMENT
HYDROGRAPHIC OFFICE



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HYDROGRAPHIC AND GEODETIC SURVEYING MANUAL

FOR THE USE OF U. S. NAVAL SURVEYS

By

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PREFACE

This Manual was compiled by Mr. John G. Kellar, Associate Hydrographic Engineer, United States Navy Hydrographic Office, and is issued to the Naval Service for the purpose of presenting the requirements of the United States Navy for the execution of hydrographic and geodetic surveys, describing the methods and equipment used for such work.

The descriptions and methods contained herein are based largely on experience gained and reports submitted by the personnel of Naval survey expeditions during the past 40 years.

Acknowledgment is made for the constructive criticisms submitted by the personnel attached to the Hydrographic Office.

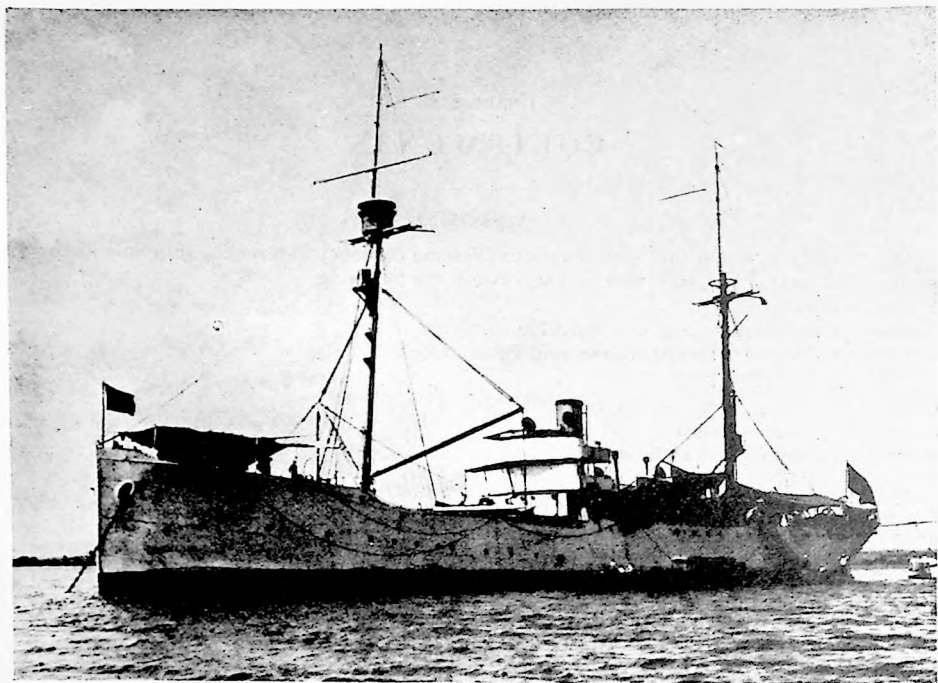
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Hydrographer.

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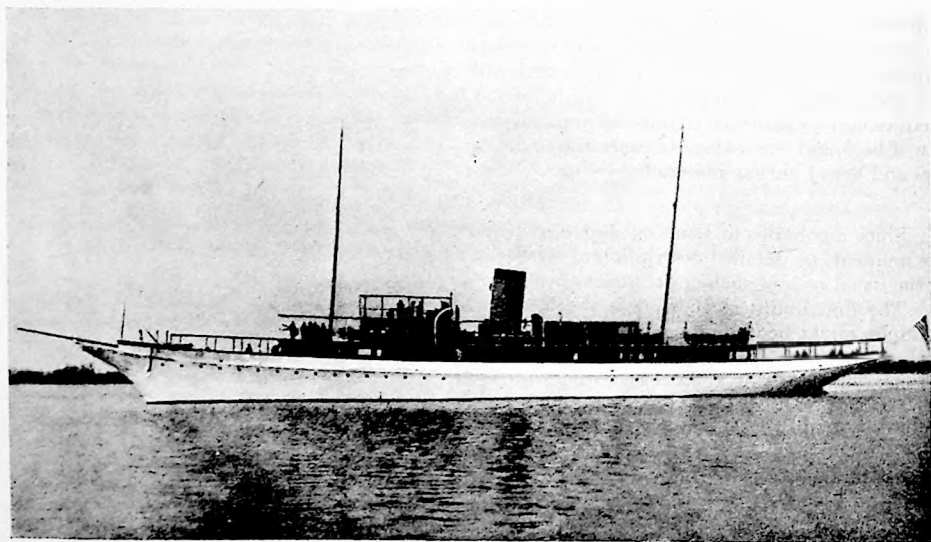
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U. S. S. Hannibal.



U. S. S. Nokomis.

CHAPTER I

EQUIPMENT

VESSELS

Among the most important considerations affecting efficiency in a survey ship intended as a base of operations for an extensive coast survey are the following:

The proper draft.

Adequate cargo space and boat carrying capacity.

Adequate space for living quarters and for drafting.

Power for towing.

Machinery for making ice and distilling water.

Special survey equipment, such as sending and receiving wireless sets, sounding machines, sonic sounding equipment, and a gyro compass.

The proper draft may be defined as the maximum that will permit the ship, not necessarily deep-laden, to penetrate the proposed area to within a short distance from the anchorages most convenient for the prosecution of work by the small-boat surveying units, in order to provide for their shelter, outfitting, and repair, for easy shifting of personnel, and for continuity of supervision.

The principal considerations affecting cargo space are the amount and kind of fuel needed, the nearness or remoteness of depots, and the difference in cost of fuel at home and in the field. Survey materials that may be stowed compactly include anchors and chain for floating signals, gravel, sand, cement, drag gear, and wire. Among materials that affect the dimensions as well as the amount of cargo space are lumber in assorted lengths, structural steel for towers, and iron pipes for beacons and water towers. Commodious cargo hatches make convenient platforms for assembling, repairing, and dismantling water signals, and for handling channel buoys.

Boat carrying capacity is important because it is chiefly by sounding launches that a ship may multiply her sounding mileage. For example, if a ship can run 60 miles of sounding per day, the addition of four launches, each good for 35 miles per day, will multiply the daily output by 3½, provided that the launches are in good running order every day. To insure this, it is well to provide a spare launch, to replace any launch that may have to be laid up for repairs.

As a rule, a ship of the proper draft for the most essential requirements of a survey is too small to carry a sufficient number of sounding launches in addition to her regular boats. Usually it will be found convenient to carry the sounding launches on the deck of a lighter ballasted by fuel and heavy survey material.

AUXILIARIES

Since auxiliaries to a survey ship must be selected from such small craft as are available at the moment, no detailed description of types can be given here; but some account can be given of the usual uses of such craft, under various conditions.

The first limiting condition is the boat-carrying capacity of the ship. A small ship, for example, might be able to carry only two small sounding launches and one or two whaleboats. Such a ship might operate economically on a bold coast having a narrow hydrographic shelf, or in harbors, but hardly on a flat coast with a wide hydrographic shelf dotted with islands and shoals. The latter, providing shelter for small craft and necessitating a high percentage of shoal water sounding, would indicate an economical employment of numerous auxiliaries, and perhaps of a smaller vessel or houseboat to serve as a sub-base.

In numbers proportionate to the size of the parent vessel and to the task, the following auxiliaries are often used:

Tenders, draft 4 to 8 feet; for dragging and sounding, freighting, towing, and tower construction.

Motor launches up to 60 feet in length; usually 60 feet for dragging, 36 to 40 feet for shoal water sounding, triangulation, and current work, and down to 24 feet for messenger service.

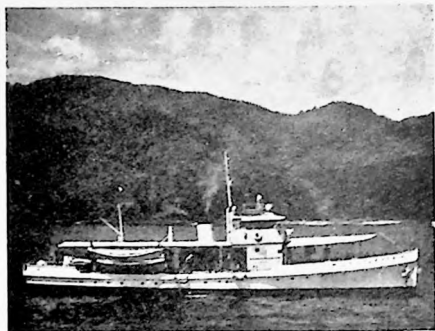


FIGURE 1.—Tenders used for sounding, dragging, towing, etc.

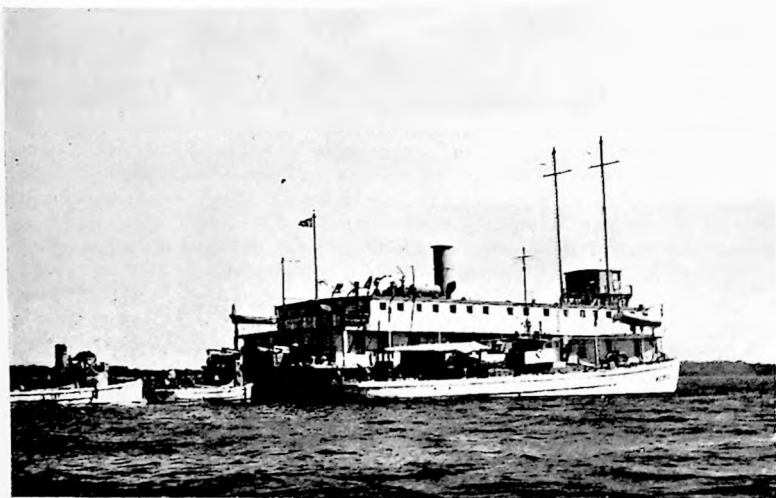


FIGURE 2.—Houseboat and small auxiliaries.

Whaleboats, dinghies, and wherries; for freighting, for landing materials and men, for shore line work, and for lifeboats of small craft.

Lighters; for the stowage of water, fuel, and signal materials, and to serve as workshops for fabricating signals and repairing boats.

Spare sounding launch; to alternate with others under repair.

Gig; for the captain's use.

INSTRUMENTS

In the following descriptions and lists of instruments, the needs of an expedition about to begin a survey of a coast of considerable extent have been kept in mind. The lists have been compiled with a view to anticipating all ordinary needs.



FIGURE 3.—Use of whaleboat for landing instruments.



FIGURE 4.—Landing on a rocky coast.

For base-line measurement two **invar tapes** or invar wires, two metric spring balances, and four or more tape thermometers should be provided. The whole apparatus should be sent to the Bureau of Standards for standardization. The usual length for invar measuring tapes is 50 meters, and the Bureau of Standards is specially equipped to mark and standardize this length of tape metal, which may be obtained from the Naval Observatory. The intended method of use should be described in the certificate.

The following standard conditions are often specified:

- (a) *Tape to be suspended.*

Tension 15 kilograms.

Suspension every 25 meters or every 12.5 meters.

Thermometers with cases weighing 45 grams to be attached to the tape at 1 and 49 meters.

Absolute error not exceeding 1 in 300,000.

Probable error not exceeding 1 in 1,000,000.

- (b) *Tape to be supported horizontally throughout its entire length.*

Tension, absolute error, and probable error as in (a).

The second tape is needed as a spare and as a standard for checking changes in the length of the first or working tape. It will be found convenient, also, to have a third invar tape 10 or 15 meters long, marked at every meter, for setting forward or backward, to avoid placing end stakes in streams or gullies, or on mounds or stumps.

Circumstances often make the use of end stakes and metal marker strips impossible or inadvisable. For substitutes it is well to provide three **portable tripods** with small heads mounted on ball-and-socket joints, and bearing short, finely divided scales, with provision for small lateral adjustment. Other accessories may be made in the field.

For graduating lead lines, stadia boards, tide boards, etc., the use of **stainless steel tapes** 50 or 100 feet in length is recommended; while for general rough use, 50-foot woven tapes or so-called "metallic" tapes are most convenient.

When it is advisable to control hydrography by precise traverse rather than by a weak extension of triangulation, **piano wire tapes** will be found to be the most practical. They handle more easily than ribbon tapes in wind and water and on rocky shores; they may be used in long lengths with light tension; and the cost of replacement is negligible in comparison with that of ribbon tapes. By comparison with a standard length laid on land or on a long wharf they may be manufactured from stock as needed.

For measurement of depths it is necessary to provide, according to the proposed scale of operations, hand leads and lead lines, deep-sea leads and sounding wire, chemical and ground-glass tubes, and apparatus required for echo sounding. If channels are to be dragged, it is essential to provide spring balances graduated up to 400 pounds for regulating the tension on the drag; also wire, floats, buoys, swivels, etc., for the manufacture and repair of the drag. For details see **Sounding**.

Outweighing the theoretical nice adjustment of weights of leads to depths, currents, speed, and height of chains, is the practical undesirability of issuing to sounding launches more than two weights of leads. In survey practice only two weights of hand leads are used in considerable quantities—a 9-pound lead for depths up to about 7 fathoms, and a 14-pound lead for greater depths; though when there is reason to expect large areas of 1- to 2-fathom water it is well to include 6-pound leads in the list. It is prudent, also, to provide a set of lead molds and a considerable quantity of pig lead for replacements.

For machine soundings 50-pound leads are commonly used. On coral or rock bottom a large loss may be expected from vessels in motion.

For hand lead lines the best material is tiller rope made of waterproof braided rope around a phosphor-bronze stranded wire core. For machine soundings stranded wire is used up to 200 fathoms, and piano wire for greater depths, in connection with checks on echo soundings.

Sounding machines.—See the chapter on **Sounding**.

Compasses.—Sounding launches and boats used for long trips should be equipped with Navy Service 7½-inch compasses graduated in degrees. With these compasses, launches and small

tenders may be steered within a degree or two of the course. Other boats may be outfitted with the smaller "boat compasses", though the latter are of little service in a choppy sea.

When the ship has to run long lines of soundings, especially offshore lines that must be plotted wholly or partly by course and engine turns, she should be equipped with a gyro compass.

Sextants and quintants.—For the use of the ship when engaged in sounding, five or more quintants and at least one 10-second sextant (in addition to that set apart for the navigator) are needed. The quintant is a favorite instrument on the bridge because of its quick-clamping device, its continuous worm-screw slow motion, and the large field of view of its telescope. But it is heavier than the sextant, and so, to avoid unduly tiring the hand, clumsy types having wide handles close to the arc should be avoided. It serves well enough for resection work in sounding, but for precise forward cuts the 10-second sextant should be used, as for example in occupying water towers and other signals too unsteady for transit occupation.

The vernier type of quintant or sextant is subject to clamp error. The so-called micrometer type is subject both to clamp error and to backlash of the screw.

Quintants have large mirrors not interchangeable with ordinary sextant mirrors. Instead of the usual horizon mirror with the lower half silvered, it is common to fit in the same holder a half-mirror with the entire surface silvered, leaving the upper part of the holder open.

Triangulation, intersection, and traverse instruments include theodolites, alt-azimuth instruments, transits, plane tables, triangulation lamps and flashlights, heliotropes, heliographs, stadia boards, ranging pikes, and fittings for attaching instruments to tower heads.

Transits and similar instruments should be fitted with prismatic eyepieces with clear and colored glasses.

When double towers are used for triangulation, one to support the instrument, the other to support the observer and to shield the instrument from wind, it is worth while to use the most precise micrometer instruments. With single towers, however, the proper instrument is one of the vernier type, suitable for observing multiple angles. (See Fig. 91.) Under these conditions, the best instrument is an 8-inch 10-second **transit-theodolite**, the outstanding features of which are a heavy, wide base with three leveling screws; a short coupling between base and plate; wide, low standards; heat-resisting covering for telescope, standards, and plates; and general simplicity of construction. These qualities insure the greatest possible resistance to vibration caused by wind, which on towers has a large lifting component, and to heat deformations. The telescope should be interior focusing, inverting, with an achromatic eyepiece of 25 to 30 diameters.

In this connection, **theodolite** is a qualitative term, referring less to the form of the instrument than to its precision, and especially to the uniformity of the divisioning of the horizontal circle, the action of the clamps and slow-motion screws, and the sensitiveness of the levels.

For two reasons a simple vertical arc without a guard is preferable to a full vertical circle—elimination of considerable wind pressure, and avoidance of the danger of warping, which would interfere with the revolution of the telescope in a truly vertical plane. Yet this handicap must usually be accepted, for in night observations requiring the inversion of the telescope the full vertical circle is needed to aid in picking up lights or stars quickly and with certainty. The vertical circle verniers of theodolites should read to 20 seconds, and of transits to 1 minute, as these instruments are ordinarily employed.

An **altazimuth instrument**, though inferior to a transit-theodolite for triangulation on towers, is suitable for that purpose in a sheltered position on the ground or on towers in calm weather; and is more convenient for star observations for time and azimuth, as well as for trigonometric leveling. Though required only occasionally for measuring altitudes up to the zenith, in case of injury to the regular triangulation instrument the altazimuth instrument might become indispensable as a spare theodolite, justifying its purchase. With this possibility in view, it would be well to dispense with some of the features desirable for star observations, namely high power, multiple threads, and axial illumination—and to purchase a 10-second transit with a full vertical circle, which would serve on occasion for astronomical observations. On the other hand, with a considerable amount of star work in prospect, it would pay to choose a high-grade altazimuth instrument.

Surveyor's transits are required for reconnaissances, for traverses, for shore-line surveying, for base-line measurement, and for subsidiary triangulation extending for only a few figures; but principally for use on towers in locating hydrographic signals by intersection. They are also used in river and harbor work for locating soundings by intersection. The principal requirements, therefore, are speed in manipulation, lightness, a tolerable precision (1 minute on both circles), clear lenses, and stadia wires with a 100-foot interval.

All things considered, in United States naval surveys the most practical transit has proved to be one weighing without the tripod about 11 pounds, with a 5½-inch horizontal circle, a 4½-inch vertical circle, a 4-inch compass circle, a direct-reading telescope, and an eyepiece magnifying about 18 diameters. The reasons for selecting a very light transit for reconnaissances, mountain trips, and traverses, a rather heavy one for tower occupations, and various other types according to individual preferences, do not outweigh the desirability of having all transits below the grade of triangulation instruments exactly alike in the same survey and in all Navy surveys, for the sake of securing interchangeability of parts, simplicity of repair at the Naval Observatory, and ease of distribution to ships.

Most instrument makers fit this type of transit with 1½- or 1¼-inch object glasses. For use in Navy surveys a somewhat larger size is more suitable, considering the great length of sights and the frequent necessity of sweeping the horizon for signals under different climatic and weather conditions.

Plane tables are sometimes of service for filling in topographic details and culture in small harbor surveys, when it is impracticable to employ aerial photography. In standard hydrographic surveys, however, the plane table method is rarely used. The instruments made by American manufacturers, with alidades 18 to 22 inches long, are suitable for work of this kind. Usually one plane table will suffice.

Signal lamps for triangulation by night are valuable when long sights are anticipated. The type developed by the U. S. S. *Niagara* during the survey of the Gulf of Panama proved to be efficient for sights up to 40 miles. They consisted of a 12-volt light, projected by a highly polished parabolic reflector, mounted with hinges in two planes to permit training in both azimuth and dip and equipped with a four-power telescope for perfecting the alinement.

A later type, now in general use in United States naval surveys, was developed by modifying an automobile headlight, equipping it with a swivel and a large filament.

Stadia rods (2), level rods (2), and range poles (6).—These instruments find their greatest use in river and harbor surveys, in running traverses, in establishing bench marks, in measuring base lines, in buoying channels, and in setting ranges for sounding or dredging. For general hydrographic surveying, the indicated quantities of precise manufactured articles will probably suffice.

Stadia boards, of ship manufacture, with whole foot spacings except the top foot, which may be subdivided into tenths, will be found more practical than precise stadia rods for the great majority of shore line traverses, in which, as a rule, transit stations at intervals are located by independent means. They may be made 15 to 18 feet high, of two light battens in a T-section, the face 4½ inches wide, the rib 3 inches deep. For sighting toward the sun it is well to mark the divisions by boring holes about ½ inch in diameter. For plumbing the board, a small pocket level should be carried.

Engineers' Y-levels are required for establishing bench marks and in connection with base line measurement. Usually one is sufficient.

Hand levels are of occasional use in reconnaissance and in sketching. The square Locke type, without a vertical arc, and the Abney type, with a small vertical arc, are favorites. The addition of stadia hairs, necessitating a longer telescope, is of doubtful advantage.

Automatic tide gages.—The C. & G. S. continuous self-registering automatic tide gage is a good type for general purposes. A roll of record paper for this instrument is 13½ inches wide and 22 yards long—enough for a lunar month.

Protractors.—For plotting three-point fixes and for laying down cuts in the field, the United States Navy three-arm steel vernier protractors are the best. As for quantities, estimate for the ship a right-hand steel protractor, a left-hand steel protractor, and a Court's protractor; for the

field drafting room the same; for each large tender, using distant hydrographic signals for fixes, the same; and for each boat engaged in sounding, triangulation, signal erection and location, shore line surveying, or in making long trips, estimate one steel protractor (with the smaller angle on either side), and one Court's protractor; add a few Court's protractors for replacing those which may be blown overboard; and if n is the number of men expected to be engaged in smooth plotting after return from the field, add $(n-2)$ steel protractors that have not been exposed to boat service, some of them left-handed and some right-handed.

For plotting any considerable amount of sounding in which positions are located by intersections, as in river and harbor work, a suitable number of Crozet protractors should be provided. The 8-inch size is convenient.

The ordinary three-arm steel protractors are inconvenient for plotting station cuts in series, sometimes as many as 100 angles on the same zero. The third arm is in the way. When cuts are observed to the nearest 10 seconds, it is hardly good practice to try to plot them with an instrument reading to only 1 minute. But the greatest objection to this use of them, contrary to their design, is the lack of straightness and rigidity of the extension arms. For the plotting of long station and topographic cuts a **triangulation protractor** having only one plotting arm, a single piece of metal 36 inches long, was designed by the senior engineer of the *Nokomis* in 1928. The circle is 20 inches in diameter and is divided to read to 10 seconds of arc on the vernier. It has been in use for some years, and has given good results with a notable saving of time in plotting.

In addition to protractors the most essential **drafting instruments** are:

Diagonal 1-meter scales and $\frac{1}{2}$ -meter scales of noncorrodible composition metal, for ratios of 1:10,000 to 1:100,000.

Straightedges, 6-foot and 4-foot, German silver or stainless steel.

Beam compasses with wooden bars of various lengths or short tubular metal bars.

Diagonal 20-centimeter scales.

Foot scales, 1 foot divided decimally.

Inch scales, 12 inches divided decimally.

Architect's scales.

Triangles—stainless steel and xylonite.

Drawing instrument sets and divider sets.

11-point spacing dividers.

Proportional dividers.

Precision pantographs.

Planimeters.

Opisometers.

Stereoscopes.

Meteorological instruments furnish data for the interpretation of tides and currents, and to that extent affect the ultimate products of a hydrographic survey—the chart and the sailing directions. The results are valuable also in the choice of the most favorable seasons for towing, sounding, triangulation, and aerial photography, and in the selection of anchorages for small boats. The most useful instruments in this class are the **anemometer** and the **barograph**, both self-registering. The **aneroid barometer**, with a maximum reading of 5,000 feet, is of occasional use in reconnaissance.

Clocks and watches for survey use may be obtained at the Naval Observatory.

A **magnetometer** and a **dip circle**, for magnetic measurements, should be included as part of the essential equipment.

A self-recording **current meter** will be required for coastal currents, and may be employed with advantage for channel currents. It should be calibrated at the Bureau of Standards before use. (See ch. V, **Tides and Currents**.)

Instruments for determining **latitude, longitude, and gravity** are not ordinarily considered to be a part of a survey ship's equipment, but are issued when occasion demands their use.

Photographic instruments and supplies for uses other than aerial photography. Photographs are especially desired for charts and sailing directions. The following is taken from General Instructions for the Compilation of Sailing Directions by Surveying Vessels (1929):

Make a proper photographic record for the Sailing Directions. Photographs should be freely used to supplement the written descriptions, etc. They may be taken from an airplane (elevation not over 50 feet) or

from the survey vessel, or from a small boat, depending on which method will best obtain a picture that will be of the greatest service to a navigator unfamiliar with the area. Each picture should be accompanied by a description of the location of the camera, expressed in terms of the bearing and distance of a known point. Prominent and useful objects, especially charted objects, that appear in the picture should be named. The photographs should be of the following character:

(a) Views of salient points of advantage to approaching and passing vessels, to be taken from the various directions of approach.

(b) Views serviceable to the navigator in approaching and passing a harbor: distant view; view of entrance; view of objects steered for on entrance course; view of leading marks, ranges, and marks that indicate turning points, used in entering.

(c) Views of landmarks in general—any picture that the survey ship's navigator may consider of value to other navigators.

It is necessary to provide also for the needs of coasting vessels for the sure identification of mountain peaks visible from sea. Photographs of ranges paralleling the coast, taken from triangulation stations and from the ship while making trips necessary for other purposes, are of great assistance to the triangulation observer, who otherwise to obtain a truthful delineation of summits must rely on sketches carried along in his notes for several years. Supplementing the general views of the ranges, special views of the principal landmark peaks should be taken on widely different bearings to give a complete idea of summit forms, and after enlargement, copies should be furnished to the triangulation observer.

Descriptions of triangulation stations should be supplemented by photographs.

Suitably illustrated descriptions of standard and improved hydrographic survey signals, instruments, and methods are of the greatest service to later expeditions.

For the purposes indicated, an 8-inch by 10-inch plate camera, with a telephoto attachment and a set of ray filters, is suggested for the ship's photographer; also an enlarging camera; also a 5-inch by 7-inch film camera for the triangulation observer, with suitable filters.

Plates are superior to films, in case it is desired to make measurements.

The Wratten K2, K3, and G filters are favorites for eliminating the effects of haze. The speed of the lens must be ample to compensate for the retarding effect of the filter. For example, with a given stop, an exposure of $\frac{1}{16}$ second without a filter is equivalent to one of $\frac{1}{4}$ second with a filter having a factor of 20.

After enlargement, views intended for the chart may be converted into line drawings suitable for re-photographing. Names and bearings are written on the enlargements. Views, enlarged prints, and line drawings should be numbered in a way to indicate their correspondence.

Range finder.—A small range finder is useful in traverse and shore line work for locating objects out of the range of a stadia board, inaccessible points on swampy shores, and controlling points of cliff lines paralleling a shore. It will save time and men whenever there is a multiplicity of detail, as in harbor surveys.

TOOLS AND MACHINERY

Among the needs of a hydrographic surveying expedition not always adequately served by the equipment of a small ship far from her base are tools and machinery for the following purposes:

- For boat repairs of all kinds in both metal and wood.
- For forging, casting, and welding.
- For shearing and punching structural steel (towers).
- For making and erecting hydrographic signals.
- For clearing land, climbing trees, and drilling rock.
- For making small fittings for instruments.
- For sewing signal coverings in quantity.
- For making ice.
- For distilling water.
- For pumping.
- For sawing, ripping, and planing lumber.

A ship engaged in reconnaissance, sounding, and towing in uncharted waters will probably need a diving outfit.

MATERIALS AND SUPPLIES

Lumber in appropriate sizes and quantities will be required for boat repairs and general ship use; for fixing up living quarters on barges, on boats, and ashore, as for example, partitions, bunks, tables, closets, ice boxes, drawing boards, sounding chains, launch canopies, and tent floors; and, of course, principally for signals, false work for foundations, and piling. The most used sizes will be indicated under the head of **Signals**.

Structural steel for towers comes in cut sizes, largely interchangeable for 60-, 80-, and 100-foot towers. Spare bolts in great quantity, spare anchor plates, and spare platforms are needed when towers are used repeatedly. For getting out material for a tower at a moment's notice, it will be found convenient to store the material for each tower in bundles or lots, tagged or painted at the ends in distinctive colors. For easy transfer to and from boats, it is advisable to carry the **sand and gravel** in bags of a uniform size, and the **cement** in sealed cans.

Anchors, chains, and shackles for large floating signals are a large item. In rough seas, many shackles are lost overboard. In stormy weather, many anchors may be lost, but this results more often than not from improper shackling. **Railroad iron** in 6-foot lengths is much used for stiffening and weighting down the legs of water towers at their lower extremities. **Telegraph wire** for guys may be estimated at the rate of 150 feet for each wooden tripod, or similar signal. **Seizing wire and annealed wire** are convenient for inland or mountain signals, for fastening wood braces to pipe iron signals, for lashing signal coverings to the legs, and for fastenings in general. Mention has already been made of **sounding wire and piano wire**, for sounding and for traverse tapes. **Galvanized iron pipes**, size 2½ inches, are convenient for beacons and water signals; and in 1-inch size for mountain signals. **Condemned boiler tubes** are needed in great quantities for stays for guys and for footings for signal masts. Short lengths of **water piping**, iron, make good center marks when set bell down and filled with concrete. **Brass piping** in assorted sizes is needed for general use. Among **other stock materials** are sheet lead, sheet zinc, sheet galvanized iron (useful for concrete forms for towers standing in water), brass and steel stock, pig iron, pig lead, bolts, spikes, nails, tacks, rope, line, spun yarn, hemp, marline, twine (for sewing signal covers), paint, shellac, and alcohol.

Black sheeting and canvas, bleached white sheeting and canvas, orange and red sheeting, are the only worth while materials for signal decoration. It pays to have signals of uniform size and design within the same class, and to make up before going to the field a considerable quantity of standard covers with suitable fastenings for each type.

Quantities of coal, gasoline, oil, water, electrical supplies, dry stores, cold storage goods, small stores, and canteen goods, will naturally depend on the type of ship and the distance from the base. Among supplies needed by camping parties are tents, mattresses, mosquito nets, fishing tackle, shoes, blankets, raincoats, rubber blankets, leggings, camp stoves, and canteens. **Medical and surgical supplies** may include antidotes and preventatives against poisonous snakes, herbs, and insects likely to be encountered.

On account of the constant danger of injury to men engaged in hoisting heavy weights and in erecting steel signals, the need of a medical officer skilled in surgery may be anticipated.

POISONOUS INSECTS, SNAKES, AND PLANTS

The following descriptions, obtained from the Bureau of Medicine and Surgery, are for the benefit of survey parties engaged in work distant from the ship.

Poisonous insect bites.—The bites of insects often cause painful and even serious injury. Multiple stings of bees may cause death, and the bites of certain species of spiders also are dangerous. The treatment of these conditions is based on the fact that the poisons injected may be destroyed or neutralized by alkali or alcohol. Therefore the application to the wound of aromatic spirits of ammonia, of sodium bicarbonate or ordinary baking powder, or of lime water or some similar alkaline solution, is very efficacious if used at once. A paste of baking soda and water is excellent, or a solution of sodium bicarbonate and water may be applied on a cloth as a dressing. The application of alcohol to the wound is excellent, also. Soap furnishes a satisfactory alkaline application for ordinary purposes, if others are not available.

Centipedes and tarantulas inflict bites of a more serious nature than those of bees and wasps, but the poison is of the same general type. The treatment is the same, except that to eliminate some of the poison it is well to suck the wound immediately (provided that there is no cut nor abrasion in the mouth), or to make a small incision at the site of the wound to encourage bleeding. When the bite is in an extremity, it will be beneficial to use a tourniquet above the bite for a few minutes, but not for long, else serious damage may result from the stoppage of the circulation. The tourniquet should be alternately released and tightened several times.

As a consequence of these bites the patient may feel weak, faint, dizzy, or nauseated. Warmth, rest in a reclining position, and the administration of 1 or 2 tablespoonfuls of whiskey, hot coffee, or aromatic spirits of ammonia, 10 drops in a quarter-glass of water, will combat these conditions.

Snake bites.—Whereas in nonpoisonous snakes no fangs are set outside the line of the teeth, which are solid, in poisonous snakes poison fangs are set outside the line of the teeth, and in addition some of the teeth are hollow and are connected with poison sacs.

Poisonous snakes are divided into two main classes according to the kind of poison produced. The snakes of the first group produce a poison (neurotoxin) that acts principally on the nervous system. The cobra and the coral snake are examples (*colubrine snakes*). Snakes of the second group produce a poison that causes the blood to dissolve, thus reducing its oxygen-carrying power. The rattlesnake, the water moccasin, and the viper are examples (*viperine snakes*). In consequence the symptoms of poisoning differ.

In bites of the colubrine snakes the symptoms at the site of the wound are less, but drowsiness, weakness, nausea and vomiting, paralysis and collapse, are prominent features. Usually the symptoms do not follow the bite immediately, but may be delayed several hours.

In bites of the viperine snakes the local symptoms are marked pain and discoloration and swelling of the wound, followed by general symptoms of weakness and depression, with labored breathing and failing pulse.

The treatment differs but little, however. The important things are:

1. *Rest*.—Make the patient as comfortable as possible, and keep him from any exertion. A reclining posture is essential.
2. If possible place a tourniquet between the wound and the heart. Ease it off at intervals of 4 or 5 minutes, tightening it again for about the same length of time.
3. Suck the wound or open it with a knife, and encourage bleeding. This washes the poison out.
4. Apply stimulants in the form of heat, warm blankets, hot water bottles, or hot water in small quantities to drink. If faintness appears, give aromatic spirits of ammonia, 10 to 30 drops in water, or a tablespoonful of whiskey. Excessive amounts of whiskey only cause depression, and do harm rather than good.
5. Apply the proper antivenin, if it is available.

Wood ticks are common in Panama and elsewhere, dropping on the passerby from trees and biting their way into the skin. There is also a fly whose eggs develop into worms that bore into the skin and grow to a length of three-quarters of an inch.

On the general subject of poisonous snakes in the tropics consult the Handbook of the Marine Corps (1930).

Poison ivy is very common in the eastern United States and Canada. It is a bushy or climbing shrub with leaves composed of three ovate leaflets, and is not to be mistaken for the ordinary harmless Virginia creeper, which it resembles somewhat. The latter, however, has five leaflets. The jingle "Let leaflets three a warning be" will keep this distinction in mind. It is well shown, also, in the Handbook of the Hospital Corps, 1930.

The treatment consists of the use of alkaline lotions, as baking soda and water, or alcohol. Both precipitate the poison. Oil or ointment should not be used, for they tend to spread the poison and make it more penetrating.

The Encyclopedia Britannica, under *Poison Ivy*, briefly describes other members of this family of plants, among them the following:

Southern poison oak, southern United States, a shrub 1 to 2 feet high, with leaves composed of conspicuously five- to seven-lobed leaflets.

Western poison oak, western United States, a shrub 3 to 9 feet high, with three roundish variously-lobed leaflets like small oak leaves.

Poison sumac (or elder or ash or dogwood), eastern and southern United States, a shrub or small tree with 7 to 13 oblong entire leaflets.

The treatment mentioned for all is a washing of the affected parts with soap and water, or alcohol, and an application of acetate of lead.

The following descriptions have been obtained from surveyors of various survey ships, as witnesses and sometimes as victims of poisoning by plants, while on duty in the tropics.

Guao, Cuba and the Island of Haiti, is a shrub resembling sumac, from 2 to 15 feet high. The small plants, usually well foliated, may be recognized by their **glossy leaflets**. The larger plants, during the greater part of the dry season at least, bear no leaves except perhaps a few leaflets at the top and at the tips of branches. The stem is seldom more than an inch in diameter, woody, crooked, and clothed with **light gray bark**. The third characteristic, in season, is the



FIGURE 5.—The manzanillo tree.

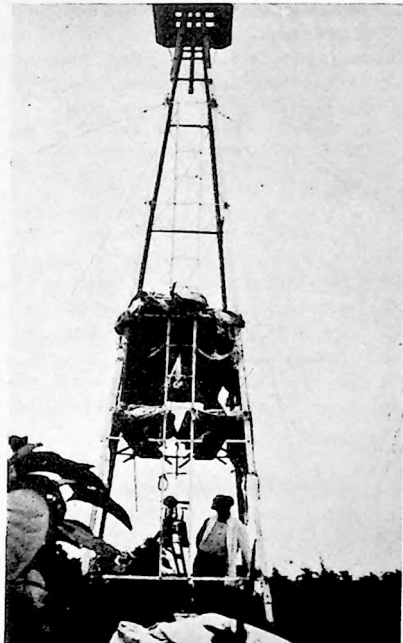


FIGURE 6.—An improvised shelter, for protection against scorpions and snakes.

loose cluster of flowers and green and purple berries. The plant is found principally where there is insufficient soil for larger plants and trees, on ridges, in crevices of rocky hills, and on low islands.

Poisoning results from bruising the stem or leaves, and even from the vapor surrounding the plant. When one is perspiring, it is well to keep to windward of it. The effects are similar to those of poison ivy, but are more violent and long continued, recurring with decreasing violence for months. Of some 20 men poisoned while clearing a base line, a few with eyes swollen shut had to be sent to the sick bay, others with swollen arms and legs were incapacitated for duty for several days, and all passed sleepless nights until the first effects had worn away.

Hot applications, as hot as flesh will bear, give temporary relief. Picric acid has been tried, with not much success. Citrate of ammonia gives considerable relief. Calamine solution with a percentage of carbolic acid has been tried with fair success.

Pica-pica, Cuba, is a plant that inflicts a mild form of itch. It is a woody vine of the pea family. Usually the malady is contracted through the medium of wet clothing laid upon the vines to dry.

Death camas, water hemlock, certain cockleburrs, and the loco weed, though dangerous to animals, are only indirectly dangerous to man, as in the use of milk.

WATER AND FOOD

Water may be clear, sparkling, palatable, and refreshing, yet carry the germs of cholera, dysentery, typhoid fever, and other serious diseases. Boating, landing, and camping parties should always be amply provided with water from the ship. When shore water has to be used it should be boiled, unless absolute certainty exists as to its purity.

Calcium hypochlorite, 1 gram to 40 gallons for half an hour, will purify water against ordinary bacteria, but will not protect against many forms of dysentery caused by small amoebae. The calcium hypochlorite is issued in capsules of 1 gram each. The contents are emptied into a small amount of water and dissolved, then this is used to sterilize a larger amount.

Foods may become the carriers of the same diseases as water. For example, cholera is often transmitted through contaminated water used to wash fruit, lettuce, etc. In addition, foods themselves, if spoiled, may cause serious poisoning. The following precautions will largely prevent danger from these sources:

1. Good refrigeration of fresh foods.
2. Particular care in the use of fish, oysters, and other sea foods, all of which spoil quickly.
3. Careful inspection of canned foods. Avoid rusty cans, bulged cans, and old cans with discolored labels. Inspect every can when it is opened. If the contents appear spoiled or if the odor is bad, the food should be rejected.
4. Thorough cooking. This prevents many diseases, as tapeworm and trichina. The germs of other types of food poisoning are also killed, and in some cases the poison itself is destroyed by heat.

In cases of food poisoning the essential treatment of the patient consists in removing the offending food from the stomach and intestines, and in rest and warmth. Nature usually does the first by causing vomiting and purging. A dose of Epsom salts (magnesium sulphate), 1 tablespoonful in warm water, may assist in this process. Quiet and warmth, warm drinks and stimulants such as hot coffee, brandy and water, or aromatic spirits of ammonia 10 to 30 drops in a little water, are often needed. Sodium bicarbonate, also, is excellent, $\frac{1}{2}$ to 1 teaspoonful. Heat should be applied to the feet and to the abdomen.

Alphabetical lists of equipage will be found in appendix II.

CHAPTER II

RECONNAISSANCE

EARLY STUDIES AND INVESTIGATIONS

The choice of the best season for field work on a particular coast and the necessity of anchorages suitable for that season are questions of commanding importance for the safety of small boats, but no less so for economy of operation and for satisfactory quality of results. The gathering of information bearing on these essentials before going to the field may well be considered as a part of the *reconnaissance*, a term used here to signify, not merely a preliminary operation, but in a broader sense a study, continuing throughout the survey, of the best methods of meeting all sorts of difficulties, such as bad weather, seasonal prevalence of winds and storms, scarcity of anchorages, difficulty of communication with the partial areas of the survey and with the outside world, wide stretches of water between possible tower sites, dangerous coasts, and swampy or excessively rugged land behind the coast line.

In planning a survey, maps of a badly charted coast are not to be trusted for bearings and distances. Nevertheless, they serve to give an idea of the nature of the task and furnish the most important local names. As a rule, they give some semblance to the reality only near towns and main channels. In the matter of least water on shoals, however, such maps are to be heeded, for though a shoal may be far out of position, the shoalest sounding will frequently be found to be correct. The quality of work done by hydrographic parties may often be judged by their success or failure in finding such critical soundings in the regular course of sounding.

GENERAL RECONNAISSANCE

Recent practice in Navy surveys involves photographing coast lines, outlying islands, and reef lines a season ahead of the hydrographic work. So far this has been done only between seasons, but the method would be particularly effective if applied also before the first season. The assembled prints, though out of scale and orientation, are capable of furnishing a reconnaissance sheet showing the most advantageous sites for triangulation towers and base lines, as well as openings in reefs and passages between islands. This method handles the especially troublesome problem presented by the unknown alinement of the salient points of a flat coast, which are strategic sites for towers. For example, if *A, B, C*, and so on, are the points of a coast, and if it is desired to place towers 12 miles apart on the points salient **for that distance**, so that, if possible, all triangulation sights may be water-borne, the proper points cannot be selected from the ship steaming along the coast, and frequently cannot even be recognized as points. Neither can they be selected by visiting the points and sighting to adjacent points, because of the curvature of the earth. The photographic sheets, however, solve the problem. These can be pantographed down to a common scale suitable for a triangulation sheet, as 1 : 60,000 or 1 : 100,000, and assembled.

In this connection it may be stated that experience in Navy surveys has demonstrated the impracticability of providing advance ground control of aerial photography. The control is preferably supplied later, with about one-third the number of signals and at no extra cost in time, by placing some of the hydrographic signals at points recognizable in the photographs, such as islets, points, and isolated trees. See **Plotting**, page 226.

When there is no photographic sheet, the most generally applicable method of obtaining first-hand information concerning anchorages, channels, islands, shoals, base line sites, and camping sites is that of trips made on dead reckoning. For this purpose, when the coast is flat, a tender of light draft having a crow's nest or other contrivance affording a sufficient height of eye, is convenient. When the coast is bold and clean, the ship may be used. When it is neces-

sary to go ashore to get the line-up of points, a reconnaissance ladder or a short tower may be used to gain height of eye. Twenty-five feet is a suitable height. Four legs will afford some stability against twist, where three will fail. The tower should be fitted to receive a transit.

RECONNAISSANCE IN FORCE, BY THE SHIP

It is convenient to classify information sought by reconnaissance as that needed by the navigator and by the executive officer, on the one hand, and that needed by the chief survey officer, on the other. The first are concerned, principally, with the safe penetration of an unknown area by the ship, with anchorages, and with the lay of the land as affecting the movements of small boats; the second principally with the location of tower sites, with the intervisibility of stations, and with locations in general. Therefore in any reconnaissance conducted by the ship in a way to gain as much of this essential information as possible with as many observers and plotters as can operate efficiently on the bridge, two sheets will be required. We may designate then as Sheet N, the navigator's sheet, and Sheet S, the surveyor's sheet.

Sheets N and S, previously prepared, should be good projections on identical scales, with meridians and parallels inked in at frequent intervals, say 4 or 5 minutes of latitude and longitude apart, with the idea of employing transparent protractors for laying down bearings quickly, and of using transfer paper or three-legged dividers to transfer features readily from one sheet to the other. The result sought by the reconnaissance is a third sheet (which may be a copy of N and S after all adjustments have been made) intended to be basic for future planning or layout sheets.

Before undertaking such a reconnaissance it will be necessary to determine the relation between engine turns and speed of the ship for the standard speed intended to be used, and perhaps for half-speed, the latter to be used for the first interval of a run while the ship is gaining velocity from zero to standard; and to determine the compass corrections on various headings.

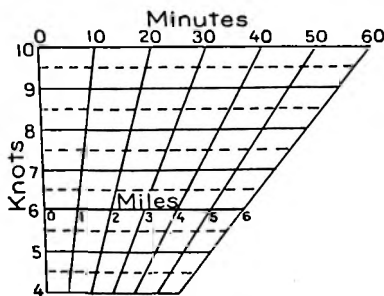


FIGURE 7.—Speed diagram.

The plotters and recorders should be furnished with tables to make this information immediately available.

Each sheet should bear a speed diagram for picking off distances readily. The unit distance is either the nautical mile or the latitude mile, equal to one minute of latitude. The diagram of the cut is not to any particular scale. The divisions on the heavy line marked "miles" must conform to the scale of the sheet. The speed, obtained from the table of engine turns and knots, is treated as an ordinate. The corresponding abscissa for the time interval from one position to the next is the required space interval on the sheet.

Many reconnaissances fail because there are too many observers and too few plotters. The observers need to be held down to essentials, and the plotters require as many aids as possible, with plenty of time between positions. A sketcher is needed for Sheet S to fill in such details as he can between located points. To illustrate "essentials", suppose that there are numerous islands on the landward side of the ship's track, and only a few scattered islands and shoals on the seaward side. The essentials on the landward side are points and large bights (anchorages) nearest the course of the ship, while on the seaward side all visible isolated islands and shoals are of commanding importance. On the landward side signals may be placed almost at will, and it is only necessary to note high trees or elevated ridges that might obscure sights, easy or difficult approaches to likely spots, and possible channels to be investigated later in detail; while on the seaward side no possible site for a signal can be overlooked, and the farther seaward it is the more important its position becomes.

The methods of conducting a reconnaissance are so various as to baffle description, depending on the nature of the region and the equipment. As an illustration, when the equipment is ample, suppose that the organization on the bridge is as follows:

The captain, as director and coordinator of operations.

The navigator or other officer in charge of Sheet N.

The survey officer in charge of Sheet S.

For Sheet N, a plotter, a recorder of soundings, a recorder of angles and bearings, and an officer at one pelorus.

For Sheet S, a plotter, a sketcher, a recorder of sextant angles, an officer for converting angles and bearings into true bearings, an officer at the other pelorus, and from two to six sextant observers.

A recorder of engine turns; a helmsman; reliefs.

With a decreased personnel some of the observers may do double duty, taking bearings for Sheet N at 5, 15, 25, etc., minutes past the hour, and cuts for Sheet S at 10, 20, etc., minutes past the hour, or at any definite minute between positions.

It is very desirable that a standard speed should be maintained. The ship's anchorage positions at morning, during the lunch hour, and in the evening may be utilized to obtain a round of angles from aloft for correcting courses previously run, for laying down cuts to distant objects ahead, and for estimating current and wind corrections. It is obvious that courses should be made as long as possible.

All compass bearings and sextant angles should be converted into true bearings, and so recorded and plotted. Positions of the ship, whether plotted on dead reckoning or with reference to objects assumed to be known in position, should include a **reference bearing** to a distant point. The latter, reduced to true by applying the compass correction, and combined with sextant angles, will reduce all observed directions to true bearings. As many as possible of the sextant angles at a position should be swung from the reference bearing, and recorded from left to right, to facilitate combining angles and bearings. When the angles become too large for the sextant, a second distant object, connected with the first by an angle, may be used for a new reference bearing. The following or any other convenient form of notes may be used:

Book S

Time a. m.	Objects— left, right	Angles	True bear- ings	To	Description
8:18	Steady on course 303° per standard compass=305° true.				
8:20	① reference	-----	276 30	①	Right tangent 8 ^m , Caballones Cay.
	2 to 1	74 15	202 15	2	Left tangent 4 ^m , Anclitas, mangroves.
	3 to 1	23 28	253 02	3	Right tangent 5 ^m , Anclitas, sandy.
	4 to 1	16 58	259 32	4	Left tangent 8 ^m , Caballones.
	1 to 5	39 50	316 20	5	Left tangent 8 ^m , Cargado, indefinite.
	1 to 6 reference	45 20	321 50	6	Tree, 9 ^m , Cargado Cay.
	6 to 7	33 28	355 18	7	Left tangent 5 ^m , Anclitas.
8:30	② reference	-----	325 00	②	Tree, 8 ^m , Cargado.
	8 to 6	141 02	183 58	8	Left tangent 5 ^m , Anclitas, mangroves.

Book N

Time a. m.	Sounding		Bottom	Bearings		To
	Fathoms	Feet		Magnetic	True	
8:18	Steady on course 303° per standard compass=305° true.					
8:20	8	3	gy M---	274 30	276 30	Position from Sheet S.
20:30	-----			St. bow--	350	Red spar 1,580 yards, by rangefinder.
21	8	3	gy M---			
23	8	2				
23:10	8	2		St. beam..	35	Red spar 1,160 yards, by rangefinder.
25	8	3				
27	8	4				
27:55	-----			St. quar..	80	Red spar 1,600 yards, by rangefinder.
29	8	5	gy M---			
8:30	8	5		323 00	325 00	Position from Sheet S.
32	9	0				

The sketcher will find a sextant convenient. For example, if the left tangent of a small island has just been observed, he can snap an angle between the two tangents, either at the "position" or at a later definite minute, to gain an idea of the size of the island. By turning angles from a distant object nearly ahead or astern he may get many extra cuts without having to worry about the exact time. As a rule he keeps his own temporary notes, and does not announce observations to the recorder.

The plotters must keep successive positions in agreement on the two sheets. From time to time, when the ship anchors or when additional information is received, adjustments in positions must be made on both sheets, and the old cuts, shown by pencil lines radiating from the old positions, must be moved to parallel cuts (since all cuts are plotted by true bearings) radiating from the new positions. It is not necessary to draw full lines, which might be confusing, but only short parts of lines near the objects cut in.

Every object is located, most effectively, by three cuts, namely, the first cut, a second cut taken on the next position to identify the object, and a third cut from a considerably later position giving a good intersection. The more important objects should receive additional confirming cuts. Distances are estimated to prevent gross errors, to assist the plotter, and to gain proficiency in estimating. Most near-by objects are cut in by bearings taken on definite minutes between positions. Only notable and characteristic soundings are plotted. Parts of channels and shoals near the ship's track may be sketched in roughly, using fathom lines.

In a running survey, which this kind of a reconnaissance resembles, three-point fixes are always available for enough of the first courses of the ship to provide bases for perpetuating the work by cutting in new signals. This kind of control is seldom available in reconnaissance, and since, with sufficient progress into the unknown, the determination of scale must eventually fall back on the always available unit of distance, which is the distance per minute the ship will travel in still water when the engines are making n revolutions per minute, it may seem best to adopt this unit for the whole reconnaissance. The advantage of a constant scale is that the result may be adjusted as a whole by changing the distances between meridians and parallels.

In addition to the two units of speed mentioned, namely speed made good under signal control during the first part of the run, and speed based on engine turns, a third method is available when there is a sharp inland peak sufficiently distant to afford good cross bearings for the whole run. A bearing to this is taken from the point of departure, and the peak is plotted on the bearing at an estimated distance or preferably at a point where a later bearing intersects the first. This fixes the scale. Since cross bearings to a distant object will give irregular space intervals from position to position, the bearings should be taken only at long intervals, and it may be well to take the mean of three or four rather than one. If the compass error is well determined, such bearings at long intervals will serve to correct the run of the ship for current (in the navigation sense) along the course. Three hours is a good interval, that being the interval of half a tide. In the absence of such an aid, the component of the current along the course must be estimated, and corrections must be made to positions every hour.

The component across the course may be taken care of by foresights and backsights, to a great extent, if there is available as a point of departure on the first long course an object that can be seen 8 or 10 miles, such as a tower, a floating signal, or an anchored barge. A backsight to this mark at the end of the course, or at the greatest possible distance on the course, will give the course made good and a correction to the course steered. At the end of the course it may be possible to set a new one directly toward a second mark, or directly away from one. The latter, obviously, is the better plan, giving a real correction, while the former must depend on an estimation of distance. Beyond these aids, the effect of outside forces must simply be estimated by the navigator or other officer familiar with the behavior of the ship, by noting the force and direction of the wind, the watching of buoys, the streaming of grass on the bottom, and the feel of the lead line.

Careful reconnaissance surveys on dead reckoning alone, by the ship and an adequate personnel, may be estimated to be good for a run of 50 miles with a scale error of about 5 percent. The method demands long courses and good water.

Astronomical sextant fixes are sometimes useful as checks, on long runs, but should be employed with caution, because of the disturbance in azimuth arising from observation errors of 1 or 2 miles, possibly more, especially should the errors chance to have opposite signs at adjacent stations.

RUNNING SURVEYS

When the initial courses are in known territory, facilitating the establishment of courses and speed made good, and when on this foundation a large part of the signals used in subsequent fixes are cut in during the run itself, the reconnaissance is sometimes called a **running survey**. With artificial and other well defined signals, there is a considerable gain in the freedom of action of the ship. Strictly, the term should be limited to apply to that form of survey ending as well as beginning under geodetic control, in which case the result ceases to be a mere reconnaissance and attains the dignity of a survey.

RECONNAISSANCE BY SMALL VESSELS

Smaller vessels, with little height of eye, and lacking a pelorus, can seldom carry distance and azimuth successfully more than 20 miles. With such vessels foresights and backsights may be employed as follows. While in known territory, a landing is made and an angle is turned from a known object to a distant unknown object in the direction of progress. A flag is left behind for a backsight, and the vessel runs to the foresight, using engine turns (tender) or an easily maintained speed, as three-quarters speed (steamer) to give distance, correcting the course by bearings on the backsight. Arrived at the foresight, by one course or several, an angle is turned from the known direction of the backsight to a new foresight. The process is repeated indefinitely. A short reconnaissance ladder is needed when the coast is flat.

In extension of the ship's reconnaissance the launch method will prove economical, and will give more definite information than any other concerning comparative distances, obstructions to sights, and ground for foundations of towers.

NAMES OF PLACES

Local names of all geographic features should be ascertained, if possible, during the early reconnaissances, taking particular care to obtain the correct official spellings. Even at best an insufficient number of names will be learned at these times, and there is always danger of attaching some of them to the wrong places. In addition, therefore, all persons working in the advanced field, especially the triangulation observers, should be on the lookout for new names and for the correct spelling of those previously learned. Finally, whenever a sheet is completed in the field it is advisable to show it to the proper local officials for verification of names.

Assigning the names of persons (not living persons) or vessels to geographical features is proper, if there is no local name, if the feature needs a name for chart purposes, and if the person or vessel has some special relation to it, such as that of discoverer or principal developer.

Next, triangulation stations should be given the name of the particular locality, if possible, and if not, that of the most prominent geographical feature nearby and under the control of the station. The preservation of this association of station names with their localities is most helpful in interpreting triangulation reports by readers of all nationalities, for interest in this matter is very general.

ESTIMATION OF DISTANCES

Distances to the sea horizon and to objects nearer may be obtained from tables when the height of eye is known. Though small vertical angles give but a feeble determination of distance, such tables are of value in the estimation of distances. By setting the sextant at angles corresponding to 1, 2, 3, etc., miles, using the horizon as a reference line for vertical angles, and noting where the imaginary points of division between successive miles lie on the land, associating each interval with a certain brightness of image, one may quickly acquire a mental scale for the estimation of distances for any particular height of eye. Or the tables may be used to obtain distances to a few well-distributed objects, and the distances to the latter may be used as bases for estimating other distances.

The tendency in estimating distances at sea is to make insufficient allowance for foreshortening, which results in underestimating distances ahead and overestimating distances across

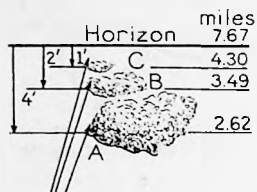


FIGURE 8.—Estimation of distances.

the line of sight. Ahead, therefore, tangent points are seen too sharp and indentations too deep, while abeam points and bights appear flatter than the reality.

In the sketch it is supposed that the angles of depression below the horizon have been measured from a height of eye of 45 feet. Since they are 4, 2, and 1 minutes of arc, to the eye *B* appears to be midway between *A* and the horizon, and *C* midway between *B* and the horizon. In reality *B* is only about one-fifth as far from *A* as from the horizon, and *C* is only about one-fourth as far from *B* as from the horizon. The distances are computed from tables 3 and 5 as follows:

	<i>A</i>	<i>B</i>	<i>C</i>
Horizon distance, table 3.....	7.67	7.67	7.67
Deductions, table 5.....	5.05	4.18	3.37
Distances, nautical miles.....	2.62	3.49	4.30

INTERVISIBILITY OF TOWERS

Concerning the intervisibility of towers in hydrographic surveys, the elements of heights of towers, distances between them, height of land and vegetation, strength of figures desired or attainable, and the connection between the main and base triangulation nets, are so interrelated and so varied in practice as to preclude a general study of the subject. By way of illustration, then, suppose that an expedition arrives in the field with 60-foot, 80-foot, and 100-foot towers, only a few of the latter, expecting to use them to the best advantage. Suppose that lines of sight approaching sea level nearer than 10 feet are to be avoided whenever possible, on account of the uncertainties of refraction and the prevalence of vapor whipped up by the wind. Further suppose that the head of each tower is fitted to receive an instrument 1 foot high, that the target on any tower sighted extends 6 feet downward from the top, and that both towers stand at sea level. Then the effective height of a tower occupied is 9 feet less than the nominal height, and that of a tower sighted is 16 feet less than the nominal height. The maximum desirable distance between any two towers is the sum of the horizon distances corresponding to the effective heights; or from table 3, at sea level, and for water-borne sights, we have:

Distances between towers at sea level
[9 feet deducted from H. T. and 16 feet from target]

Tower occupied	Tower sighted		
	60 feet	80 feet	100 feet
60 feet.....	15.76 miles.....	17.32 miles.....	18.65 miles.....
80 feet.....	17.23 miles.....	18.79 miles.....	20.12 miles.....
100 feet.....	18.50 miles.....	20.06 miles.....	21.39 miles.....

At a distance of 20 miles a target 6 feet wide subtends an arc of 10 seconds.

NOTE.—In this book the term “mile” means a geographic mile, equal to the length of 1 minute of arc on the equator, 6,080 feet, approximately. It is an angular unit. The linear unit most nearly corresponding to it is the nautical mile (used in the tables), which has approximately the same value.

To consider a more general case, suppose that between two tower sites *A* and *B*, not necessarily at sea level, there is a point of land, *I*, which is *a* miles from *A* and *b* miles from *B*; and that the heights of *A*, *I*, and *B*, respectively, are *h*₁, *h*₂, and *h*₃ feet above sea level. When are the stations intervisible?

Case 1. *Intermediate point lower than either station.*—In this case the effective heights of the stations may be found by subtracting from their elevations the elevation of the intermediate object. Thus, in the figure, if the elevations are reduced by *h*₂, in effect replacing

the ray of light MN by $M'N'$, it will be seen that two conditions must be satisfied for the intervisibility of A and B ; namely, that the horizon distance corresponding to h_1-h_2 must be equal to or greater than a ; and the horizon distance corresponding to h_3-h_2 must be equal to or greater than b . The horizon distances may be found from table 3, within its limits; or by applying the formula

$$d=1.144\sqrt{x};$$

or mentally by applying the rule:

The distance in miles is equal to the square root of the height in feet, plus one-seventh of the same.

Of several objects on the line of sight between two stations, it is not necessarily the highest that is most likely to obstruct the line of sight, for the element of curvature, which varies as the square of the distance, may remove a distant point of higher elevation farther below the line of sight than a nearer point of lower elevation. Thus, in the figure, the elevation of I , which is a lower point than B' , is the critical elevation.

Case 2. Intermediate point intermediate in elevation between the two stations.—The preceding figure will serve to illustrate this case, if modified to make A lower than I , and if the arc of reduced heights, $M'N'$, no longer possible, is removed. If it is assumed that the elevations of A , I , and B , and the distances between them are known, then the angles of elevation at A , I , and B may be computed from the formulæ

$$\begin{aligned} h_2 &= h_1 + 6080 a \tan E_2 + C_2 \\ h_3 &= h_1 + 6080 (a+b) \tan E_3 + C_3, \end{aligned}$$

and

in which h_1 , h_2 , h_3 are the known elevations in feet, 6080 is the number of feet in a nautical mile, a and $(a+b)$ are the distances in miles, E_2 and E_3 are the angles of elevation required, and C_2 and C_3 are corrections in feet for curvature and refraction.

By formula (2), **Trigonometric Leveling**, the correction for curvature and refraction expressed in meters is equal to 0.0675 times the square of the distance expressed in kilometers. Modifying the formula to express the correction (C) in feet and the distance (d) in miles, we have

$$C=0.76d^2$$

After applying this to the given equations there will remain in each only one unknown quantity, the elevation angle, E_2 to the intermediate object I , and E_3 to the distant object B . Solve the equations and compare E_3 with E_2 . If E_3 exceeds E_2 , then under ordinary atmospheric conditions, B will be visible above I ; otherwise, B will be obscured by I .

If the elevations and distances are not known, at least approximately, the only remaining method of settling the question of intervisibility will be that of trial at one of the stations, or the measurement of angles of elevation at intermediate points of critical elevation.

Problems.

1. A hill 420 ± 25 feet high is in line between a low flat cay and a mountain $2,500 \pm 100$ feet high. The distance from A to the hill is 5 ± 0.2 miles, and to the mountain 25 ± 2 miles. It is proposed to erect an 80-foot tower on the cay at sea level. What are the prospects of seeing the mountain from the tower?

In the most favorable case, $a=5.2$ miles, $a+b=23$ miles, $h_1=81$ feet, $h_2=395$ feet, $h_3=2,600$ feet, $C_1=21$ feet, and $C_2=404$ feet. Therefore, if the elevation angles at A , to the hill and to the mountain, are E_2 and E_3 ,

$$5.2 \times 6080 \tan E_2 = h_2 - h_1 - C_2 = 293,$$

and

$$23 \times 6080 \tan E_3 = h_3 - h_1 - C_2 = 2115,$$

whence

$$E_2 = +0^\circ 31' 51'' \text{ and } E_3 = +0^\circ 51' 59'',$$

which indicate success by a considerable margin.

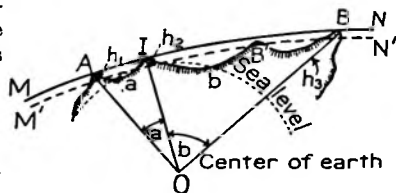


FIGURE 9.—Intervisibility, Case 1.

In the most unfavorable case, $a=4.8$ miles, $a+b=27$ miles, $h_1=81$ feet, $h_2=445$ feet, $h_3=2,400$ feet, $C_2=18$ feet, and $C_3=554$ feet. The elevation angle equations are

$$4.8 \times 6080 \tan E_2 = h_2 - h_1 - C_2 = 346,$$

and

$$27 \times 6080 \tan E_3 = h_3 - h_1 - C_3 = 1761,$$

whence

$$E_2 = +0^\circ 41' 56'' \text{ and } E_3 = +0^\circ 36' 58'',$$

which indicate failure by a small margin.

2. A is a small shoal awash, in line with the summits, I and B , of a ridge. At I a transit mounted on a tripod 5 feet high gives angles of elevation of -1° to A , 5 miles distant, and of $+1^\circ$ to B , 8 miles distant. If a 60-foot tower were placed at A , with the instrument 58 feet above sea level, how much would the line of sight to B clear the summit of I ?

The curvature and refraction corrections for 5, 8, and 13 miles are, respectively, 19.10, 48.64, and 128.44 feet. The elevation equations for A and B seen from I are

$$h_1 = 0 = (h_2 + 5) - 5 \times 6080 \tan 1^\circ + 19.00,$$

and

$$h_2 = (h_3 + 5) + 8 \times 6080 \tan 1^\circ + 48.64,$$

whence

$$h_2 = 506.63 \text{ feet and } h_3 = 1,409.28 \text{ feet.}$$

The part of 1,409.28 feet due to the angle of elevation to B from a 58-foot height at A will be

$$1409.28 - 58 - 128.44 = 1222.84 \text{ feet.}$$

At 13 miles this corresponds to an elevation angle of $0^\circ 53' 10.9''$. A line of sight with this angle of elevation from the head of the tower will attain an elevation at I equal to

$$58 + 5 \times 6080 \tan 53' 10.9'' + 19.00 = 547.32 \text{ feet,}$$

and will clear the intermediate summit by 40.69 feet.

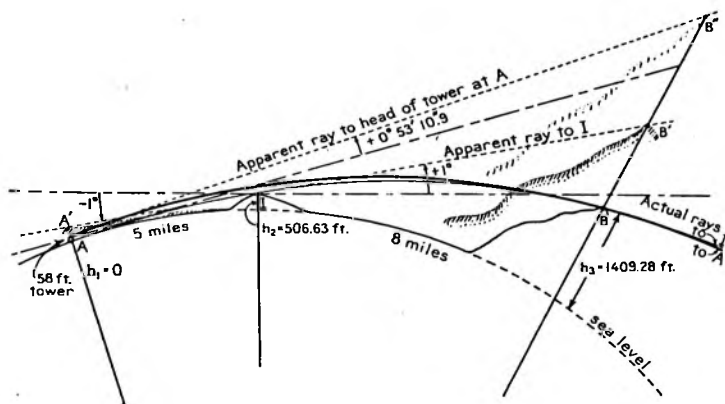


FIGURE 10.—Intervisibility, Case 2.

NOTE.—The correction for curvature and refraction is always positive, regardless of the sign of the angle of elevation.

3. The instrument supports at the ends of a 5-mile base line are 10 feet above the ground, and it is desired to maintain this height for the line of sight over the base line. How high above the points of instrument support must the targets be placed?

From instrument supports to bottoms of targets, 19 feet.

4. A low flat ray with trees rising to 40 feet above sea level is situated midway between two similar cays 12 miles apart, on which towers are to be erected at sea level. The heights available are 60, 80, and 100 feet,

to be given preference in the order named, subject to the requirement that the line of sight must clear the tree tops by at least 8 feet. Which towers shall be used? (Solution left for the reader.)

REFRACTION AND ATMOSPHERIC DISTURBANCES

In selecting tower sites the possible effects of refraction and atmospheric disturbances that affect visibility must be considered. Vertical refraction is in general a favorable factor, and is counted on heavily to make long sights possible between tops of towers. Its irregular effects, often noted when stations are occupied close to the ground, are due to local causes, but are largely avoided by using towers.

Lateral refraction is often troublesome when the line of sight is along the coast, mostly over land with stretches of water between, and also when the line of sight passes close to the side of a hill or cliff. In these situations favorable weather must be chosen in order to obtain satisfactory triangle closures. In many cases night observations will be found the only remedy. The unfavorable conditions may be avoided largely in the first place by a better selection of tower sites, on salients rather than in heights, and on summits rather than on hillsides or plateaus.

Small sandy beaches or plots at tower sites often cut down the visibility many miles during the heat of the day by causing the air to boil. The experience of being able to see a distant tower easily from the bridge of a ship anchored near a tower, and then going ashore and failing to get the sight from the tower, is a common one, disclosing this unfavorable atmospheric effect. It is a local effect, intensified by the high power of the telescope, and may often be avoided by placing towers on soil protected by vegetation rather than on bare sand plots.

ECONOMIC CONSIDERATIONS

Desirable average length of side.—It is desirable to adopt an average-length triangulation side as ideal, and to avoid alternate expansions and contractions of the net, which involve the introduction of figures weaker than the average.

On a flat coast the upper limit of length is determined largely by the curvature of the earth's surface and by the height of towers. The lower limit is determined by the distances to be spanned between islands, when they are few; by the necessity of observing at an elevation to avoid refraction effects; and by considerations of economy.

An average length of side that might be maintained easily in clear weather may have to be reduced during a season of low visibility.

It is desirable to avoid spacing the triangulation stations too widely and thereby losing intimate control of the numerous signals close to the hydrography. If possible, every sounding signal should be placed so as to permit two or more cuts from computed stations.

The question of strength of figures must be given due consideration in a practical way. In general, each new triangulation station should be so placed that the angle there, called the **receiving angle**, shall lie between 30° and 150° .

The typical figures composing a triangulation net are quadrilaterals with sides and diagonals, central-point polygons with sides and radials, and simple triangles, all lines in each case being observed over in both directions. On a flat coast with an average width of hydrographic shelf, when the towers provided are 60 feet or more in height, the best length of side is usually about as follows:

For quadrilaterals, in clear weather, shortest side 9 miles, longest side 12 miles, longer diagonal 16 miles. In the hazy part of the season, reduced dimensions, with the longer diagonal not longer than 10 miles.

For central-point polygons, shortest radial 8 miles, longest radial 16 miles, longest side 14 miles, in clear weather.

The average length of side increases when the coast is hilly. In the survey of the Gulf of Panama, the U. S. S. *Niagara*, using sights up to 40 miles long, found 14 miles to be the ideal length of sides in that climate, for the following purposes:

(a) To avoid delays. Good visibility for much over 14 miles was encountered only 3 days per month, on the average.

(b) To avoid the interference of low-lying clouds on lines of sight from the hill stations.

(c) For ease in training night lamps and in communicating with light keepers by blinker.

(d) For intimate control of hydrography.

ECONOMY OF SPACING TOWERS

The relative economy of quadrilaterals and central-point polygons may be judged by comparing 10 squares 12 miles on a side with 4 regular central-point hexagons 15 miles on a side, the figures being placed end to end. (Draw the two nets on cross-section paper).

Each net requires 22 towers. The squares cover a belt 120 miles long and 12 miles wide, the hexagons a belt 104 miles long and 26 to 30 miles wide, the actual areas covered are as 1,440 to 2,338. With an extra width of control of 5 miles on the landward side of each net and 10 miles on the seaward side, the areas intimately controlled are as 3,240 to 4,680. The polygon system, then, is nearly 50 percent more economical than the quadrilateral system in cases where each can be used advantageously, a narrow offing being more advantageous for the first and a wide one for the second. The polygon system is much more flexible than the quadrilateral system, and is especially adaptable when islands are widely and irregularly spaced. Circumstances will usually dictate a combination of the two systems, and will often compel the use of simple triangles to connect them.

Since moisture, wind pressure, and heat cause the heads of towers to move around, in order to render errors from these sources negligible, tall towers should not be spaced closer than about 6 miles, unless ground targets are provided.

THE LAY OF TRIANGULATION

When circumstances permit, triangulation control will be obtained most economically by leading the main net through the middle of the area to be surveyed, employing subsidiary transit stations on both sides, or occasional spurs from the main net, to control outlying portions. The only essential function of the main triangulation is to carry forward distance and azimuth with the least possible accumulation of error from one end of the survey to the other. The necessity for extreme accuracy does not hold in the outlying portions of the survey, unless there is a probability of another extensive triangulation net originating there.

In the case of a bold coast with a narrow shelf it may be necessary to place the net almost wholly on the mainland. In such a case, practical considerations such as inaccessibility of station sites, difficulty of transport, cost of reconnaissance, cost of clearing lines of sight, and difficulty of measuring check bases, or the reverse of these conditions, may dictate a choice or a compromise between a wide net with long sides and few check bases, and a narrow coastal net with short sides and several check bases. The expedient of employing a line of towers in the sea paralleling a line on shore, even in as little as 1 fathom of water quiet on the surface, is futile, because of the rotary uplift and the great weight of water in wave action. In such circumstances triangulation may well be abandoned in favor of a precise traverse, to provide a single line of control stations on shore.

EXPANSION FROM THE BASE

That part of the triangulation net which begins with the measured base, and ends with the first side of the main net of approximately the average length of side, is called the **base net**. It affords a means of expanding a short unit of length, the measured base line, into a computed unit of length of serviceable magnitude. Whenever possible, the base net should be located so as to provide a good connection with the main net. The primary consideration is the best position and the greatest strength of the main net.

In expanding from the shorter unit of length to the longer, the receiving angles at the unknown stations are necessarily smaller, and the figures are weaker, than they are on the average in the main net. Consequently more than ordinary care should be taken to select as few and as strong figures as possible.

A quick expansion in a single figure is desirable from the viewpoint of simplicity, economy, and avoidance of excessive azimuth errors, but may give a weak figure unless the base is of adequate length. A gradual expansion through several figures, though stronger, may result in a complicated base net, involving unnecessary expense and accumulation of azimuth error. On the whole the best solution seems to come from a measured base one-third to one-half as long as a side of the main net, with one or two expansion figures in the base net. The following types may serve as illustrations.

Figure (a) represents a single-expansion trapezoidal base net having the shorter parallel side as the measured base, or **known side**, of the figure, and the longer parallel side as the **required side**. The expansion angles are 90° and 30° each, and the ratio of expansion is 2. The base line is perpendicular to the direction of progress.

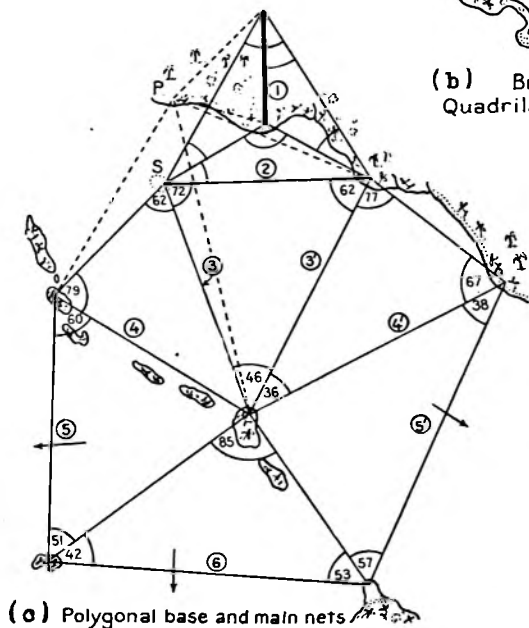
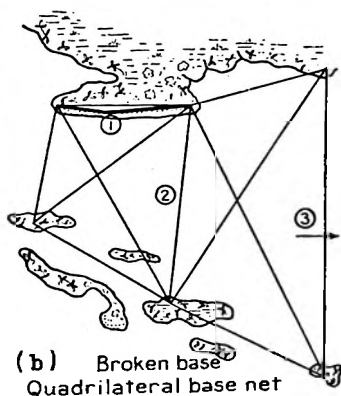
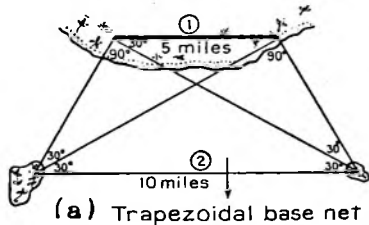


FIGURE 11.—Expansion from a base net.

NOTE.—In Figure 11 the arrows indicate the direction of progress. The ringed numbers, beginning with 1, the measured base, indicate successive bases for going forward.

Figure (b) represents a trapezoidal base net, with one leg, the projected length of a **broken base**, as the first known side. The successive bases are indicated by numbers. The base line is parallel to the direction of progress.

In Figure (c) the base net is a central-point equilateral triangle. Any radial may be the base, and any side may become the required side. In this case the required side, no. 2, is a side of the larger central-point polygon, with successive bases as numbered. The larger polygon, in addition to covering a large area and providing excellent control for a harbor survey, offers considerable choice in the direction of progress. It also carries the main net away from the main-

land, along which sights are likely to be difficult, and toward the middle of the area to be surveyed.

All the figures illustrate expansion with progress. Another form of base net, employing the principle of expansion without progress, is typified by an elongated rhombus with the short diagonal as the base and the long diagonal as the required side. It may be used when only a short extent of land is available for a base line site, or for convenience in turning the base net 90°. For example, suppose that in Figure (a) a graded road, favorable for base measurement, extended southward across the middle of the line shown as the base, while the land on both sides was swampy. A short base could be measured on the road, and the first figure could be made rhomboidal with the longer diagonal, now shown as the base, converted into the first required side.

STRENGTH OF FIGURES

The probable error in length introduced in passing from the known side of a figure to the required side is given by

$$e^2 = \frac{4}{3} d^2 \left(\frac{D-C}{D} \right)^2 (k^2 + kq + q^2),$$

or

$$e^2 = \frac{4}{3} d^2 R,$$

in which e is the probable error in the logarithm of the required side, the unit being 1 in the sixth decimal place; d is the probable error, in seconds, of any observed direction; D is the number of observed unknown directions to be adjusted to make the figure geometrically consistent; C is the number of conditions that may be expressed in the form of independent equations for adjusting the D directions; and k and q are the rates of change of the log sines of the angles opposite the known and required sides, respectively, in any component triangle, the units being 1 second and 1 tabular unit in the sixth decimal place. The rates k and q are positive for acute angles and negative for obtuse angles, corresponding to increasing and decreasing log sines, respectively.

The factor d is concerned with observers, instruments, and methods of observing. It is unknown, and can only be estimated. It may be kept small by taking the proper precautions for minimizing observing errors.

The factor R is an index of the inherent strength or weakness of the figure, depending on the form and arrangement of the component triangles. If R is small, the figure is strong; if R is large, the figure is weak. There is an R for each chain of triangles through which the required side may be computed from the known side. The strength of the figure is judged by the smallness of its least and next to least indices, R_1 and R_2 .

In any triangle the angles opposite the known and required sides, are called the **distance angles**, their sines being proportional to the two sides. The third angle, called the **azimuth angle**, does not enter into the computation of the required side from the known side. In the accompanying table, the values of the function $k^2 + kq + q^2$ of the rates of change of the log sines of the distance angles are given for various combinations of distance angles. The distance angles involved in the following problems, which refer to figure (c), are indicated by small arcs.

Problems

1. Find R_1 and R_2 for the central-point equilateral triangle, the base net in figure (c).

The number of unknown sides is 5; of observed directions to be adjusted, 10; of conditional equations, 4, comprising three triangle equations and one side equation. Therefore $(D-C) \div D = 0.6$.

The best chain of triangles consists of the isosceles triangle on the left, say, in which the distance angles are 30° and 120°, and the equilateral triangle, in which each distance angle is 60°. The corresponding values of the two rate functions are 10 and 4. (Enter the table in the first column with the difference of the angles, and in the first line with the smaller angle.) The sum multiplied by 0.6 gives 8 for the value of R_1 .

The next best chain consists of two of the isosceles triangles. The distance angles are 30° and 30° for the first and 30° and 120° for the second. The values of the rate function are 40 and 10; and their sum multiplied by 0.6 gives 30 for the value of R_2 .

2. What would be the effect of placing a station at P instead of the station at S , as indicated by the dotted lines?

The distance angles in one chain, beginning with the isosceles triangle on the right, are 30° and 120°, followed by 68° and 75°. The functions are 10 and 2, and 0.6 of their sum is 7.2. The distance angles in another

chain, beginning with the new triangle on the left, scale 62° and 73° , followed by 36° and 75° . The functions are 3 and 10, and 0.6 of their sum is 7.8. The new base no. 2, therefore, is more strongly determined than the old one; but it is not so well carried forward to the next base, no. 3.

3. What is the strength of no. 6 determined from no. 2?

All directions except those of no. 2 are unknown. Therefore $D=22$. Six triangles and one side equation obtained by equating the two values of no. 6 give seven independent equations; whence $C=7$. Therefore $(D-C) \div D=0.68$. The rest of the solution may be written conveniently thus:

Left-hand chain

$$46 \div 62 = 8$$

$$79 + 62 = 2$$

$$51 \div 60 = 7$$

$$53 + 85 = 2$$

$$19 \text{ by } 0.68 = 12.9 = R_1$$

Right-hand chain

$$46 + 72 = 6$$

$$67 + 77 = 2$$

$$57 + 38 = 14$$

$$42 + 85 = 6$$

$$28 \text{ by } 0.68 = 19.0 = R_2$$

TABLE 1.—*Strength of figure index numbers*
[Values of the function k^2+kq+q^2 (the unit being 1 in the sixth decimal place)]

Difference	Smaller distance angle																					
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	21	22	23	24	25	26	27	28	29	30	33	36	39	42	45	50	55	60	65	70	80
0	100	90	81	74	67	61	56	51	47	43	40	32	25	20	16	13	9	6	4	3	2	0
1	95	86	78	70	64	58	53	49	45	42	38	31	24	20	16	13	9	6	4	3	2	0
2	91	82	74	67	61	56	51	47	43	40	37	29	23	19	15	12	9	6	4	3	2	0
3	86	78	71	64	59	54	49	45	42	39	36	28	23	18	15	12	8	6	4	3	1	0
4	83	75	68	62	57	52	48	44	40	37	34	27	22	18	14	12	8	6	4	2	1	0
5	80	72	66	60	55	50	46	42	39	35	33	26	21	17	14	11	8	5	4	2	1	0
6	77	69	63	58	53	48	45	41	38	35	32	26	21	17	14	11	8	5	3	2	1	0
7	74	67	61	56	51	47	43	40	37	34	31	25	20	16	13	11	7	5	3	2	1	0
8	72	65	59	54	50	45	42	39	36	33	30	24	19	16	13	10	7	5	3	2	1	0
9	70	63	58	53	48	44	41	37	35	32	30	24	19	15	12	10	7	5	3	2	1	0
10	68	62	56	51	47	43	39	36	34	31	29	23	18	15	12	10	7	5	3	2	1	0
11	66	50	55	50	46	42	38	35	33	30	28	22	18	14	12	9	7	4	3	2	1	0
12	61	58	53	49	45	41	37	35	32	29	27	22	18	14	11	9	6	4	3	2	1	0
13	63	57	52	47	44	40	37	34	31	29	27	21	17	14	11	9	6	4	3	2	1	0
14	61	56	51	46	42	39	36	33	30	28	26	21	17	13	11	9	6	4	3	1	1	0
15	60	54	50	45	41	38	35	32	30	27	25	20	16	13	11	8	6	4	2	1	1	0
16	59	53	48	44	41	37	34	32	29	27	25	20	16	13	10	8	6	4	2	1	1	0
17	57	52	47	43	40	36	33	31	28	26	24	19	16	12	10	8	6	4	2	1	1	0
18	56	51	47	43	39	36	32	30	28	26	24	19	15	12	10	8	5	3	2	1	1	0
19	55	50	46	42	38	35	32	30	27	25	23	18	15	12	10	8	5	3	2	1	1	0
20	54	49	45	41	37	34	31	29	27	25	23	18	15	12	9	7	5	2	2	1	0	
30	47	42	38	35	32	29	27	25	23	21	19	15	12	9	8	6	4	2	0	1	0	
40	42	38	34	31	28	26	24	22	20	18	17	13	10	8	6	5	3	2	1	1		
50	38	35	31	28	26	23	21	19	18	16	15	11	9	7	5	4	3	2	1			
60	36	32	29	26	23	21	19	18	16	15	13	10	8	6	5	4	2					
70	33	30	27	24	22	19	18	16	15	13	12	9	7	5	4	3	3					
80	31	28	25	22	20	18	16	15	13	11	11	9	6	5	4	4						
90	30	26	24	21	19	17	15	14	12	11	10	8	6	5	5							
100	28	25	22	20	18	16	13	13	12	10	10	8	7	6								
110	26	23	21	19	17	15	14	13	12	11	11	9										
120	25	23	20	18	17	16	15	14	14	13												
130	26	24	22	21	19																	

For any figure the tabular values are to be multiplied by $(D-C) \div C$. This is about 0.7 for the commonest figures. The heavy black line, corresponding to $k^2 + kq + q^2 = 36$ and $(D-C) \div D = 0.7$, indicates in a general way the dividing line between figures of first- and second-order triangulation, as usually classified, with R_1 equal to 25. (See p. 131.)

Values of the conditional factor, $(D-C) \div D$

Triangle, central-point or exterior-point:	
All stations occupied.....	0.65
Central or exterior point not occupied.....	0.71
Triangle, simple:	
All stations occupied.....	0.75
Two stations occupied (no conditional equation).....	1.00
Quadrilateral, with two diagonals:	
All stations occupied.....	0.60
One unknown station unoccupied.....	0.71
One known station unoccupied.....	0.75
Central-point polygon, all stations occupied:	
4 sides, 0.64; 5 sides, 0.67; 6 sides, 0.68; n sides.....	$(3n-3) \div (4n-2)$
Central-point polygon, unknown center not occupied:	
n sides.....	$(3n-4) \div (3n-2)$

USE OF SIMPLE TRIANGLES

The table of **conditional factors** indicates that a simple triangle with all stations occupied is about 80 percent as strong as a quadrilateral equally well-conditioned. The use of simple triangles is not permissible when well-conditioned quadrilaterals are available, the objection to triangles being that side equations for arriving at the length of the required sides in two ways are lacking.

In a hydrographic survey, however, the distribution of islands may be such that the only quadrilateral available is one composed of two well-conditioned triangles on the shorter diagonal and two ill-conditioned triangles on the longer diagonal, with the usual added circumstance that towers of exceptional height would be required at the ends of the longer diagonal. Expressed mathematically, R_1 would be small, indicating strength coming from the good triangles, while R_2 would be large, indicating weakness due to the bad triangles. In such circumstances, it is the part of economy and good judgment to forego the use of the doubtful quadrilateral, and to employ the well-conditioned triangles only, to carry forward length and azimuth. In some cases a shoal may be found, within or without a simple triangle, on which a pipe or beacon may be placed to serve as an additional station, unoccupied, converting the triangle into a central-point triangle with a gain in strength of about 5 percent.

Triangles are often indispensable to span great distances, to connect quadrilaterals and polygons, and to change the direction of a triangulation net when rounding a cape. There should be no hesitation in selecting them when necessity or good judgment dictates.

BASE LINE SITES, LENGTHS, AND FREQUENCY

Except for convenience in computing geodetic positions, there is no advantage in placing the base line near the astronomical station; nor near one end of the survey area. The only essential requirements are that the length and position of the base line should be such as to afford a strong connection with some side of the main net, with as few and simple expansion figures as possible.

In United States Navy practice, base lines of coastal surveys are made about 4 miles long, whenever that extent of suitable land can be found. The frequency of check bases may be taken as 1 in every 75 to 100 miles, or 1 for every 15 to 20 figures. Shorter bases, 1 to $1\frac{1}{2}$ miles long are suitable for detached harbor surveys. A rough rule is that the principal base should be one-fiftieth of the extent of the survey in length. With the use of invar tapes it is sometimes more economical to measure a base long enough to dispense entirely with the base net, and so save towers remote from the sounding.

A stadia traverse run over the proposed site will enable one to lay the base on the best ground, with the terminal stations so chosen that all important irregularities of elevation, such as gullies and streams, may be spanned by single tape lengths in the regular succession. No tape length should be inclined more than 3° , which corresponds to a 5 percent grade; and no deflection angle

in a broken base should much exceed 5°. Within these limits rough ground is not unsuitable for base measurement if the footing for stakes is good. For other details, see **Base Line Measurement**.

ASTRONOMICAL AND AZIMUTH STATIONS

When the survey is charged with the duty of establishing its own observation spot, and when the methods and instruments to be employed are competent to fix the apparent zenith within 5'', it is highly desirable to bring together the origin of the survey, its controlling azimuth, and its working unit of length, by adopting for the observation spot a station of the fully expanded triangulation net; and to observe the azimuth there, thus freeing the system of the azimuth errors of the base net. Such a junction of primary elements constitutes a **La Place station**.

When the origin of the survey is an old observation spot, or a lighthouse or other reference point originally determined therefrom, it may be found impracticable to convert it into a **La Place station**. If it is inaccessible, or unsuitable for occupation, or inconveniently situated for junction with the main net, the necessary connection may be made by intersection or by traverse. The main net should not be deformed and weakened merely for the sake of a rigid connection with the observation spot.

Azimuths previously observed at the observation spot for the purpose of locating reference marks are not acceptable for orienting triangulation.

The following tables are computed from formulae given in Chauvenet's *Spherical and Practical Astronomy*, Volume I. They are based on an index of refraction of 0.0784, barometer 760 millimeters, and temperature 10° C.

TABLE 2.—*Dip of sea horizon, D*
HEIGHT ABOVE SEA LEVEL IN FEET

Feet, units.....	0	1	2	3	4	5	6	7	8	9
Feet, tens	" "	" "	" "	" "	" "	" "	" "	" "	" "	" "
1.....	3 06	0 59	1 23	1 42	1 58	2 12	2 24	2 36	2 46	2 56
2.....	4 23	4 30	4 36	4 42	4 48	4 54	5 00	5 06	5 11	5 17
3.....	5 22	5 27	5 33	5 38	5 43	5 48	5 53	5 58	6 03	6 07
4.....	6 12	6 17	6 21	6 26	6 30	6 35	6 39	6 43	6 48	6 52
5.....	6 56	7 00	7 04	7 08	7 12	7 16	7 20	7 24	7 28	7 32
6.....	7 36	7 39	7 43	7 47	7 51	7 54	7 58	8 01	8 05	8 09
7.....	8 12	8 16	8 19	8 23	8 26	8 29	8 33	8 36	8 39	8 43
8.....	8 46	8 49	8 53	8 56	8 59	9 02	9 05	9 09	9 12	9 15
9.....	9 18	9 21	9 24	9 27	9 30	9 33	9 36	9 39	9 42	9 45
10.....	9 48									

$D = 58'' \cdot 82 \sqrt{x}$, D in seconds of arc, x in feet

TABLE 3.—*Distance to sea horizon, d, in nautical miles*
HEIGHT ABOVE SEA LEVEL IN FEET

Feet, units.....	0	1	2	3	4	5	6	7	8	9
Feet, tens	0. 00	1. 14	1. 62	1. 98	2. 29	2. 56	2. 80	3. 03	3. 24	3. 43
1.....	3. 62	3. 79	3. 96	4. 12	4. 28	4. 43	4. 58	4. 72	4. 85	4. 99
2.....	5. 12	5. 24	5. 37	5. 49	5. 60	5. 72	5. 83	5. 94	6. 05	6. 16
3.....	6. 27	6. 37	6. 47	6. 57	6. 67	6. 77	6. 86	6. 96	7. 05	7. 14
4.....	7. 24	7. 33	7. 41	7. 50	7. 59	7. 67	7. 76	7. 84	7. 93	8. 01
5.....	8. 09	8. 17	8. 25	8. 33	8. 41	8. 48	8. 56	8. 64	8. 71	8. 79
6.....	8. 86	8. 93	9. 01	9. 08	9. 15	9. 22	9. 29	9. 36	9. 43	9. 50
7.....	9. 57	9. 64	9. 71	9. 77	9. 84	9. 91	9. 97	10. 04	10. 10	10. 17
8.....	10. 23	10. 30	10. 36	10. 42	10. 48	10. 55	10. 61	10. 67	10. 73	10. 79
9.....	10. 85	10. 91	10. 97	11. 03	11. 09	11. 15	11. 21	11. 27	11. 33	11. 38
10.....	11. 44									

$d = 1.144 \sqrt{x}$, d in nautical miles, x in feet above sea level

TABLE 4.—*Dip of object nearer than the horizon, D''*

Distance to water line in nautical miles	Height of eye above sea in feet									
	5	10	15	20	25	30	35	40	45	50
0.1.....	28.2	56.3	84.4	112.5	140.6	168.7	196.8	224.9	253.0	281.1
0.2.....	14.1	28.1	42.2	56.3	70.4	84.4	98.5	112.5	126.6	140.6
0.3.....	9.5	18.9	28.2	37.6	47.0	56.3	65.7	75.1	84.5	93.8
0.4.....	7.2	14.2	21.3	28.3	35.3	42.3	49.4	56.4	63.4	70.4
0.5.....	5.8	11.5	17.1	22.7	28.3	33.9	39.6	45.2	50.8	56.4
0.6.....	4.9	9.6	14.3	19.0	23.7	28.4	33.1	37.7	42.4	47.1
0.7.....	4.3	8.3	12.3	16.4	20.4	24.4	28.4	32.4	36.4	40.5
0.8.....	3.9	7.4	10.9	14.4	17.9	21.4	24.9	28.5	32.0	35.5
0.9.....	3.6	6.6	9.8	12.9	16.0	19.1	22.2	25.4	28.5	31.6
1.0.....	3.2	6.0	8.9	11.7	14.4	17.2	20.1	22.9	25.7	28.5
1.1.....	3.0	5.6	8.1	10.7	13.2	15.8	18.4	20.9	23.5	26.0
1.2.....	2.9	5.2	7.5	9.9	12.2	14.6	16.9	19.2	21.6	23.9
1.3.....	2.7	4.9	7.0	9.2	11.4	13.5	15.7	17.9	20.0	22.2
1.4.....	2.6	4.6	6.6	8.6	10.6	12.6	14.7	16.7	18.7	20.7
1.5.....	2.5	4.4	6.3	8.1	10.0	11.9	13.8	15.6	17.5	19.4
1.6.....	2.4	4.2	6.0	7.7	9.5	11.2	13.0	14.7	16.5	18.3
1.7.....	2.4	4.0	5.7	7.3	9.0	10.6	12.3	14.0	15.6	17.3
1.8.....	2.3	3.9	5.5	7.0	8.6	10.1	11.7	13.3	14.8	16.4
1.9.....	2.3	3.8	5.3	6.7	8.2	9.7	11.2	12.6	14.1	15.6
2.0.....	2.3	3.7	5.1	6.5	7.9	9.3	10.7	12.1	13.5	14.9
2.5.....	2.2	3.3	4.4	5.6	6.7	7.8	8.9	10.1	11.2	12.3
3.0.....	2.2	3.2	4.1	5.0	6.0	6.9	7.8	8.8	9.7	10.7
3.5.....	2.1	3.1	3.9	4.7	5.5	6.3	7.1	7.9	8.7	9.5
4.0.....	2.4	3.1	3.8	4.5	5.2	5.9	6.6	7.3	8.0	8.7
4.5.....	2.5	3.2	3.8	4.4	5.0	5.7	6.3	6.9	7.5	8.2
5.0.....	2.7	3.3	3.8	4.4	4.9	5.5	6.1	6.6	7.2	7.8
5.5.....	2.9	3.4	3.9	4.4	4.9	5.4	5.9	6.4	7.0	7.5
6.0.....	3.0	3.5	4.0	4.4	4.9	5.4	5.8	6.3	6.8	7.3

$D'' = 25.65d + 33.73 \frac{x}{d}$, D'' in seconds of arc, d in nautical miles, x in feet.

Divide by 60 to derive the tabular dips, expressed in minutes.

For values below the heavy lines the objects are beyond the horizon.

Problems illustrating use of horizon tables.

1. From a height of eye of 46 feet a tangent point of a cay nearer than the horizon is observed to be $1^{\circ}30''$ below the horizon. How far is the cay from the observer?

By table 3 the full horizon distance is..... Miles 7.76

By table 5 the reduction is..... 3.87

The distance is..... 3.89

2. The depression angle, below the horizon, of a buoy, with the water line plainly seen, is $0^{\circ}3'$, and the height of eye is 35 feet. Required the distance to the buoy.

By table 3 the horizon distance is..... Miles 6.77

By table 5 the reduction is..... 4.30

The distance to the buoy, therefore, is..... 2.47

3. A triangulation observer from a height of eye of 87 feet observes the elevation of the apparent horizon as $-9^{\circ}20''$ and the elevation of a peak nearly in the same direction as $+2^{\circ}20''$. The least count of the vertical circle vernier is 20 seconds, but 10 seconds may be estimated. What is the correct angle of elevation of the peak, within the power of the instrument, referred to the mean temperature and mean index of refraction that must be assumed in computing the elevation?

Table 2 gives the mean dip as 9'09". Therefore the instrument reads 11 seconds too low; and the proper correction, considering the limitations of the instrument, is +10 seconds, making the angle of elevation +2'30".

4. How much is gained in distance by using 100-foot towers instead of 80-foot towers, at sea level, for water-borne sights, under the following conditions?

(a) The line of sight must not approach sea level nearer than 10 feet.

(b) The target at the distant tower must be at least 6 feet in vertical section, and it is not to be raised above the tower.

For 80-foot towers, the effective heights are 70 and 64 feet; the corresponding horizons are 9.57 and 9.15 miles; and the total distance is the sum of the latter, 18.72 miles.

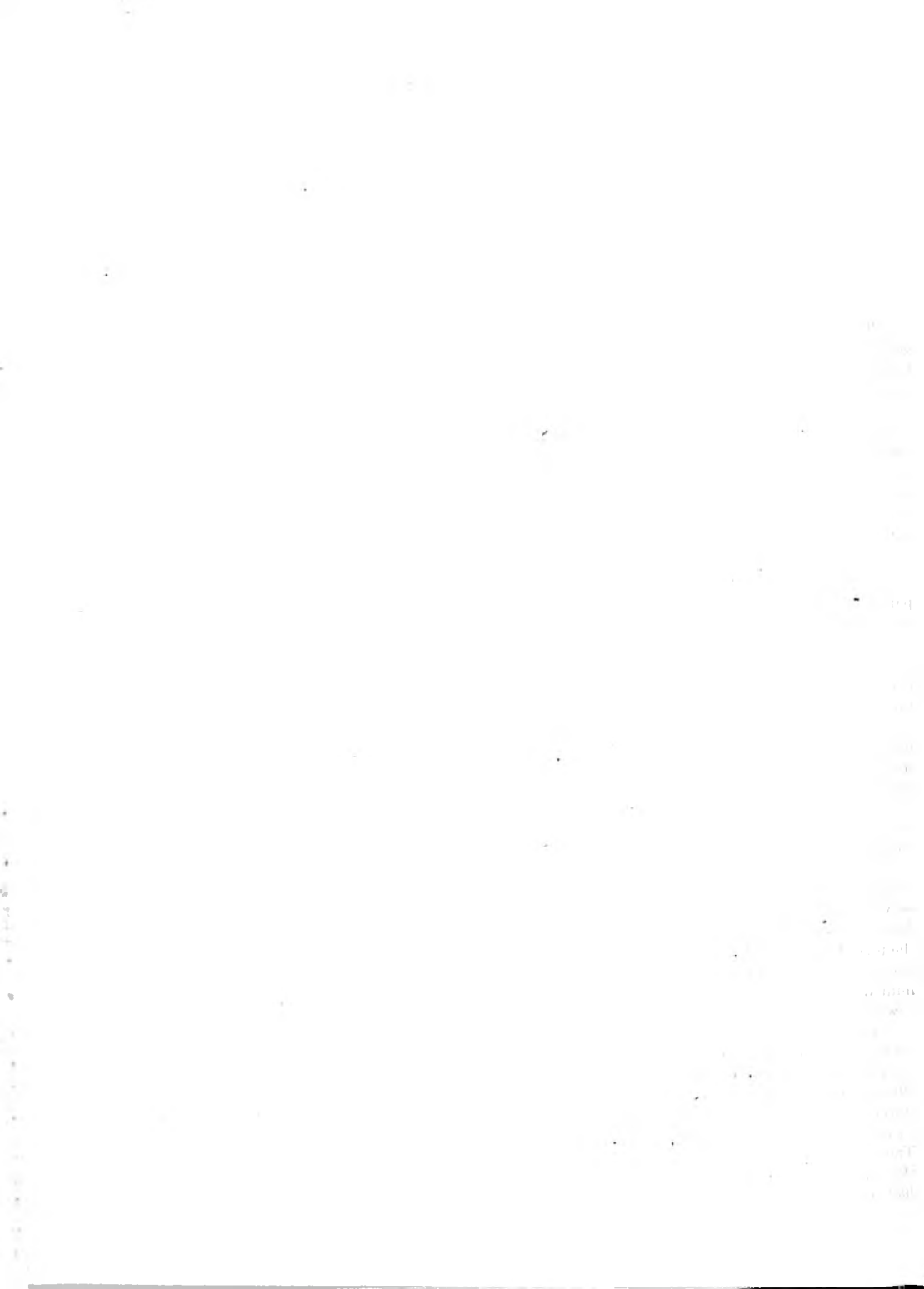
For 100-foot towers, the effective heights are 90 and 84 feet; the horizons are 10.85 and 10.48 miles; and the total distance is 21.33 miles.

The advantage amounts to 2.61 miles.

TABLE 5.—Deduction for object nearer than the horizon

	Height of eye, z , in feet									
	10	20	30	40	50	60	70	80	90	100
	Dip of sea horizon, D , in seconds									
	186	263	322	372	416	456	492	526	558	588
Depression below sea horizon, D'	Distance to horizon, d , in nautical miles									
	3.62	5.12	6.27	7.24	8.09	8.86	9.57	10.23	10.85	11.44
	Deduction, d' , from horizon distance, for object at sea level D' seconds below horizon									
"										
10.....	1.15	1.48	1.71	1.90	2.06	2.21	2.35	2.47	2.59	2.70
20.....	1.45	1.81	2.08	2.30	2.48	2.65	2.79	2.93	3.06	3.18
30.....	1.63	2.06	2.40	2.61	2.71	2.99	3.15	3.30	3.44	3.57
40.....	1.78	2.26	2.59	2.86	3.08	3.27	3.45	3.61	3.76	3.90
50.....	1.91	2.41	2.78	3.07	3.30	3.51	3.70	3.87	4.03	4.18
60.....	2.01	2.52	2.95	3.25	3.50	3.73	3.91	4.11	4.27	4.43
70.....	2.10	2.69	3.09	3.41	3.68	3.91	4.13	4.31	4.49	4.66
80.....	2.18	2.80	3.22	3.56	3.84	4.08	4.31	4.50	4.69	4.86
90.....	2.25	2.89	3.34	3.69	3.99	4.24	4.47	4.68	4.87	5.05
100.....	2.31	2.99	3.45	3.81	4.11	4.38	4.62	4.84	5.03	5.22
110.....	2.37	3.06	3.54	3.92	4.27	4.51	4.76	4.98	5.19	5.38
120.....	2.43	3.13	3.63	4.02	4.35	4.55	4.88	5.12	5.33	5.53
180.....	2.65	3.48	4.05	4.50	4.88	5.21	5.51	5.78	6.02	6.25
240.....	2.80	3.71	4.34	4.84	5.26	5.63	5.96	6.26	6.53	6.79
300.....	2.91	3.88	4.56	5.10	5.55	5.95	6.31	6.63	6.93	7.21
360.....	2.98	4.01	4.73	5.30	5.79	6.21	6.59	6.95	7.25	7.55
420.....	3.06	4.11	4.87	5.47	5.98	6.42	6.82	7.19	7.53	7.84
480.....	3.11	4.20	4.98	5.61	6.14	6.60	7.02	7.40	7.74	8.08
540.....	3.16	4.27	5.08	5.73	6.27	6.76	7.19	7.58	7.95	8.29
600.....	3.19	4.34	5.16	5.83	6.39	6.89	7.33	7.75	8.12	8.47
1200.....	3.38	4.66	5.61	6.39	7.05	7.64	8.17	8.66	9.11	9.52
1800.....	3.47	4.80	5.80	6.63	7.34	7.97	8.55	9.08	9.57	10.03
2400.....	3.49	4.87	5.88	6.76	7.50	8.16	8.76	9.32	9.83	10.32
3000.....	3.51	4.92	5.97	6.84	7.60	8.28	8.90	9.48	10.01	10.51
3600.....	3.52	4.95	6.02	6.91	7.68	8.37	9.00	9.59	10.13	10.65
$d' = 1.144\sqrt{z} - \frac{D+D'}{50.3} + \sqrt{\left(\frac{D+D'}{50.3}\right)^2 - 1.315x}$										

CAUTION.—Note that D' is not a true depression angle, such as would be measured by a transit from a level line of sight, but is a *relative* depression angle, measured downward from the horizon, as with a sextant.



CHAPTER III

SURVEY SIGNALS

STANDARD TYPES AND SIZES

The principal survey signals perfected by the United States Navy comprise an improved steel triangulation tower, a four-legged wooden water tower that can be fashioned on deck and lowered into the water with the ship's boom, a 40-foot tripod, and a 4-drum, 3-anchor floating signal. In recent surveys there is a tendency toward uniformity in other types of survey signals.

Standardization in form and size results in a large saving in time, which is a prime consideration when the season of good weather for sounding is limited. If a ship goes to the field with a supply of triangulation towers, tripods, tetrapods (usually called "quadrupods"), and floating signals of standard form, **knocked down** and stowed in bundles, with the dressings cut to the correct form, sewed, fitted with eyelets and lashings in place and labeled, upon arrival at the field it will usually require only a few days to signal out an area effectively and to cut in the signals, ready for sounding.

MATERIALS AND DRESSINGS

In the construction of signals, rough commercial lumber serves for general purposes. When parts are intended to be used repeatedly, such as masts of floating signals, only the best material for the purpose is economical. Iron pipes and steel parts of old towers are useful for semi-permanent signals, such as beacons intended to mark shoals.

Considering the cost of labor and fuel, and above all the cost in time due to interruptions and extra trips for the redecoration of signals, it pays to use only substantial cloth and standard drapings and fastenings.

For white cloth, the **count**, or the number of threads to the inch, warp and filling, is an index of the light reflecting power. Sheeting with a count of about 72 by 68 has proved to be the best material for white dressings. Bunting, with a count of 28 by 26, only, is unsatisfactory. Cheese cloth and mosquito netting may be classed as worthless.

For black cloth the most convenient index of efficiency is the weight, and it is necessary also to insure that the color is fast. It is difficult to obtain good black cloth, and for this reason lightweight canvas or duck is commonly used.

Colored dressings have generally proved to be ineffective except at short distances. As light reflectors they are effective only in the degree of their resemblance to white; as light interrupters, only as they resemble black. A red cylindrical molasses tank with a conical roof affords a familiar example of the color value of red. The object is large, the color is solid and bright, and the form is such as to reflect light in many directions. If seen by reflected sunlight it is conspicuous, but far less so than if it were painted white. If the sun is obscured by a cloud, the tank immediately merges into the background; or if it interrupts the sky line, its color value is that of a weak black.

In decorating a large signal for distance, **mass of color** is more effective than variety. For such signals, white alone, or black alone, or a wide spread of both, are commonly used. Turkey red is good for a moderate distance. In 1924 the U. S. S. *Hannibal* made an investigation of the effectiveness of other colors, in comparison with the usual ones, at Guanimar, Cuba. An 80-foot tower was covered from top to bottom with cloth of various bright colors, and a large black flag was mounted at the top. The background was land in the dry season, of a faded green color. The village of small houses, mostly unpainted, near which the tower stood, was in plain sight of the anchorage, a few miles distant. The result of the test was that the village and the black flag could be seen at any time. But most of the time, even at a short distance, binoculars were

required to find the tower, which was, in effect, not decorated but camouflaged. It was also found that using cloth in narrow alternate strips, even black and white cloth, resulted in camouflage.

The general rule is to employ white against the land and black against the sky, but there are exceptions. Though white against the land is most conspicuous by reflected light, the light being behind the observer or on one side, it is equally inconspicuous when the observer faces the light, and is easily hidden, early and late in the day, by haze hanging over the water. It is safer to use black to outline that part of the signal considered as the target, thus avoiding phase error; and to cover an expanse below the target with white, to be seen when the sun is shining from the right direction; or to mount a white flag above, to flash in the sunlight. This will meet the needs of the triangulation observer and the sextant observers in sounding boats, and will avoid a condition sometimes encountered when all the sounding signals are decorated in white, namely, that in the morning all signals in the east, and in the afternoon all signals in the west, are invisible.

For the spread of decorations below a black triangulation target, orange with either white or black is fairly effective, one color being used to outline the other.

In certain situations, as when a tower in the water is viewed from a high hill, or when the tower stands in front of trees higher than itself, the background of the head of the tower is dark. In such cases the best decoration is white above the platform and black below.

TOWER SITES, DECORATIONS, AND CENTERS

In selecting the site of a triangulation tower, in preparing the foundations and center mark, and in decorating the tower, all considerations of convenience and of possible uses of the tower for other purposes than the accurate extension of triangulation and the permanence of the mark should be rigidly subordinated to the main purpose. If there is firm land a little way inside of a muddy point, that is the place for the tower rather than the point. If the lower part of the tower is obscured by trees, that is so much the better for the safety of the tower and the precision of the angles to be observed thereon. The visibility of the tower naturally desired by sounding parties should be given no weight whatever. The top decorations should be black, to avoid phase. They should be substantial, well fitted, and easily removable and replaceable. Before the completion of the triangulation observations at that point there should be no decorations below about 30 feet from the top.

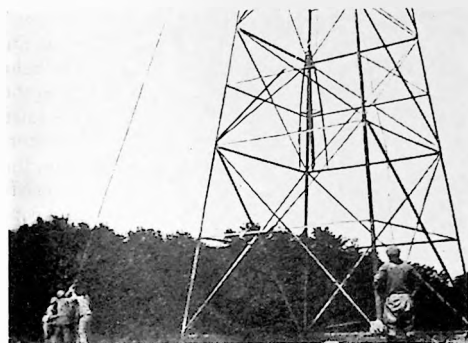
ULTIMATE CONTROL FOR SUBSEQUENT SURVEYS

Stacks, steeples, conspicuous trees, and other landmarks are cut in as a matter of routine, and are often useful in long-range work. In boat work, however, they cannot be counted on heavily. Usually the signals closest to the sounding are laid out tentatively, in geometrical pattern, in straight lines diagonal to the sounding lines, and are later modified in position according to the lay of the land. Often the boats could get along without any land signals. Nevertheless, a certain number of land signals and of spots permitting occupation with a transit should be included in the scheme, to bring the control of the towers closer to the sounding and to provide stations for obtaining a greater body of tangents. It is well to mark all transit stations with boiler tube centers, especially near channels, in order to afford a permanent intimate control of the hydrography, enduring after the removal of towers and principal signals.

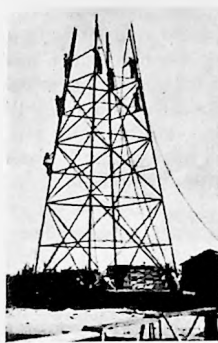
Landmarks, tangent points of islands, beacons, and center-marked transit stations thus become points of ultimate control, where subsequent harbor improvement surveys may begin. Such points are essential, also, for checking the positions of beacons and buoys moved as a result of repairs or storms, and for locating new navigation marks. The control points, therefore, should be located in such a way as to afford at least one line of stations connected by distance and azimuth.

TRIANGULATION TOWERS

Prints and specifications of steel triangulation towers are on file at the Hydrographic Office. The Navy standard tower is of the four-post, sectional type, galvanized steel. It is built up from footplates bolted to concrete foundations to a maximum height of seven sections, or about 100 feet. By omitting the first or lower section, as in the drawing, and moving the footplates



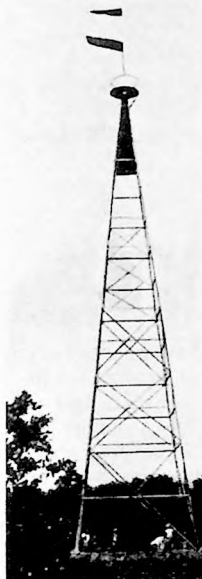
Hoisting the top section in one piece.



Hoisting pieces with a gin pole.



Laced skirts, wind vents.



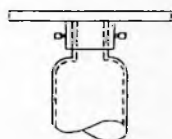
100-foot tower, complete.



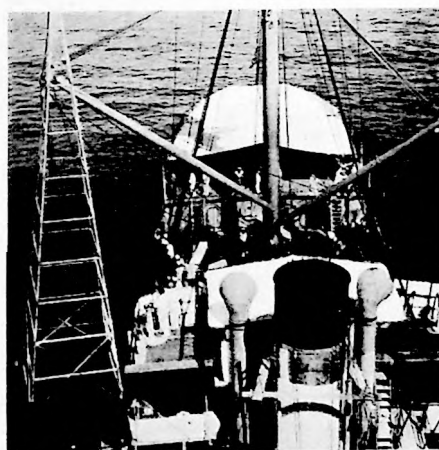
Mixing concrete for a tower on a shoal.



Passing pieces up by hand.



Interchangeable instrument support.



Planting a tower in the water. The base is weighted.

FIGURE 12.—Triangulation towers.

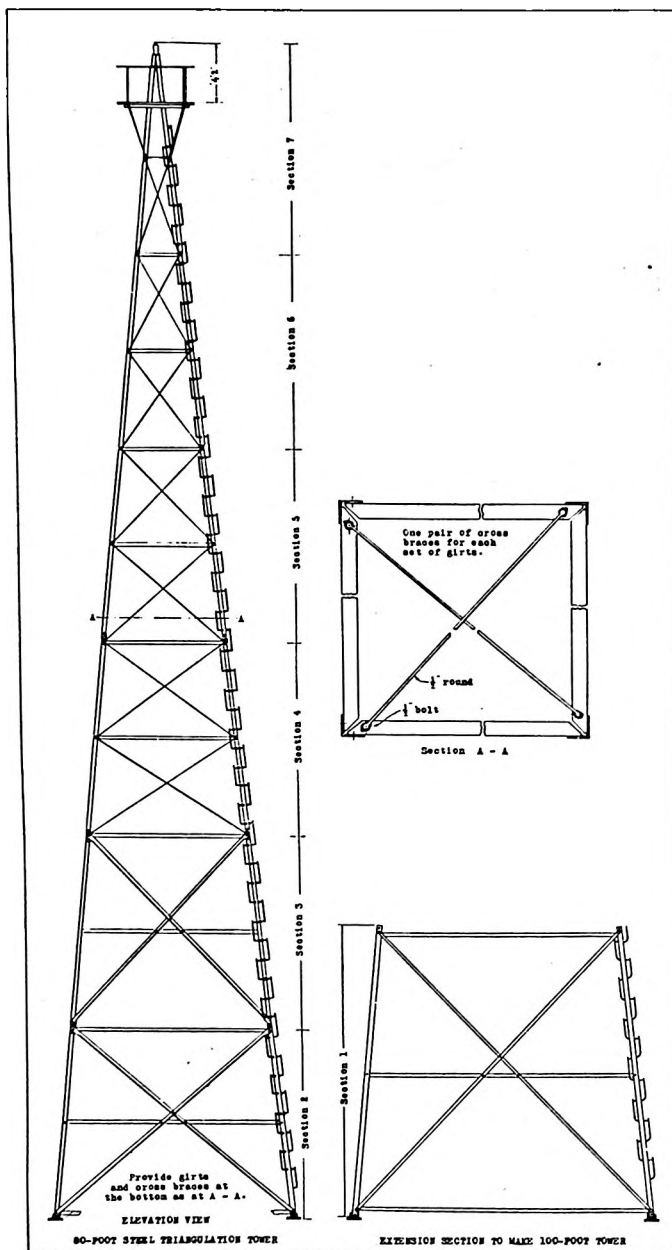


FIGURE 13.—Triangulation tower, elevation.

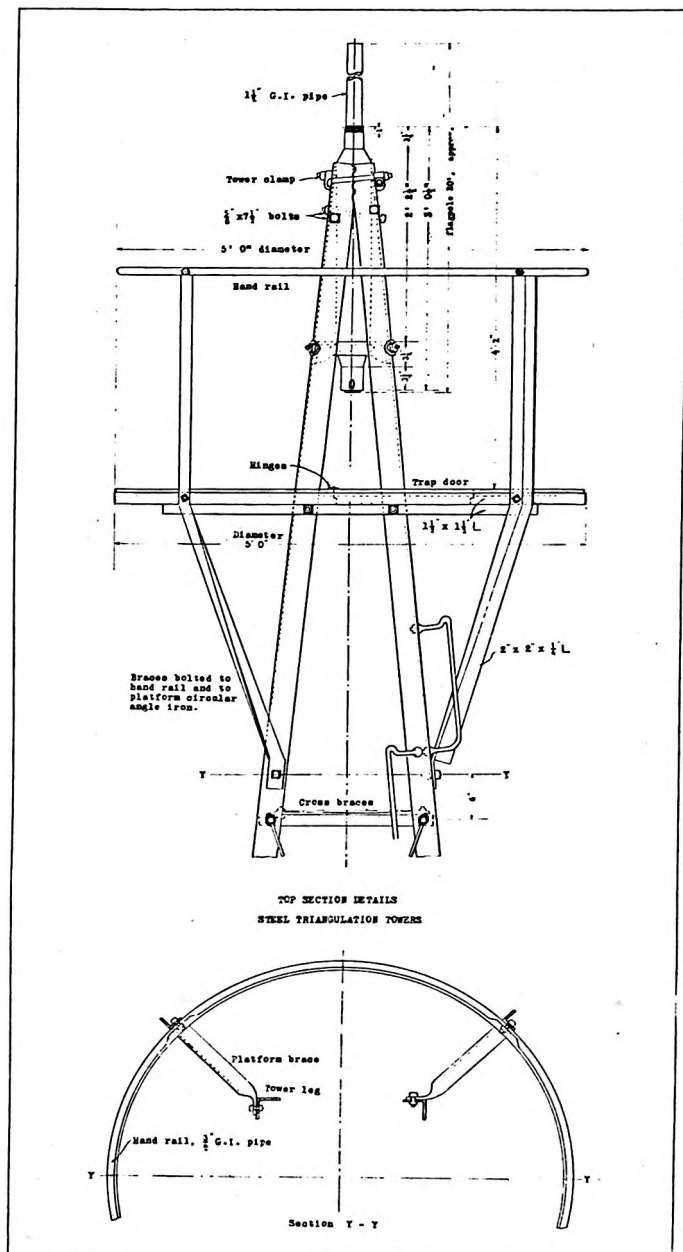


FIGURE 14.—Triangulation tower, details.

inward, an 80-foot tower may be constructed. A 60-foot tower is obtained by omitting two sections, and a 40-foot tower by omitting three sections. All the parts of a section, and the sections also, are joined by bolts. The separate pieces are transported to the tower site by boat, truck, pack animals, or manpower, according to the necessities of the case, and are there assembled section by section.

As concrete requires a day to set, or possibly 2 days for the more important and permanent structures, at least 2 days should be allowed for erecting each tower. When access to the site is not difficult, materials and gear may be landed the first day, the site cleared, and the foundations prepared, up to the point where the foot-plates, joined by the bottom chords, are anchored by bolts embedded in wet concrete. (See Fig. 15.) Also, if there is no danger of theft or loss, the remaining parts of the tower may be arranged in systematic order for erecting. On the next day the tower is erected, the center form is placed, the dressings are lashed on, and the flagpole is secured in the pipe at the head of the tower, in the raised or in the lowered position, as may be required.

Gin poles of spruce or other light timber 30 feet long are commonly used for hoisting the parts of the tower. This is regarded as a safer practice than that of passing pieces upward from hand to hand, by men ranged along the corner posts of erected sections. It requires fewer men, but more time, than the passing method. On the side of both safety and speed is the employment, in the tower erection party, of a nucleus of experienced tower men carried over from a preceding season.

MATERIALS AND TOOLS FOR FOUNDATION WORK

Materials:

- 16 94-pound bags of cement.
- 3,000 pounds of sand (may be available at the site).
- 100 gallons of water (may be available at the site).
- 6 1-inch by 12-inch by 16-foot rough boards.
- 2 1-inch by 4-inch by 16-foot rough boards.
- 16 3-inch by 3-inch by 16-foot scantlings, for piling, if necessary.

The boards for forms are best cut in advance and transported in bundles.

Only a little of the piling should be cut and sharpened in advance, for tests made at the site may show that longer pieces are needed than has been anticipated.

The interior dimensions of the concrete forms for the footings of 100-foot and 80-foot towers should be 40 by 40 by 12 inches. The corresponding weight of foundations is about 3.2 tons, or 1,600 pounds at each leg. The weight of a 100-foot steel tower is about 3.8 tons. Footings for smaller towers may be somewhat reduced in dimensions.

Tools and accessories:

- | | |
|---|--|
| 1 mixing board for concrete, 5 by 5 feet. | 2 machinist's hammers, 2-pound. |
| 1 tape, 50-foot, steel or "metallic." | 1 hacksaw and spare blades. |
| 1 level or transit, with tripod. | 1 ratchet brace. |
| 1 leveling rod. | 1 wood bit, $\frac{1}{2}$ -inch. |
| 1 carpenter's level, 24-inch. | 1 wood bit, $\frac{1}{4}$ -inch. |
| 2 sledge hammers, 10-pound. | 1 steel square, 18-inch. |
| 4 machetes. | 6 galvanized buckets. |
| 3 axes. | 1 cold chisel, $\frac{1}{2}$ -inch. |
| 2 clawhammers. | 1 cape chisel, $\frac{1}{2}$ -inch. |
| 1 crosscut saw. | 2 5-gallon cans. |
| 2 shovels, D-handle. | 2 spud (open-end) wrenches, $\frac{1}{2}$ -inch. |
| 2 shovels, long handles. | 1 monkey wrench, 10-inch. |
| 2 picks. | 1 Stillson wrench, 10-inch. |
| 15 pounds nails, 5 pounds each of 8-penny, 10-penny, and 20-penny. | |
| 1 tower base, consisting of 4 girts, 2 cross braces, 4 footplates with their bolts, and 16 machine bolts $\frac{1}{2}$ inch by 10 inches, each with nut and 2 cut washers—for setting the foundation. | |

For erecting the tower (considering that the foundation and construction parties may be distinct):

All tower parts, as per the list given later.

1 steel or "metallic" tape, 50-foot.

1 level or transit, with tripod.

1 leveling rod.

2 machetes.

1 ax.

2 cold chisels, $\frac{3}{4}$ -inch.

2 machinist's hammers, 2-pound.

1 hacksaw and spare blades.

1 ratchet drill, with $\frac{1}{8}$ -inch, $\frac{3}{8}$ -inch, and $\frac{1}{2}$ -inch drills.

1 brass tailblock to fit 3-inch diameter gin pole.

1 quart red lead in oil.

1 paint brush.

40 fathoms 21-thread manila line.

1 set of tower dressings.

1 flagpole and flag.

1 ratchet brace.

1 wood bit, $\frac{1}{4}$ -inch.

1 monkey wrench, 10-inch.

2 pounds $\frac{1}{4}$ -inch cut washers.

1 Stillson wrench, 10-inch.

8 spud wrenches, $\frac{1}{4}$ -inch.

8 spud wrenches, $\frac{3}{8}$ -inch.

4 spud wrenches, $\frac{1}{2}$ -inch.

1 can machine oil, corked

1 tool chest with padlock, for keeping smaller tools together.

The foregoing applies to the hand-to-hand passing method. When the gin-pole method is used, four gin poles will be required, each of 3-inch diameter and 30 feet long, each with a tail block and 40 fathoms of $2\frac{1}{2}$ -inch manila line and a $\frac{1}{4}$ -inch hook.

When most of the tower sites are accessible by boat, a regularly constituted "tower party" organized for both foundation and erection work, and provided with a tender, can save time and fuel by taking aboard several towers at a time, and all equipment permanently, for trips, lasting possibly a week, anchoring near each tower until it is completed.

It is advisable to maintain records, on the tender and on the ship, of towers on hand, spare parts on hand and spare parts needed, and equipment of all kinds, including camping requisites. A "check off" list or lists will be found convenient in loading boats, to the end that no needed item shall be left behind.

ORDER OF LOADING AND UNLOADING BOATS

To avoid leaving on the ship parts that will be needed in the beginning, and landing parts that will not be needed for some time, the material is handled in a certain sequence. In the following list, not only the sections but also the items in each section, are named in the order of loading them into boats so that they may be unloaded in the order needed. Materials and tools for foundation work are not included.

Boards for working platforms will be needed on soft muddy shores.

Six-thread manila is often used for skirt fastenings instead of wire.

Seventh section (last section), not painted

Number	Description	Mark	Approximate length
1	Flag, 12 by 12 feet		
8	Skirts, black canvas, including 1 for railing		
1	Pipe, $1\frac{1}{2}$ inch diameter, galvanized, 20 feet long. Drill holes at both ends to allow $\frac{1}{4}$ -inch eyebolts to fit loosely.		
1	Pipe ring, $\frac{1}{4}$ -inch diameter, railing		5 feet diameter.
1	Platform with trap door		5 feet diameter.
1	Pipe, 4-inch, and base		3 feet.
2	Cross braces, $\frac{1}{4}$ -inch	#1	2 feet 2 inches.
8	Diagonal braces, $\frac{1}{4}$ -inch	T2841	7 feet 1 inch.
4	Angle girts	T2821	1 foot 8 inches.
4	do	T2822	3 feet 0 inches.
3	Corner posts	T3400	13 feet 8 inches.
1	Corner post, ladder	T3401	Do.

SURVEY SIGNALS

Sixth section, painted yellow and green

Number	Description	Mark	Approximate length
4	Cross braces, $\frac{1}{2}$ -inch	#2, #3	3 feet 11 inches, 5 feet 9 inches.
8	Diagonal braces, $\frac{1}{2}$ -inch	T2842	7 feet 8 inches.
8	Diagonal braces, $\frac{1}{2}$ -inch	T2843	8 feet 4 inches.
4	Angle girts	T3423	4 feet 4 inches.
4	do	T3424	5 feet 8 inches.
3	Corner posts	T3402	13 feet 8 inches.
1	Corner post	T3403	Do.

Fifth section, painted red and white

4	Cross braces, $\frac{1}{2}$ -inch	#4, #5	7 feet 8 inches, 9 feet 6 inches.
8	Diagonal braces, $\frac{1}{2}$ -inch	T2844	9 feet 2 inches.
8	Diagonal braces, $\frac{1}{2}$ -inch	T2845	10 feet 1 inch.
4	Angle girts	T3425	7 feet 0 inches.
4	do	T3426	8 feet 4 inches.
3	Corner posts	T3402	13 feet 8 inches.
1	Corner post	T3403	Do.

Fourth section, painted green

4	Cross braces, $\frac{1}{2}$ -inch	#6, #7	11 feet 4 inches, 13 feet 3 inches.
8	Diagonal braces, $\frac{1}{2}$ -inch	T2846	11 feet 1 inch.
8	Diagonal braces, $\frac{1}{2}$ -inch	T2847	12 feet 2 inches.
4	Angle girts	T3427	9 feet 8 inches.
4	do	T3428	10 feet 11 inches.
3	Corner posts	T3402	13 feet 8 inches.
1	Corner post	T3403	Do.

Third section, painted yellow

4	Cross braces, $\frac{1}{2}$ -inch	#8, #9	15 feet 1 inch, 17 feet 0 inches.
4	Angle braces	T3461	8 feet 9 inches.
4	do	T3460	Do.
4	do	T3459	9 feet 3 inches.
4	do	T3458	Do.
4	Angle girts	T3429	12 feet 3 inches.
4	do	T3430	13 feet 7 inches.
3	Corner posts	T3402	13 feet 8 inches.
1	Corner post	T3403	Do.

Second section, painted red

6	Safety belts		
4	Straps, manila, to be spliced on board		
4	Blocks, single sheave. 6-inch		
4	Hoisting lines, 2 $\frac{1}{2}$ -inch		40 fathoms.
4	Cross braces	#10, #11	18 feet 10 inches, 20 feet 10 inches.
4	Angle braces	T3469	9 feet 8 inches.
4	do	T3468	Do.
4	do	T3467	10 feet 3 inches.
4	do	T3466	Do.
4	Angle girts	T3431	14 feet 11 inches.
3	Corner posts	T3404	13 feet 8 inches.
1	Corner post	T3405	Do.

First section, painted white

Number	Description	Mark	Approximate length
4	Cross braces, $\frac{1}{2}$ -inch	#12, #13..	23 feet, 28 feet.
4	Angle braces, Tw	Ex. 115..	13 feet 4 inches.
4	Angle braces	Ex. 114..	Do.
4	Angle girts	Ex. 112..	16 feet 3 inches.
3	Corner posts	Ex. 102..	9 feet 8 inches.
1	Corner post	Ex. 103..	Do.
4	Angle braces	Ex. 117..	13 feet 4 inches.
4	do	Ex. 116..	Do.
4	Angle girts	Ex. 113..	18 feet 2 inches.
4	Angle girts	Ex. 113 $\frac{1}{2}$..	20 feet 1 inch.
3	Corner posts	Ex. 104..	10 feet 4 inches.
1	Corner post	Ex. 105..	Do.
4	Braces for platform		
1	Clamp, $\frac{1}{2}$ -inch diameter, for tying in tower; in box for bolts.		
80	Ladder rungs, 40 left, 40 right; in box for bolts		
4	Footplates		
16	Anchor bolts		$\frac{3}{8}$ x 8 inches.
1	Box for bolts		
1	Tool chest		

The total weight of all sections of the 100-foot tower is approximately 3.5 tons, the heaviest single steel part weighing about 100 pounds.

PROCEDURE FOR SELECTING SITE AND ERECTING TOWER

The senior hydrographic engineer should choose the definite site of each triangulation tower, and mark it with a small flag or stake, having in mind not only technical requirements but conditions facilitating the later work of landing and construction parties. He should also mark the best landing place, if the tower officer has not previously visited the site.

Arrived at the tower-site anchorage, the usual procedure is as follows:

First are landed men, tools, and materials for clearing the tower site, and for excavating and laying the foundations. These operations go on simultaneously. The concrete forms are assembled and put into approximate position by eye, transit, and tape. A rubber band around a square rod about 6 feet long will serve for establishing points on the same level, for example, the bottoms of the excavations and later the footplates.

The bottom horizontal girts (c) are bolted to the footplates (a), using washers (g) in number to correspond to the thickness of the tower leg. The girts thus bolted to the footplates will serve as a base template for the anchor bolts (b). Using some definite point of a footplate as a datum for levels, the rod is carried to each footplate, and the concrete forms (e) are set by trial and error at such depths as will bring the four footplates into the same horizontal plane. Before the final securing of the forms the squareness of the tower base must be insured by applying the test of equal diagonal distances between the footplates.

The tops of the forms are kept level with a carpenter's square. As shown in Figure 15, the piece (d), 1 inch thick, serves to suspend the anchor bolts (b) while the concrete is being poured, and also supports the weight of the footplates and girts. The nuts (f) are secured to the bolts by the first two threads, or only enough to carry their weight. With the concrete filling the forms to the under sides of the distance pieces (d), and the footplates level, the level of the concrete in all forms should be the same. Ease in the subsequent erection of the tower is proportionate to the care exercised in getting the foundations truly level and the base exactly square.

ERECTION OF THE TOWER ON CONCRETE FOUNDATIONS

The next day the distance pieces (d) are removed and the foot-plates (a) are let down to rest on the concrete. At this point check levels should be taken on the footplates to guard against unequal shrinking of the concrete or inequalities of its surface under the footplates. The differences in level should be well under a quarter of an inch. If they are not, the high foundations may be chiseled out easily under the footplates, until by trial the levels are satisfied. The footplates may now be firmly bolted to the footings, with a washer under each nut. Next the corner

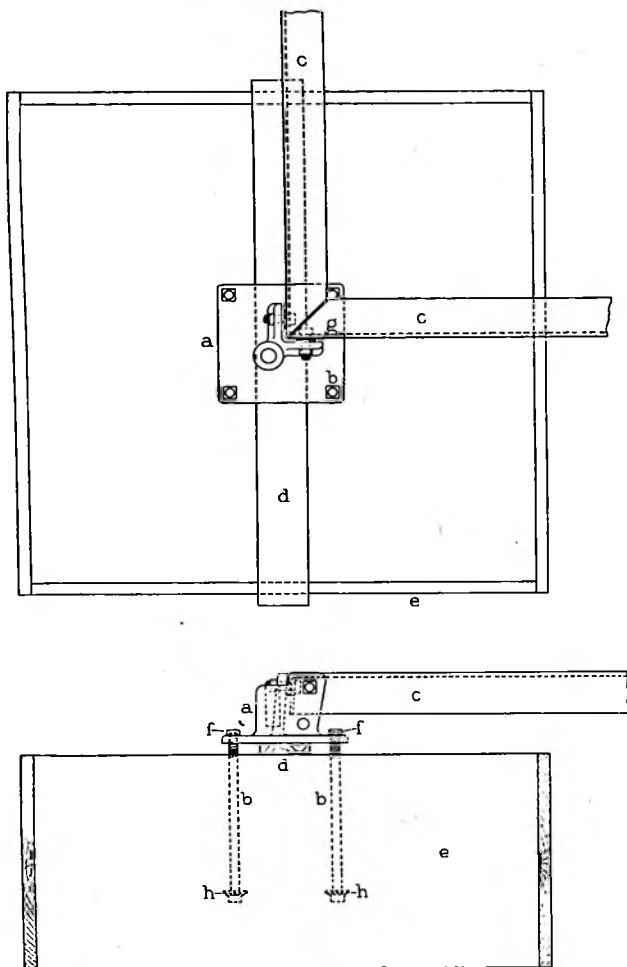


FIGURE 13.—Tower base and footing, ready for pouring concrete.

a, Footplate; b, anchor bolts; c, lower chord of base, or bottom girt; d, distance piece, removed after setting of concrete; e, form for concrete; f, nuts, now on two threads, washers removed; g, spacing washers, the thickness of the tower leg; h, washers to hold the anchor bolts.

pieces, or legs, are bolted to the footplates, and then the girts and finally the diagonals are bolted on.

Each section is built up in the order:

1. Corner pieces, or legs.
2. Girt angles, or chords (horizontal).
3. Diagonals (angle irons in lower sections, tie rods above).
4. Cross braces, or tie rods (horizontal).

In the lower part of the tower, where the diagonals are angle bars, no particular checking of the sides of sections is necessary. Higher up, however, where the diagonals are rods with eye ends and fitted with turnbuckles, it is necessary to check each section when completed, else the succeeding sections will become more and more out of line, and it will be extremely difficult to get the headpiece on. If when a stage is assembled the turnbuckles on the cross

braces are so adjusted that the distance between the north and south legs is exactly equal to that between the east and west legs, the tower cross section at that height is square.

Turnbuckles must be used with care, especially in erecting old towers with buckled tie rods.

The tower erecting party should put in the center monument also—a task for which the small triangulation party is necessarily ill-prepared. The exact center, however, cannot be located at this time, for it requires some time for the tower parts to come to their final bearings under varying winds and temperatures, and there may be some slight settling of the foundations. The triangulation party, then, should be the one to make the measurements for accurately establishing the center mark and referencing it to nearby permanent objects, for sketches to be incorporated in the "Description of Station" cards.

When the standard Navy station center is to be used, it is a good plan for the construction party to fill the center mark form to within 6 inches of the top, with small reinforcing rods extending upward and not too near the center. The top of the central boiler tube should be about 10 inches below the top of the form. The triangulation party can then complete the center monument with a little additional concrete, and at the same time establish the station center accurately in fresh concrete.

OTHER FOUNDATIONS FOR TOWERS

On hard or rocky ground, inland towers intended to be taken down soon are often provided with simple concrete pads for footings. These, in the main, have proved to be satisfactory.

On marshy ground the use of large wooden platforms, or mattresses, is always unsatisfactory for observing purposes, and endangers the safety of the tower. Large mattresses are more objectionable than small ones, if the surface dries, because of warping. When the soil is underlain by a stratum of soft material, mattresses merely cause the structure to float, and tying them down by means of timber piles at the corners does not prevent quaking. To obtain proper support, the piles must be capped with concrete, if the site is on land. In several feet of water the container of the concrete may be an empty barrel or drum, or a form made of galvanized sheet iron.

In making foundations on marshy soil, or under water in mud, coral, or sand, it is best to ascertain at the outset the nature of materials at various depths by sinking a long pipe. If the material is soft for a sufficient depth, the foundation may be made of 4- by 4-inch timber piling pushed down vertically, braced against flexure by piping driven on a slant and wired to the piles, the whole topped by a concrete cap. But if gravel, sand, or hardpan is encountered only a little way below the surface, it will then become necessary to ascertain whether the crust is thin or thick. If it is thin, long pipe piling can probably be forced through it. If it is thick, the constructor will be confronted with the dilemma of having to drive all piles to hardpan or else all of them considerably short of that depth, lest the sinking of some of the piles cause the tower to topple. In most cases the problem may be solved by using a large number of short piles. In doubtful cases borings should be taken at each footing.

Towers erected on foundations laid on coral under several feet of water are subject to unfavorable conditions similar to those which may cause a dam to fail because of the permeation of subsurface water. On the other hand, if the water is deep enough and if the bottom is soft mud, there is a good chance of stability, if the base of the tower is sufficiently weighted with iron.

The following instances partially illustrate the difficulties and the principles involved in simple foundation work without a pile driver:

1. A tower erected on small concrete foundations on boiler-tube piling, in a foot of water on mud and a thin crust of hardpan was found standing 5 years after erection, but was taken down because of rusted, weak, and missing parts. A new tower was erected on the same spot, on mattress foundations. This was blown down in a fairly high wind before the end of the season, and a third tower had to be erected.

2. A tower erected on mattresses on a small sandy beach, the only beach on a large marshy island, was wrecked within a few weeks. The failure was probably due to soft material beneath the sandy crust. The tower, even when new, was unsteady for observing angles.

3. A tower built on a coral shoal in about 3 feet of water, on concrete and boiler tube foundations, near the center of the shoal and but little subject to wave action, was steady for observing angles, and served for a beacon for many years. A year after erection, the legs, attacked by salt spray, showed considerable pitting, and were given a cement coating. Later occupations found the tower still steady.

4. Several beacons consisting of a mast in a cubic yard of concrete were used to mark coral heads in about a fathom of water. They served admirably as hydrographic signals, but badly as permanent beacons, for a year later they were found to have been undermined by wave action and rolled away from the original sites. Their location was in a sound, far from any barrier reef, and the wave action, except during occasional storms, must have been only moderate.

5. A 100-foot tower was erected at the end of a sandspit in about a fathom of water and not far from the edge of a barrier reef. The bottom was hard, and piling could not be driven. Good metal forms were made for the four footings, and 15 tons of concrete were poured into each. Some years later the tower was found to be on the point of failure because of the undermining of the foundations by wave action.

6. A tower erected on small concrete foundations on a barrier reef in about 4 feet of water proved to be too unsteady to occupy. As the sights were short, it was decided to occupy the center mark, which consisted of a barrel of cement on top of three others, the whole bound together and fastened down by driven pipes. This support, though steadier than the tower, was still unsatisfactory. The next year found the tower down and the center mark partly undermined, whereupon a concrete monument 11 feet high and about 4 feet square at the base was built at the center. Strangely, this proved to be less steady for observing purposes than either of the two previous structures. Apparently the pounding of the seas caused the whole reef to quake. As possibly bearing on this point, the mainland, a few miles distant, is noted for its large caves extending to near the sea coast.

7. Thirty- and forty-foot towers planted in 1 to 3 fathoms of water on soft mud and well weighted have sometimes proved to be good triangulation supports in dead calms. One such tower, however, in 1 fathom of water, on soft mud, showed a periodic motion of the top whenever there was a slight swell, though unaccompanied by wind. From indications elsewhere in the region, it is believed that the bottom consisted of a thin layer of sand over about 3 feet of ooze, the latter resting on hardpan or old coral.

8. Sixty- to eighty-foot towers in 5 to 8 fathoms of water on soft mud bottom, and well weighted at their bases, proved to be serviceable for triangulation in calm weather. They were used in considerable numbers during one season (see Fig. 12), and with good success, in conjunction with a line of towers on shore fronting a tropical forest. Their employment saved a vast amount of heavy clearing.

9. A similar chain of offshore towers, 60-foot, in 5 fathoms of water on hard sand bottom, proved to be unstable for observing, and by later observations were found to have changed their locations by inching themselves along the bottom.

The advantages obtainable by placing a tower in the water offshore, even a tower too unsteady to be occupied successfully, are not lightly to be foregone. Its position may be the key to the solution of the troublesome problem of getting around a cape. Again, in the middle of a body of water too wide to see across, it may permit the use of a polygon instead of a chain of encircling figures.

FORTY-FOOT TRIPODS

In 1926 the U. S. S. *Niagara* developed a tripod of great visibility, of materials that could be landed through the surf, with a simple framework of 4- by 4-inch timbers 40 feet long, not too heavy to be set up by manpower without the aid of a gin pole and tackle, and utilizing lumber purchasable in the region, with a minimum of waste. Painting and cutting were done on board the ship. Pieces of similar dimensions were cleated together, rolled overboard, towed to near the line of breakers, and cut loose and allowed to drift to shore, allowance having been made for current. The officer-in-charge and 15 men, perforce, had to make the best of their way through the surf in a whaleboat, with tools and the lighter pieces of lumber. After casting off the whaleboat, the motor launch, with its full crew, moved farther offshore and anchored, using line rather than chain, for line would give warning before breaking, whereas chain would not. It was found that borrowing any of the crew to help in erecting the signal resulted in danger to the launch, especially when there was a great range of tide.

The design of the tripod shown in Figure 16 is modified from that of the *Niagara* in the direction of greater strength, by using three built-up beams instead of the three bottom boards (a 4- by 6-inch piece could be used) and by adding short reinforcing members to the legs. Without reinforcement, the 4- by 4-inch legs are not stiff enough to bear wind pressure, and may even fail during the process of erection. It is important to select timbers without knots. The flagpole must have the right dimensions to support the weight of the legs at one stage of the erection. See (a), Figure 17.

In point of visibility there is little to choose between 12-inch sidings, as in Figure 16, and 10-inch sidings with 2-inch spaces between. The narrower boards may even increase the effective strength, for the spaces act as wind vents.

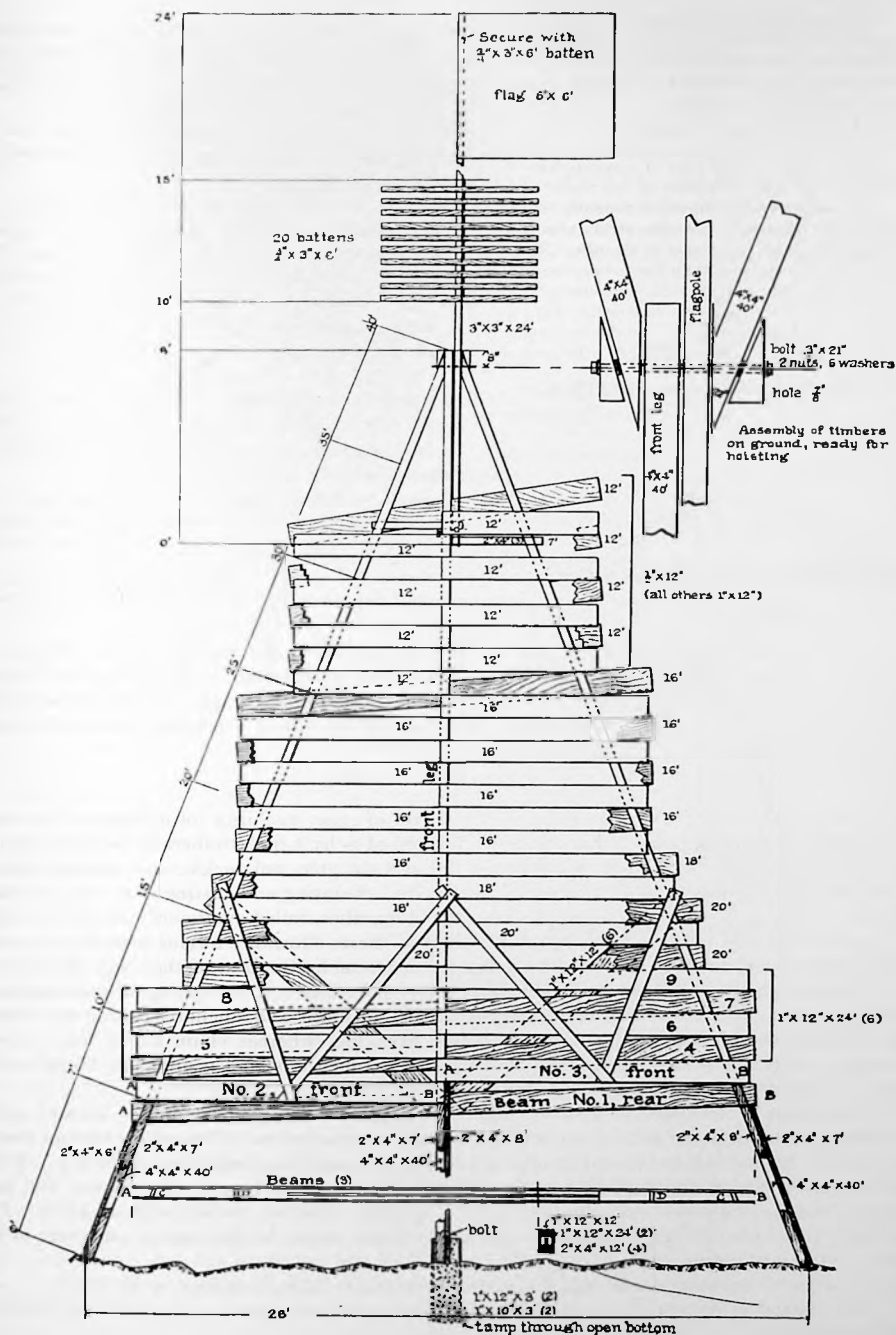


FIGURE 16.—Forty-foot tripod.

When visibility at great distances is not a prime consideration, the sidings longer than 18 feet may be omitted. However, it will still be desirable to reinforce the legs at about three-fourths the distance down from the top, using scantling or piping. In some cases most of the sidings on the landward side of the tripod may be omitted, substituting a few chords of 2- by 4-inch or 2- by 6-inch material.

When the footing is mud or loose sand it is prudent to spike short pieces of scantling to the lower ends of the legs, one inside and one outside, to act as shoes.

The top of the tripod is usually occupied with a 10-second sextant for cuts to objects not visible from the ground, where a transit is used. Sometimes a transit is taken to the top, not for cuts, but to pick up indistinct distant objects, and thus to furnish rough finder angles,

which are afterward refined by using the sextant. For this purpose the transit may be mounted on its base board attached to a bracket or small platform at the head of the tripod.

If the station is a triangulation station, it should have an accurately centered center mark, as good or better than that shown. If it is only an important intersection station, a well-driven boiler tube will suffice to mark the station. *In all cases the center mark should be referenced.*

The timbers are assembled as shown in Figure 16, with nuts near the ends of the 21-inch bolt, in order that the legs may have the necessary play during the erection of the tripod. On sloping ground there is a lifting advantage in laying the front leg directly up the slope; and, other things being equal, in leading the legs toward trees that may serve for stays for guy wires, as legs and guys should extend nearly in the same directions.

In (a), Figure 17, the tripod has been raised by a concentration of manpower near the center and working outward, only two men being reserved to prevent the rear legs from slipping, by using long-handled shovels or crowbars or loose turns of line about driven boiler tubes. Between (a) and (b) the manpower is concentrated on the front leg only and the flagpole. At (c) the tripping line is made fast, the legs are shifted to their

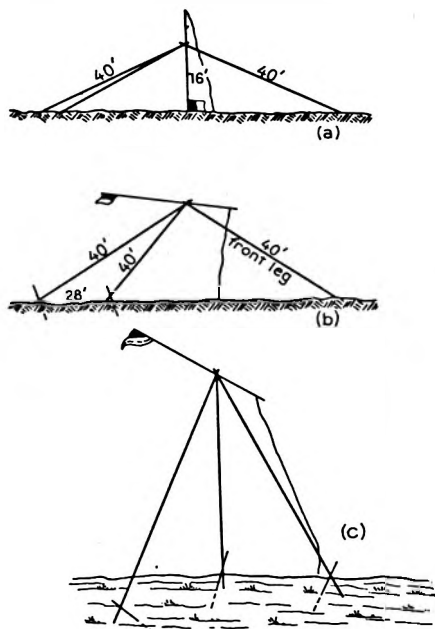


FIGURE 17.—Steps in the erection of a tripod.

final positions and set into shallow holes, and the bottoms are secured by wire and boiler tubes.

The bottom beams are now nailed in place, and also the blocks C bearing against the insides of the legs. Next, with men standing on the ends of the beams and others thrusting outward on the legs at about the 15-foot level, the boards at this level and their corresponding braces are nailed in place. Next, the remaining boards, up to this level, are nailed on, in order from the bottom, as each is secured, the braces passing over its outer face are nailed to it and the nails are clinched.

The siding is continued to near the top. The flagpole is then inclined downward to a convenient position and the battens on the beacon head are nailed on. The flagpole is brought to the vertical by hauling down on the tripping line, the nuts at the head of the tripod are set up, and the lower end of the flagpole is secured to the legs by short lengths of scantling. A platform is added, if desired, and the siding is completed. The remaining peak of the tripod is outlined, for a triangulation target, by wrapping black muslin tightly around it, or by fitting to it a standard 6-foot tripod skirt. The guy wires are attached, led out, and tightened, if single, by turnbuckles near the top, if double by Spanish windlasses near the bottom.

The tripod is decorated for sounding by means of cloth, paint, or whitewash. Paint is most effective for the beacon head and the 12-foot sidings, and should be applied to one side

of each board a day or two ahead of the time of its intended use. Cloth being relatively expensive (cheesecloth and bunting are worthless), whitewash is generally preferred below the 25-foot level, as showing against the land. In rare cases, as when a tripod stands on a bare islet, a blackwash may be used, made of lampblack and linseed or fish oil, a pound to the gallon. It dries slowly, and so should be applied in place, with a pump or spray.

The bill of materials, less extras mentioned, is as follows:

LUMBER

Number	Inches	Feet	Use	Number	Inches	Feet	Use
3	4 by 4	40	Legs, no knots.	4	1 by 12	20	Sidings.
1	3 by 3	24	Flagpole.	3	1 by 12	18	Do.
12	2 by 4	12	Spacers in beams.	12	1 by 12	16	Do.
1	2 by 4	8	Reinforcements and fastenings.	6	1 by 12	12	Diagonal braces.
6	2 by 4	7	Do.	2	1 by 12	3	Concrete form.
1	2 by 4	6	Do.	2	1 by 10	3	Do.
3	1 by 12	12	Webs in beams.	21	$\frac{1}{2}$ by 3	6	Battens, painted.
12	1 by 12	24	Sidings.	12	$\frac{1}{2}$ by 12	12	Sidings, painted.

OTHER MATERIALS

- 1..... Flag, 6 feet by 6 feet, hemmed.
- 1..... Bolt, $\frac{1}{2}$ inch by 21 inches, each end threaded for 6 inches; with 2 nuts and 6 washers.
- 1..... Bolt, 12 inches or longer, for center mark.
- 1..... Skirt, standard, 6-foot, black canvas (light weight), edged with twine and fitted with grommets and lashings.
- 50 pounds..... Cement.
- 10 pounds..... Spikes, 40-penny.
- 20 pounds..... Nails, 20-penny.
- 15 pounds..... Nails, 12-penny.
- 50 fathoms..... Light telephone wire, for guys.
- 3..... Turnbuckles, for tightening guys.
- 6..... Boiler tubes, condemned, for stays.
- Spun yarn and seizing wire, for lashings.
- Paint for battens and 12-foot sidings.
- Lime, slaked, for whitewash.
- Lampblack and oil, 1 pound to a gallon, for blackwash.

Tools and accessories will vary with the locality. Not all of the following will be needed in all cases:

Number	Articles	Remarks
42 fm.	Manila line, 18-thread.....	For tripping and steadying lines and miscellaneous use.
2.....	Saws, crosscut.....	Hand.
4.....	Hammers, claw.....	At least 3 needed simultaneously.
1.....	Brace, ratchet.....	If holes have not been bored previously.
1.....	Bit, $\frac{1}{2}$ inch, or auger.....	Do.
1.....	Wrench, $\frac{1}{2}$ inch.....	Open-end or monkey.
2.....	Pliers, side-cutting.....	
2.....	Axes (one is a spare).....	Suitable for felling trees.
2.....	Machetes.....	For clearing brush and killing snakes.
1.....	Pick.....	For digging holes for feet of tripod.
2.....	Shovels, long-handled.....	For digging; for holding feet of tripod from slipping.
2.....	Sledges, 10-pound.....	For driving boiler tubes; one is a spare.
1.....	Cold chisel.....	For cutting crosses or footholes.
4.....	Individual tool bags.....	For tools and nails.
2.....	Pails, galvanized iron.....	For use with concrete and whitewash.
1.....	Transit and tripod.....	For centering; possibly for taking cuts.
1.....	Binoculars or long glass.....	For reading wigwag signals between shore and boat.
1.....	Spraying outfit.....	For spraying whitewash or blackwash.
1.....	First-aid kit.....	
.....	Emergency rations.....	Boat may be unable to return through the surf until the tide changes.

THIRTY-FOOT QUADRUPODS

Light quadrupods, of which two types are shown in Figures 18 and 19, are usually preferred to the heavy 40-foot tripod of Figure 16, whenever access or construction work would be difficult. The first is made of 4- by 4-inch posts with light bracing, the second of the upper part of a steel tower, with light timbers and cloth added to widen the top. The tower head may be occupied with a transit. Center marks are added whenever possible.



FIGURE 18.—Thirty-foot wooden quadrupod.



FIGURE 19.—Thirty-foot steel quadrupod, converted tower.

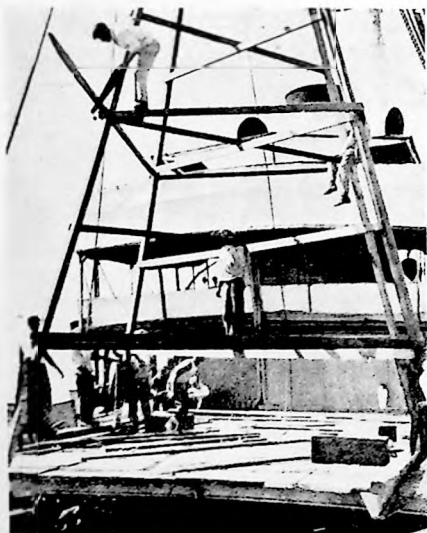


FIGURE 20.—Building water tower on deck.

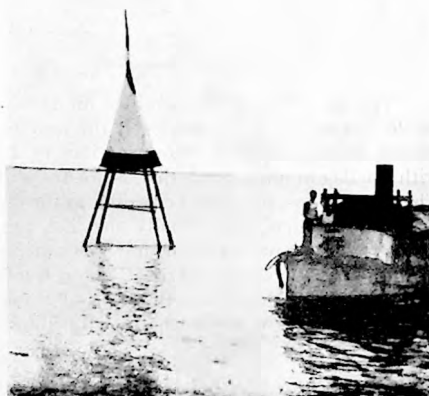


FIGURE 21.—Water tower in 3 fathoms of water.

WATER TOWERS

The dimensions of the 38-foot water tower, commonly called a 40-foot quadrupod because the legs are of that length, are given in Figure 22.

These water towers are intended to be occupied with a 10-second sextant. They cannot be occupied successfully with a transit even in the calmest weather at sea. Difficult sextant angles, between objects 6 or more miles distant, may be obtained by employing a board support for the sextant, 6 inches square or larger, nailed to the top of the tower alongside the foot of the flagpole. In windy weather it may be necessary to furl the flag.

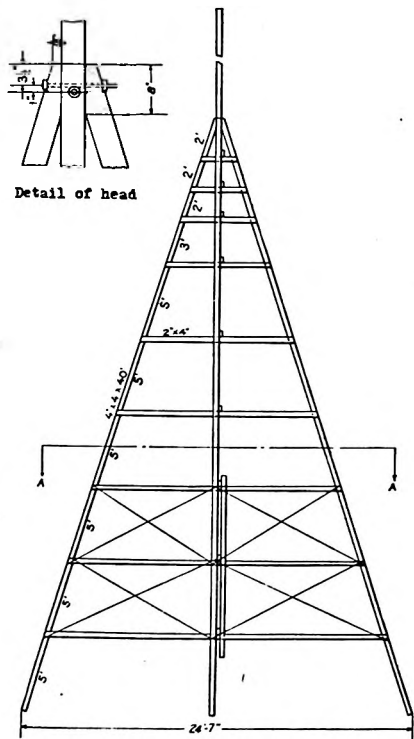


FIGURE 22.—Water tower

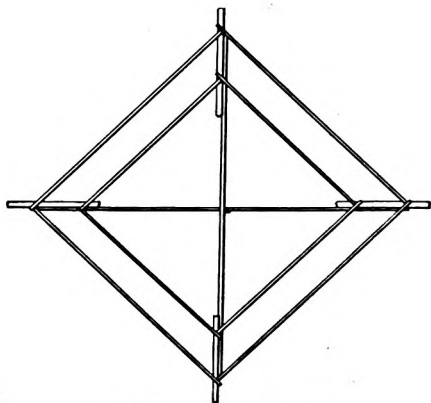


FIGURE 23.—Section A-A of figure 22.

The water tower is assembled on deck. After stiffening the legs with iron pipes 2½ inches by 20 feet lashed in the angles at the junctions of legs and braces, and after weighting the lower section of each leg with short sections of I-beams or railroad iron, wired on, the whole tower, with the flag in place but furled, is hoisted with the boom and lowered over the side into the water. The decorations, made up according to the standard pattern, are added by the crew of a follow-up boat. The skirts, and usually the flag, are nearly always black, as the signal shows against the sky. The tower is stable in 18 feet of water, as a rule, and has been used in 22 feet of water. In sounding, the signal "carries" about 6 miles; that is, it may be seen that far, with the naked eye in clear weather, from the deck of a sounding launch. The flag (if black), but not usually the peak of the tower, can be seen 14 miles with a transit mounted on an 80-foot tower at sea level.

SURVEY SIGNALS

Bill of materials for 38-foot water tower

LUMBER

Number	Size	Use	Number	Size	Use
2	2 by 4 inches by 21 feet, 7 inches.	Cross brace.	2	2 by 4 inches by 4 feet, 9½ inches.	Cross brace.
2	2 by 4 inches by 18 feet, 7 inches.	Do.	2	2 by 4 inches by 3 feet, 7 inches.	Do.
2	2 by 4 inches by 15 feet, 7 inches.	Do.	2	2 by 4 inches by 2 feet, 3½ inches.	Do.
2	2 by 4 inches by 12 feet, 7 inches.	Do.	1	2 by 4 inches by 12 feet.	Vertical brace.
2	2 by 4 inches by 9 feet, 7 inches.	Do.	4	2 by 4 inches by 15 feet, 6 inches.	Girt.
2	2 by 4 inches by 6 feet, 7 inches.	Do.	4	2 by 4 inches by 11 feet, 6 inches.	Do.
			4	4 by 4 inches by 40 feet.	Legs.
			1	4 by 4 inches by 12 feet.	Mast.

HARDWARE AND OTHER MATERIALS

- 2 ½-inch by 10-inch galvanized-iron bolts, each with 2 washers.
- 500 feet of telegraph wire for horizontal and vertical diagonals.
- 4 2½-inch by 20-foot galvanized-iron pipes for leg stiffeners.
- 4 125-pound weights, I-beams, railroad rails, or grate bars.
- 40-penny and 20-penny spikes and nails.
- 1 Standard skirt, light canvas, black usually, fitted with grommets and reeving line.
- 1 Flag, width about half the height, to avoid fouling the mast, fitted with grommets and hoisting line.

THIRTY-FOOT TRIPODS

Thirty-foot tripods may be used with advantage where trees would not permit lower shore signals to be cut in from towers or transit points. Five men can erect one, on land, but several more men are required to erect one from a boat in 3 to 8 feet of water. As in the case of other shore signals, they show better when they are placed some distance offshore—a method which also saves landing materials and wading in the mud. In the lee of a shore or shoal they may be erected from a cargo boat. Suitable materials are 3- by 3-inch or 2- by 4-inch scantlings for legs, 6-inch boards or 2- by 4-inch scantlings for braces, and white cloth for decorations. Guys are sometimes necessary. If the mud is very soft, footplates made of short sections of 2- by 4-inch may be required to prevent the overturning of the signal by the sinking of one of the legs.

If 2- by 4-inch material is used for the whole length of leg, it will need stiffening below the middle part. For the top, at least, it is convenient to use 2- by 4-inch material, in order to avoid beveling. With this size at the top, the three legs and the flagpole may be united by a bolt 10 or 12 inches long, permitting play of the parts while erecting the signal. The flag or beacon head, which is the target for a signal of this size, may be erected by a downhaul on the foot of the pole, which extends 3 or 4 feet below the head of the tripod. Collapsible beacons have been used with success.

These signals are not economical for a height of target much less than 35 feet. They may be occupied with a sextant.

MAST SIGNALS

The mast signals developed by the U. S. S. *Hannibal* are economical in time and labor, and are effective for a height of about 20 feet. They cannot be occupied. They are much used for mainland signals where the control is offshore. One of them can be erected in about half an hour by two or three men. They may be dismantled quickly, without damage, and used repeatedly.

The mast signal consists of a 3- by 3-inch by 30-foot mast, three-wire guys 120° apart, and three white triangular skirts. One edge of each skirt is lashed, by a line running through



FIGURE 21.—Triangulation tripod.

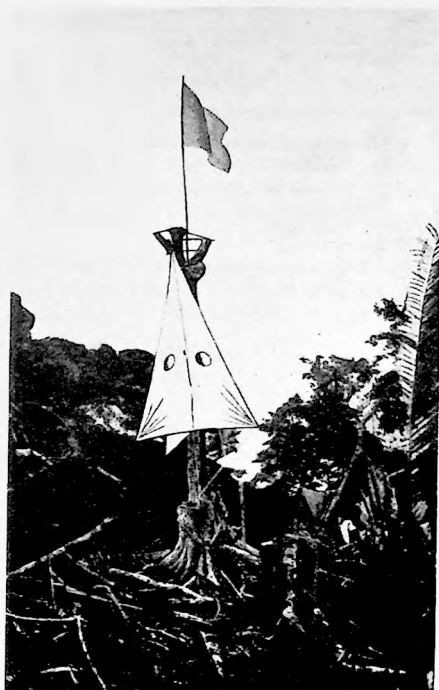


FIGURE 25.—Tree signal.

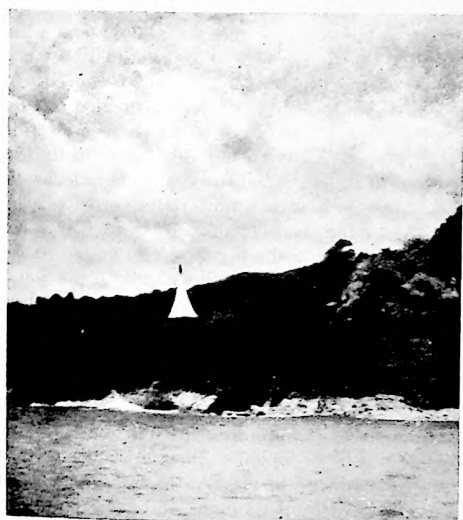


FIGURE 26.—Mast signal.

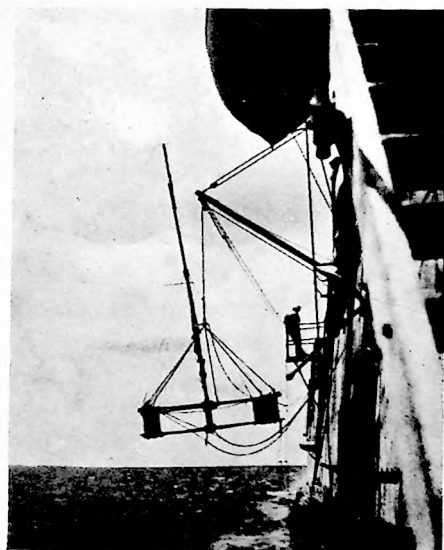


FIGURE 27.—Floating signal hoisted out, ready to lower away

eyelets, to the mast; and the outboard lower corners are made fast to the guys, which likewise run through eyelets in the upper edges of the skirts. A hole in the ground, or a boiler tube, provides footing for the mast. If the latter is used, it remains as a center mark when the signal is removed. Figure 26 shows the usual type.

FLAG SIGNALS

Ordinarily, flags in trees are expensive in time and labor, considering their small range, the danger of their fouling, and their worthlessness during the calmest weather. They are most useful in surveying a labyrinth of small islands. In such cases there is good reason for the use of many designs and colors, to assist in the identification of islands seen from distant towers. Occupation is difficult.

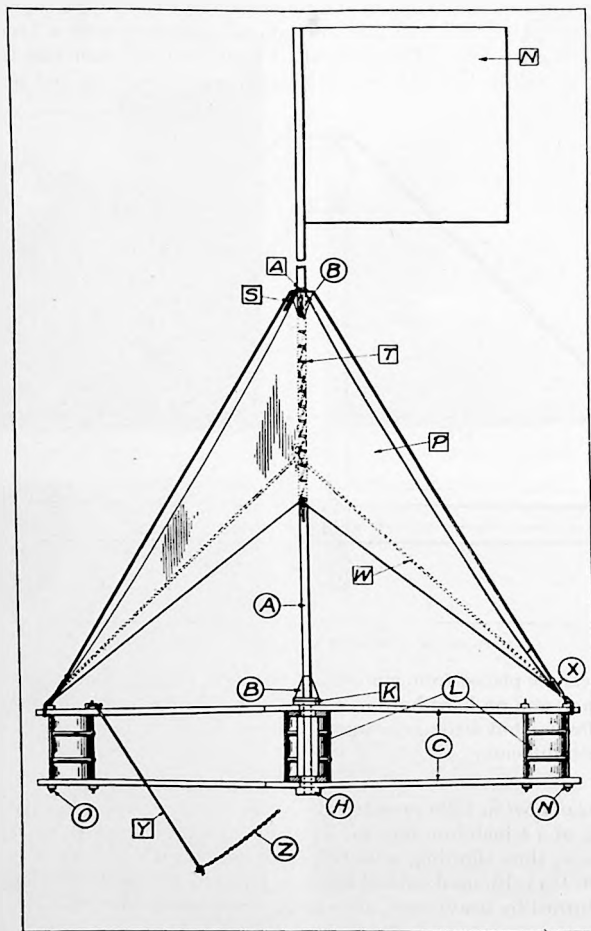


FIGURE 28.—Four-drum floating signal, elevation.

The description of the four-drum floating signal, Figures 27 to 33, is begun on page 53.

BEACON HEADS

In areas marked by many islands with shoals extending from their points, one of the most convenient signals is the beacon head, which consists of a 2- by 4-inch scantling about 12 feet

long, to the sides and edges of which are nailed battens for exposing two areas of cloth at right angles, each about 8 by 10 feet. (See Figs. 16 and 18.) The beacon head may be made up before arriving at the station or enroute between stations. In wading-water it may be lashed to a boiler tube and guyed. In deeper water, if the bottom is soft, it may be lashed to two pipes, which are pushed down 3 or 4 feet vertically, 2 to 4 feet apart, and then drawn together with a hand line. The range of visibility is about 4 miles. The station can be occupied before the beacon head target is put on, with a sextant and sometimes with a transit. Stations of this kind are useful in the vicinity of channels, and have the merit of considerable permanency.

BEACONS

In water up to 10 feet a signal intended for a beacon as well as for a sounding signal is sometimes constructed by using three iron pipes or railroad rails joined by a bolt at the top. It is seldom advisable to add a flag or a beacon head. For decorations, each side from the peak down may be covered with half-inch boards or slats interwoven with wire, and painted.

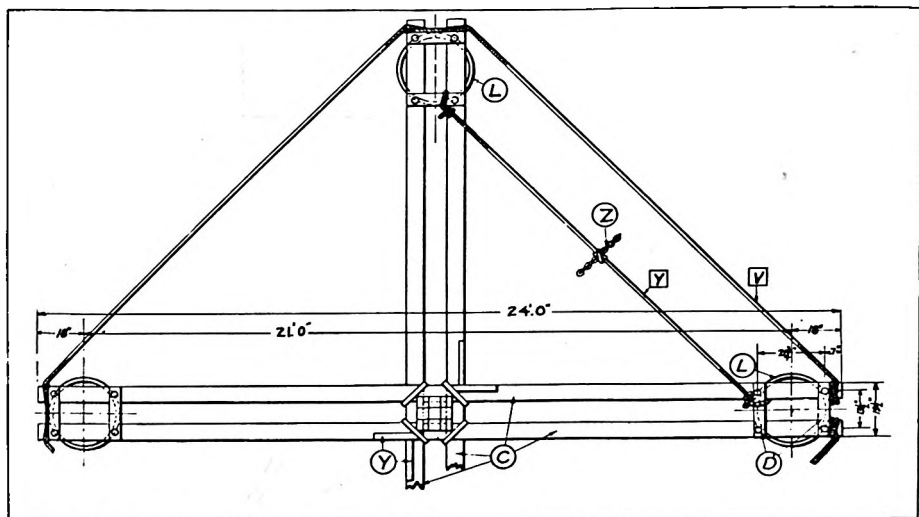


FIGURE 29.—Half-plan of floating signal. (See pp. 53, 54.)

These signals can be placed from a launch in somewhat deeper water than would be possible with a wooden signal, and on hard bottom as well as on soft bottom. They will last 6 years or longer, the legs offering but little resistance to wave action. They may be occupied with a sextant, though with difficulty.

REEF SIGNALS

The U. S. S. *Hannibal* in 1929 reported the satisfactory use of a reef signal in the form of a pyramid, consisting of a 4-inch iron pipe set 3 feet into a concrete pyramidal base and projecting 2 feet above the base, thus affording a footing for a 3-inch by 3-inch by 30-foot flagpole. The U. S. S. *Paducah*, in 1914-16, used cubical bases, about 3 feet on a side, with less success. Some of them were overturned by heavy seas, after being undermined by wave action.

MOUNTAIN SIGNALS

The erection of mountain signals presents difficulties in the transportation of materials, food, and water. Usually the party must be small. Whenever possible, all the work should be performed in one trip, the triangulation observer accompanying the erecting party to make sure of the intervisibility of stations and to observe. If the peak is bare, so that it is known

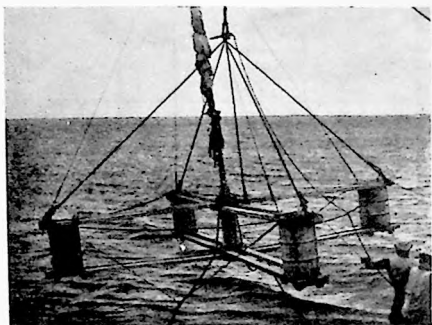


FIGURE 30.—Floating signal ready to be anchored.



FIGURE 31.—Anchored and dressed

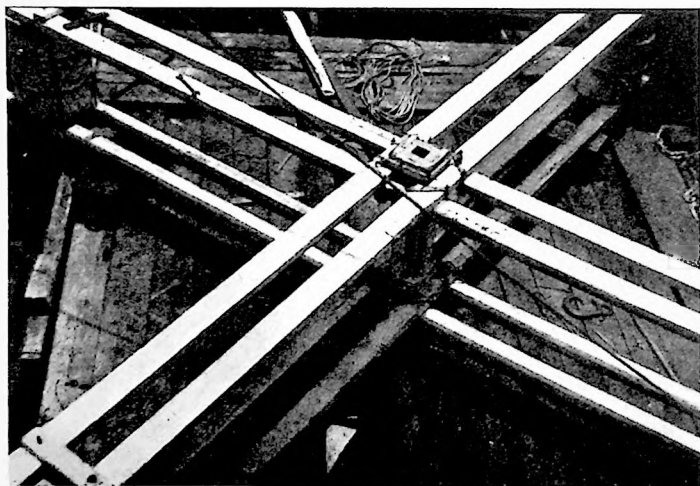


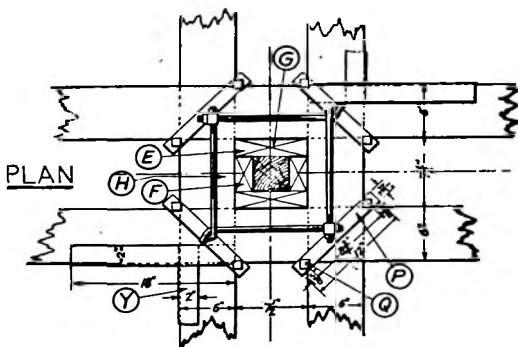
FIGURE 32.—View of framework

in advance that materials will have to be transported to fashion a symmetrical signal of size sufficient to carry the greatest distance required, subtending, say, 10 seconds of arc or more in both width and height, the best materials for the purpose are 1-inch iron piping in short lengths, threaded at both ends, black canvas with grommets for making permanent seizings, and seizing wire for fastenings and guys.

INLAND SIGNALS IN WOODED COUNTRY

When there is reasonable assurance that signal material may be found near at hand, only tools, fastenings, and dressings will be required. In the triangulation of the north coast of Cuba (U. S. S. *Nokomis*) many of the tripods on inland hills were made from trees felled at the spot. Saplings were used for braces, and sheeting for dressings.

In the royal palm belt, or in other situations where there is a large tree overtopping all others, a secondary signal, suitable for occupation, may often be made by lopping off the branches and improvising an observing platform, as illustrated in Figure 25. The dressings here used are skirts of the standard mast signal. They are strung on wire guys. But tree signals of this kind, however well braced, are not satisfactory for main triangulation purposes.



FOUR-DRUM FLOATING SIGNALS

The four-drum floating signal perfected by the U. S. S. *Hannibal* in 1915 is of substantial construction, stows in a small space, can be assembled or dismantled on deck, can be planted or picked up with the ship's boom, or may be towed by a launch, and is available for use offshore in depths up to 17 fathoms or more. Its range of visibility is about 10 miles. It is not designed for occupation, even with a sextant (Figs. 27-33).

In deep water it is customary to anchor it with three anchors 120° apart, and in shoal water with two anchors 180° apart.

Figure 27 shows the standard floating signal suspended from the ship's boom by a sling fitted with a tripping line. The anchor chains are suspended in loops along the ship's side. To drop any loop while the ship is under slow way or drifting, it is only necessary to cut a lashing at the top of the festoon. After the floater is anchored, the flag and skirts are unfurled, and the skirts are spread upon the guy wires leading to the drums (Figs. 30, 31).

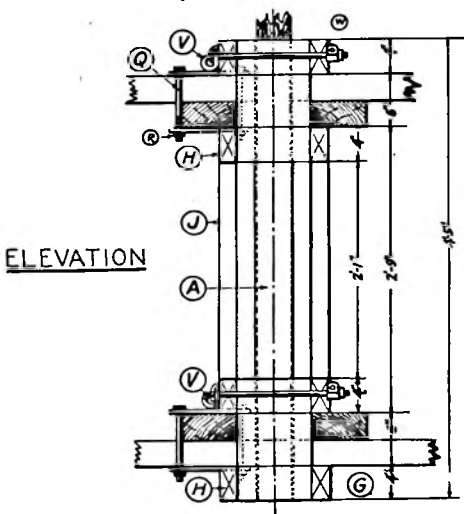


FIGURE 33.—Details of step for mast.

Bill of material—Four-drum floating signal

Item	Number	Size	Specifications
LUMBER			
(A)	1	3½ by 3½ inches by 36 feet	Yellow pine, S4S mast (Figs. 28, 29, and 33).
(B)	12	3½ by 3½ by 12 inches	Spruce, S4S mast cleafs (wedge shape).
(C)	8	3 by 6 inches by 24 feet	Spruce, rough. Frame timbers.
(D)	16	2 by 4 by 19½ inches	Spruce, rough. Frame ties.
(E)	2	2 by 7½ by 57 inches	Spruce, rough. Tabernacle sides.
(F)	2	2 by 3½ by 57 inches	Do.
(G)	8	2 by 4 by 7½ inches	Spruce, rough. Tabernacle collars.
(H)	8	2 by 4 by 11½ inches	Do.
(J)	4	2 by 4 by 25 inches	Spruce, rough. Tabernacle collar spacers.
HARDWARE			
(L)	4	50-gallon, 23 by 36 inches	Gasoline drums, galvanized, U. S. Navy standard, floats.
(M)	16	½ inch by 48 inches	Eyebolts, shouldered galvanized iron.
(N)	16	½ inch	Washers, galvanized, for ½-inch bolts.
(O)	16	½ inch	Nuts, square, galvanized, for ½-inch bolts.
(P)	16	½ by 1½ by 9½ inches	Flat galvanized iron, drilled as shown.
(Q)	16	½ inch by 10 inches	Bolts, square head, galvanized, threaded for 2 inches.
(R)	16	½ inch	Nuts, square, galvanized, for ½-inch bolts.
(V)	4	½ inch by 28 inches	Round bars, galvanized, threaded both ends and bent.
(W)	4	1½ by 1½ by 1½ inches	Blocks, cast iron, galvanized.
(X)	4	½ inch by 12 inches	Turnbuckles, galvanized, right- and left-hand threads, hook and eye.
(Y)	4	Angle, bent to 2 by 3 by 18 inches.	Plate steel, 20 gage, U. S. Standard.
(Z)	3	½ inch by 30 fathoms	Chain, galvanized iron, open link.
(A)	4 lengths		Rope, ⅝-inch 6-strand galvanized iron or steel, stays.
(B)	4 fathoms		Rope, ⅝-inch 6-strand galvanized iron or steel, buoy ropes.
(C)	4	⅝ inch	Thimbles, galvanized oval, for wire rope.
(D)	1	3 feet long	Telegraph wire lashing.
(E)	⅛ pound	4-penny	Nails, galvanized iron, roofing.
(F)	½ pound	10-penny	Nails, galvanized iron, barbed.
(G)	3½ pounds	20-penny	Do.
(H)	4	30-penny	Do.
(I)	3	¾ inch by 21 inches	Bars, iron, square.
(K)	3	½ inch	Screw shackles, galvanized iron, for anchor chain.
(L)	3	200 pounds	Anchors, Cape Ann type.
CANVAS, CORDAGE, ETC.			
(N)	1	12 feet by 12 feet	Flag, muslin.
(O)	1 yard		Cotton sheet, heavy, for parceling.
(P)	35 yards	No. 8, 72 inches wide	Cotton canvas for sail.
(Q)	32 pairs	No. 4	Spur grommets and washers.
(R)	5 pounds		Marline.
(S)	8 lengths	9-thread	Manila rope, 1 fathom each, head lashings.
(T)	2 fathoms	9-thread	Manila rope, 1 fathom each, sail lacing.
(U)	8 lengths	15-thread	Manila rope, 4 fathoms each, foot lashings.
(V)	13 fathoms	2½-inch	Manila rope, grab rope.
(W)	18 fathoms	3½-inch	Manila rope, slings.
(X)	30 fathoms	2½-inch	Manila rope, buoy ropes.
(Y)	6 fathoms	2-inch	Manila rope, sling, for anchor chain.
(Z)	2	6 by 6 by 36 inches	Spruce, upper buoy. Paint white, bore 1-inch hole 3 inches from end.
(U)	2	3 by 3 by 18 inches	Spruce, lower buoy. Paint white.

CHAPTER IV

SOUNDING

PLANNING

The division of work among sounding units is usually about as follows:

Between 1 and 10 to 20 fathoms, launches.

Between 10 and 100 fathoms, tenders (subchasers, gunboats, tugs, etc.).

Beyond 100 fathoms, the ship.

This allotment will naturally be modified by circumstances. Subject to considerations affecting safety of navigation, vessels of the second and third classes may sound with advantage any open areas of sufficient depth, leaving to the launches all detailed work such as inshore sounding, sounding around islands and shoals, and particularly the close development of channels, both by sounding lines and by dragging.

All vessels are equipped with port and starboard sounding chains; with bridges for observers, recorders, and lookouts; with plotting boards or tables; with hand lead lines; and in some cases



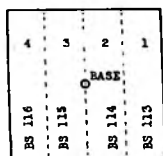
FIGURE 34—Launch engaged in sounding.

with voice tubes from bridge to plotting board. Each tender is also equipped with a hand or electric sounding machine for taking vertical wire soundings, and with echo sounding apparatus, as well. The ship carries one or two electrically driven sounding machines, with booms and sheaves for wire and pressure tube soundings; deep sea sounding machines (when necessary); and echo sounding equipment.

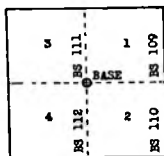
Sounding launches usually are 30-to 50-foot launches of the Navy, varying in type according to the available supply, and converted into sounding vessels by the addition of 7½-inch steering compasses, sounding chains, canopies or housings, observers' platforms, drawing boards, and so on. The hull should be sturdy for towing purposes and to resist damage in grounding. Towing bitts are needed, for in practice a considerable part of the work of sounding launches consists in moving lighters, towing floating signals, and dragging channels. The power should be ample and under instant control. This is necessary when working near the edges of reefs, when holding the launch stationary while taking deep soundings, when keeping a constant tension on a wire drag, and in other circumstances. Quiet action of the engines is very desirable, also, to avoid interference with the oral transmission of observations and orders. Except in river work, the amount of which is usually negligible, flat bottoms, shallow drafts, and absence of well formed keels are disadvantageous in steering good courses.

The personnel usually consists of the officer-in-charge, the coxswain, four leadsmen, two sextant observers, a recorder, a plotter, and an engineer. On occasion all these men have to sleep on the launch, and accordingly provision should be made for cots or bunks. On long trips it is often necessary to carry extra fuel and water; and when sounding parties are used to erect hydrographic signals there is need of space for carrying lumber, gravel, and cement. The 36- and 40-foot launches satisfy most of the foregoing requirements fairly well.

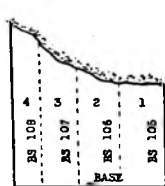
The division of work into sounding areas.—According to practice, the unit sounding area is a quadrangle of 10 minutes of latitude and longitude. Usually two units are sounded simultaneously, four launches dividing one unit as indicated by the cuts, while the ship and tenders sound adjoining units farther seaward. Under good weather conditions the launches can sound



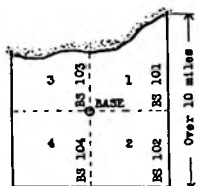
(a)



(b)



(c)



(d)

FIGURE 35.—Subdivision of sounding areas, numbering of boat sheets.

a unit area per week, including moving to the area from an adjoining area, erecting and locating hydrographic signals, and sounding. This supposed rate implies only a general hydrographic development.

Plan (a) has the advantage of sounding lines 10 miles long, with few end turns as compared with plan (b). Long lines tend to give a more even development than short lines. For parties 1 and 4, there is some waste of time in plan (a), which is less in plan (b), in getting to the place of work in the morning and in returning to the base in the evening. Plan (b), which admits of a quick return to the base, or afterward to work, is often employed in seasons when only a certain part of each day may be expected to be calm, or in seasons when sudden squalls may occur.

Fractional areas occur adjacent to the mainland or to lines of islands, as illustrated in (c) and (d).

When foul bottom and reefs are encountered, it has sometimes been found advisable for the launches to operate in pairs, in order that one may assist the other in case of accident.

DAYMARKS AND FIELD RECORD NUMBERS

At the beginning of each day on which sounding is to be done, the draftsman assigns to all sounding vessels a day letter, or "daymark", the same letter for all, chosen chronologically from the series of 24 letters, *A* to *Z*, omitting *I* and *O* lest they be mistaken for numerals. The ship uses simple letters, and the various sounding parties use letters with appropriate subscripts.

When the first series is completed, a new series called the *A* series is begun. It is formed by prefixing *A* to the successive letters of the alphabet. This, in turn, is followed by a *B* series, then by a *C* series, and so on, as illustrated in the accompanying Daily Record.

Boat sheets have to be numbered as issued. Reserving the numbers 1 to 99 for a later numbering of smooth sheets and miscellaneous records, it is suggested that boat sheets be numbered chronologically and in the series 101, 102, etc. This will avoid renumbering them in the Archives as subnumbers under a blanket number for all records of the season. Field books may be numbered, at any convenient time, in the series 201, 202, etc., and sounding books in the series 301, 302, etc., to the end, which will often be beyond 400. The best time to do this, and to enter the numbers both in the Record and on the boat sheets, is at the completion of each boat sheet.

The Daily Record is purely a field record and its form is not important. It may be expanded easily into a Surveying Log, by supplying the missing dates in the first column, adding columns for other kinds of work, adding more extensive remarks, and using opposite pages of a book. It will be found useful in compiling reports and statistics.

Daily record of daymarks and boat sheets

Date 19—	Ship		No. 1 Party		No. 2 Party		No. 3 Party		No. 4 Party		Remarks
	Day	B. S.	Day	B. S.	Day	B. S.	Day	B. S.	Day	B. S.	
January											
7			A ₁	101	A ₂	102	A ₃	103			Boats began Area ①.
8			B ₁		B ₂		B ₃		B ₄	104	
9			C ₁		C ₂		C ₃		C ₄		
10			D ₁		D ₂		D ₃		D ₄		
11			E ₁		E ₂		E ₃		E ₄		
12			F ₁		F ₂		F ₃		F ₄		ML #1 under repair. No. 1 Party in ML #5.
14					G ₃		G ₃		G ₄		
15			H ₁		H ₂		H ₃		H ₄		
16	J	105	J ₁		J ₂		J ₃		J ₄		
17	K		K ₁		K ₂		K ₃		K ₄		
18	L		L ₁	102	L ₂	104	L ₃	101	L ₄	103	Check lines. Moving to ③. Signaling ③. Cutting in signals. Boats began ②, ship in ①.
19	M										
21	N										
22	P										
23	Q		Q ₁	106	Q ₂	107	Q ₃	108	Q ₄	109	
24	R		R ₁		R ₂		R ₃		R ₄		No. 3 Party in ML #5.
25			S ₁		S ₂		S ₃		S ₄		
26			T ₁		T ₂		T ₃		T ₄		
28			U ₁		U ₂		U ₃		U ₄		
29	V		V ₁		V ₂		V ₃		V ₄		
30	W		W ₁		W ₂		W ₃		W ₄		Ship completed ①.
31	X		X ₁		X ₂		X ₃		X ₄		
February											
1	Y	109	Y ₁		Y ₂		Y ₃		Y ₄		Ship in ②.
2	Z		Z ₁		Z ₂		Z ₃		Z ₄		
4	AA		AA ₁		AA ₂		AA ₃		AA ₄		Check lines. Signaling ③. Cutting in signals. Boats began ②. Ship completed ③.
5	AB		AB ₁	107	AB ₂	108	AB ₃	109	AB ₄	106	
6	AC										
7	AD										
8	AE		AE ₁	110	AE ₂	111	AE ₃	112	AE ₄	113	
9	AF		AF ₁		AF ₂		AF ₃		AF ₄		No. 4 helping No. 2 Check lines. Boats began ④.
11			AG ₁		AG ₂		AG ₃		AG ₄		
12			AH ₁		AH ₂		AH ₃		AH ₄		
13			AJ ₁		AJ ₂		AJ ₃		AJ ₄		
14			AK ₁		AK ₂		AK ₃		AK ₄		
15			AL ₁		AL ₂		AL ₃		AL ₄		Ship completed ③.
16			AM ₁		AM ₂		AM ₃		AM ₄		
18			AN ₁		AN ₂		AN ₃		AN ₄		
19			AP ₁		AP ₂		AP ₃		AP ₄	111	
20			AQ ₁	112	AQ ₂	113	AQ ₃	110	AQ ₄	111	
25			AR ₁	114	AR ₂	115	AR ₃	116	AR ₄	117	Ship completed ④.
26	AS	109	AS ₁		AS ₂		AS ₃		AS ₄		
27	AT		AT ₁		AT ₂		AT ₃		AT ₄		
28			AU ₁		AU ₂		AU ₃		AU ₄		
March											
1			AV ₁		AV ₂		AV ₃		AV ₄		Check lines. No. 3 and No. 4 dragging.
2			AW ₁		AW ₂		AW ₃		AW ₄		
4			AX ₁	117	AX ₂	116	AX ₃	115	AX ₄	114	
5			AY ₁	120	AY ₂	121	AY ₃	118	AY ₄	119	
6			AZ ₁		AZ ₂		AZ ₃		AZ ₄		
7	BA	122	BA ₁		BA ₂		BA ₃		BA ₄		Ship, sonic lines.
8	BB		BB ₁		BB ₂		BB ₃		BB ₄		

Daymarks are written on smooth sheets as well as on the rough work sheets, called boat sheets. For this purpose black ink is avoided, and on the smooth sheets yellow and orange ink are avoided because they photograph black. To further guard against mistaking daymarks for soundings or other features, they are written small, considerably off the line of soundings to which they refer, and in a direction perpendicular to that of all figures, names, and notes on the sheet.

To avoid confusion, daymarks are assigned from day to day by some one person.

MASTER SHEET AND BOAT SHEETS

It will be found convenient to make a projection, on the scale proposed for sounding, on a large sheet limited principally by the dimensions of the drawing board, to cover several areas like (a), for plotting triangulation stations by latitude and longitude and all other stations from them as the necessary angles come in from day to day. From such a master sheet, sectional boat sheets may be prepared quickly by pricking through stations and intersections of parallels and meridians, ringing the station points with red ink, drawing the meridians and parallels, adding the titles by means of rubber stamps, and drawing pencil guide lines proposed to be run by the sounding boats.

Master sheets are convenient also for laying out work, partitioning areas, showing special features, and receiving daily installments of shore line, either laid down roughly from tangents observed at towers, or transferred from shore-line sheets. In this way, all information essential for an intelligent prosecution of the work is collected on a single sheet in a way to be available at a moment's notice.

To facilitate transferring from sheet to sheet, either directly or by pantograph, it is well to adopt a uniform rule of inking in the meridians and parallels having **even-numbered minutes**, 2 minutes apart for ordinary scales and 4 or 8 minutes apart for small scales. Sounding, reconnaissance, topographic, bridge, and triangulation sheets are thus easily compiled from a variety of other sheets of the same season or of several seasons. The transfer of shoal soundings and fathom lines to drag sheets from sheets of a previous season is a familiar example.

At the conclusion of the smooth plotting, master sheets that have been used only to transfer stations to other sheets should be destroyed, as they are an incumbrance to the files. But if they show extra soundings, shore line, or topography, they should be folded into two parts if they are large, given new titles appropriate to the added matter, and submitted with the smooth sheets, with numbers following those of the smooth sheets.

SMOOTH SHEETS

Smooth sheets should be designated by simple numbers 1, 2, 3, etc., without the addition of letters A, B, etc. The use of letters should be avoided on all sheets, smooth or rough. If it is desired to retain the correspondence of sheets and field areas (which is of no assistance in office compilation), the field area numbers should be subordinated, thus: Sheet 4 (Areas 4, 6, 7 in part).

TRAINING SOUNDING PARTIES

The following method of individual and class instruction has been found to be interesting and effective.

For instruction in reading sextant angles, in which nothing less than perfection is desired, construct a large wooden arc and an index arm held by light friction, with limb and vernier divisions painted broad in enlarged imitation of the sextant arc and vernier proposed to be used. Taking the observers of all parties in a single class, drill them particularly, and all hands incidentally, in reading angles, also in setting off angles. When this has become familiar, each officer may concentrate on his own observers with the actual sextants assigned to them, first in reading set angles, then in reflecting large plain objects and reading their own angles. With facility gained in manipulating the sextant, again assemble all observers for a competitive drill, including the recorders, one of whom should be detailed to give the signals, by whistle or otherwise, for simultaneous angles by all observers.

Collect the observers in one spot from which two distant fixed signals, *L* on the left and *R* on the right, can be seen plainly; and for a central signal *C*, moving in order to vary the angles, use an upright flagpole on a launch maneuvering perhaps half a mile away, on lines limited by the lines of sight to *L* and *R*, which should be at least 60° and preferably 180° or more apart. The instructor should measure the sum angle *L* to *R* from a position at the left of the group, and again from a position at the right, to ascertain the small difference between the two values.

At the word "mark", with the launch moving, each pair of observers measures *LC* and *CR*, the angles in pairs are recorded and added by the recorders, and the results are announced. Within the small difference ascertained by the officer, they should agree. Their failure to agree will show up slow observers, mistakes in reading verniers, mistakes in adding angles, systematic errors in judging the coincidence of images, index errors, and even defects in sextants, such as curved mirrors. Variety may be obtained by changing sextants, positions, and objects.

Illustration of practice notes.

No. 1 Party	Time	Position no.	Objects and angles Left—Center—Right				Sum
Personnel			Tom—Launch—Bee				
	h m		°	'	°	'	° '
In charge.....	8 00	[1	65	32	81	10	146 42
Recorder.....	01	[2	60	00	86	42	146 42
Left angle.....	02	[3	55	30	91	13	146 43
Right angle.....	03	[4	50	02	96	40	146 42

Though seamen in the Navy are familiar with the use of lead lines, compasses, and launch engines, they will need additional training for surveying. For example, the requirement that the lead line shall be "up and down" at a certain "marked" instant is far different from the usual one that the leadsman shall begin, when directed, to wind up for heaving the lead. The reading of feet instead of fractional fathoms is new. The height of chains is less than on the ship, and the rolling and pitching of the vessel is beyond previous experience. The engineer must practice the art of maintaining a constant speed on the same line and on adjacent lines, whether the vessel is heading into the wind or running before it. The coxswain will be expected to hold his course very closely, and in some cases to keep on a certain range despite cross winds and currents.

LEAD LINES

Too much stress cannot be laid on the necessity of careful marking and checking of lead lines. To mark them accurately for their intended use, and to keep them so, requires constant vigilance on the part of boat officers. Boat officers should personally inspect their lead lines while laid out under tension across the deck marks, every morning, before accepting them for the day's work.

The amount of tension required depends on the material of the line, its condition (it should always be wet), and the amount of usage it has had. Before sounding in channels, or in critical depths, or in connection with drag work, the proper amount of tension should be found by actual measurement in water by comparison with wire lines for a depth as great or somewhat greater than the critical depth. These extreme precautions, applying to common cotton line, are unnecessary in the case of tiller rope line, such as is now generally used, having a phosphor bronze stranded wire core and a hard braided cotton covering, well waterproofed. It has been found that the proper amount of tension for a new line of this kind is just enough to straighten out the bights. After use it will stretch a little, and accordingly the original marks may have to be moved. The following is a good system of **marking the fathoms**:

Fathoms	Markings
1, 11, 21....	1 strip of leather, $\frac{1}{4}$ by $3\frac{1}{2}$ inches.
2, 12, 22....	2 strips of leather, $\frac{1}{4}$ by $3\frac{1}{2}$ inches.
3, 13, 23....	3 strips of leather, $\frac{1}{4}$ by $3\frac{1}{2}$ inches.
4, 14, 24....	Blue bunting, cut $3\frac{1}{2}$ by 6 inches.
5, 15, 25....	White bunting, cut $3\frac{1}{2}$ by 6 inches.
6, 16.....	Yellow bunting, cut $3\frac{1}{2}$ by 6 inches.
7, 17.....	Red bunting, cut $3\frac{1}{2}$ by 6 inches.
8, 18.....	Green bunting, cut $3\frac{1}{2}$ by 6 inches.
9, 19.....	White and blue bunting, 1 piece of each, cut $3\frac{1}{2}$ by 6 inches.
10.....	Strip of leather, $\frac{1}{4}$ by $3\frac{1}{2}$ inches, with one $\frac{1}{8}$ -inch hole.
20.....	Strip of leather, $\frac{1}{4}$ by $3\frac{1}{2}$ inches, with two $\frac{1}{8}$ -inch holes.

Up to 7 fathoms **foot marks** are attached, as follows:

At one foot: Leather, one strip, $\frac{1}{4}$ by 2 inches.

At two feet: Leather, two strips, $\frac{1}{4}$ by 2 inches.

At three feet: Leather, three strips, $\frac{1}{4}$ by 2 inches.

At four feet: Blue bunting, 2 by 2 inches, folded to $\frac{1}{4}$ -inch width.

At five feet: White bunting, 2 by 2 inches, folded to $\frac{1}{4}$ -inch width.

These foot marks are smaller than the fathom marks, and are secured to the line at their mid-point; the fathom marks are secured at one end.

From 7 to 12 fathoms **half-fathom marks** are usually the only intermediates. They consist of three strips of leather $\frac{1}{4}$ by 2 inches.

The bunting used to mark fathoms is folded to a size of $\frac{1}{2}$ by 6 inches, and one end is turned down an inch for the lashing. The colors for 9 and 19 fathoms combine those for 4 and 5 fathoms.

For **securing the marks to the line** flax thread from stranded halyard stuff is used. With a number 16 sail needle pass the thread under one strand of the braid and then draw tight a clove hitch around the line, whipping on the marks with opposite turns of single thread, drawing tight each turn. Cross-lash the leather marks, but not the bunting marks. Care must be taken not to pierce the braid, for the line, once cut, will soon break. Toggles may be lashed on, or secured with a clove hitch of the line if allowance for the length of the hitch has been made.

At the lead end of the line is a $\frac{1}{4}$ -inch galvanized thimble for attaching a 14-pound lead with a $\frac{1}{4}$ -inch shackle. The lead should be shackled on before the markings are set.

ARMING MIXTURE FOR LEADS

After much experimentation the following arming mixture was developed on the U. S. S. *Nokomis* by Chief Quartermaster S. W. Ramsey. It was found to adhere well to the lead and to retain bottom specimens better than either soap or tallow.

Formula for one gallon of the mixture:

2½ pounds, or 2 quarts, of tallow.

4 pounds of white salt-water soap, about 1½ bars.

3 ounces of **medicated** absorbent cotton.

NOTE.—Ordinary cotton will discolor the mixture.

Grate the soap and mix it with the tallow. Boil thoroughly. Tear the cotton into fine shreds, and add it gradually to the boiling mixture, stirring the mixture meanwhile. Remove the mixture from the fire, and stir it briskly until it has cooled.

DUTIES OF THE RECORDER

The quality of work done by a sounding unit is largely dependent on the alertness, intelligence, and faithfulness of the recorder. Referring to the standard sounding book, the following suggestions for keeping notes are offered:

1. At the beginning of the day, record the date; the name of the place, as Balandras Channel, or Area 3, or Levisa Bank, or the approximate latitude and longitude; the name of the vessel, of the officer-in-charge, of the observers, of the leadsmen, and of the recorder; and the daymark.

2. At the beginning of the day and of the afternoon period, record that sextants and leadlines have been checked and found to be correct; or that they have been found to be incorrect and have been adjusted or have not been adjusted, as the case may be; or that they have not been checked. In case of errors, record instrument, time, error, and correction, thus:

Left sextant, 1255, coincidence reads $-2'$, correction $+2'$, adjusted.

Right sextant, o. k.

Lead line no. 1, 1255, reads 31 feet between 0- and 5-fathom deck marks, correction to soundings -1 foot.

Used a. m. but not p. m.

Leadline no. 2, 1255, o. k. Used p. m.

3. Column 1 is for entering the civil time of each sounding in hours and minutes and fractions of a minute, or in hours, minutes, and seconds. When a position is on time this is indicated by writing its number opposite the proper time. But if the position is appreciably early or late, its number may be written opposite the time of the nearest sounding, with an added note to bring it to the correct time, as: 10 seconds early, or 10 seconds late. Usually it is convenient to use the seconds column for this purpose.

In plotting positions and soundings it is assumed that every numerical event, recorded in the same horizontal line as a time entry, took place at that precise instant.

It is part of the recorder's duty to study the action of the observers and leadsmen in order that he may give the "stand by" whistle at the proper interval in advance of soundings and angles to cause them to occur at the right time; and to insist on uniformity of performance. With leadsmen the advance interval varies with the depth. With anglers a warning of 10 or 15 seconds should be ample. When the objects are difficult to see, the anglers should bring the images together several times between positions in order to be able to get the angles on signal, with a short run of the tangent screw.

The team work of officer-in-charge, recorder, observers, and leadsmen at the time of taking a position may be illustrated as follows:

The launch, headed north true, is east of the line, as shown by the last position. The officer-in-charge, wishing to get back on the line, orders a position on the next minute. He should not order a change in course before that time, nor before the angles are snapped, even if they are late, lest the turning of the boat cause them to be missed. If necessary, he should stop the boat until the angles are obtained, before changing course. Suppose that the position is to be taken at 8:18, that the leadsmen requires 5 seconds warning and the observers 10 seconds. The officer says "position next minute." At 8:17:50 the recorder blows two whistles for the observers, at 8:17:55 one whistle for the leadsmen, and at 8:18 one long blast for all. Unless the observers announce failure to get the angles, course is changed immediately.

4. When the distance from the end of a line to the beginning of the next line is considerable, the notes should state the approximate distance and course to the latter.

5. Bottom obtained with an armed lead should be marked "specimen", or (sp.), at the top of each page. In recent years it has been the practice to arm the lead only on check lines. For each boat sheet, however, there should be, in addition, a general note regarding the nature and appearance of the bottom. Any unusual local characteristic may be noted opposite the position where it is encountered.

For charts, the material and appearance of the bottom, not merely its hardness or softness, are required. When the bottom over large areas is mud, the lead may be armed only on the

Time	Pos	Original Lead	Observed Angles		Stations or Objects Observed			Notes
			Left	Right	L	C	R	
8 20		116 3 5	58-02	6-51	Tom	B.g	Bee	
same			cut	20-10	Bee	Pelham Rk		Center
20 30		4 1						
21		4 2						
30		4 4						
22		4 4						
30		4 4	cut	52-05	Bee	Pelham Rk		5 fr high
23		3 5	cut	4-15				
30		4 1						
24		4 3						
30		4 5						
8 25		117 4 4	5-42	100-10	Tom	B.g	Bee	
same			cut	25-00	Pelham Rk	Tom		
same			103-52		Tom		Bee	Sum angle

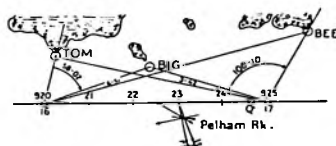


FIGURE 38.—Three-point fixes and additional angles for cuts.

check lines. But when it is irregular, especially when it is dotted with coral heads, specimens should be obtained also on the regular lines at intervals sufficiently close to enable the plotter to outline, by lines similar to fathom lines, the principal areas presenting different characteristics.

6. It is important to record all courses. For a launch the new course may be set down on the line of the time entry corresponding to the time of beginning to change course. For a larger vessel it will be necessary to record both the time of changing course and the time when the ship is steady on the new course.

The notes and the sketch in Figure 36 illustrate the following points.

The signals are mostly effectively placed a little off their respective points of land.

Convex fixes are good even when one of the angles is small.

Visible rocks, near at hand and presenting definite points to cut upon, can be located by cuts in passing them, if a good course is maintained. The cuts will be accurate in proportion to the distance of the signal from which the angle is swung. One or more angles between tangents gives the size of the visible part of the rock. The height, if small, is estimated.

The boat officer should occasionally check his anglers by observing the sum of their angles, or the more difficult angle.

The oral part of the work in the Figure 36 might be somewhat as follows, so far as it relates to locations:

- Before 9:20. Officer. To anglers: "Use Big for center." To coxswain: "Steady on course 90." To extra observer: "Cut in Pelham Rock, on the starboard bow."
- After 9:20. Left angler. "Left, fifty-eight, Oh two."
Right angler. "Right, six, fifty-one, objects Tom, Big, Bee."
Third angler. "Cut, twenty, ten, Bee to Pelham Rock."
Third angler. "Give me a mark at 2½ minutes, and another at 3 minutes."
- At 9:22½. Recorder. "Stand by,—mark."
Third angler. "Cut, fifty-two, Oh five, Bee to Pelham Rock."
- At 9:23. Recorder. "Stand by,—mark."
Third angler. "Tangents, left to right, six, fifteen."
- After 9:25. Left angler. "Left, three, forty-two."
Right angler. "Right, one hundred, ten, same objects."
Recorder. "Right, one hundred ten degrees."
Right angler. "No. One hundred degrees, ten minutes."
Officer. "Sum angle, one hundred three, fifty-two."
Recorder. "Sum checks position angles."
Third angler. "Cut, twenty-five degrees, flat, Pelham Rock to Tom."

When a launch has noisy engines, the recorder should repeat the angles, and the anglers should pronounce 50 as **five-ty** and 15 as **five-teen**. Sometimes there is trouble with the words twenty and forty.

As a rule only isolated rocks between sounding lines are located by cuts. Sometimes coral shoals awash, when outlined by visible rocks are located in this manner, the boat officer acting as a third angler. But usually larger rocks and shoals are located by going to the edges with the launch, taking positions, and sketching the outline from position to position.

In ship work there is often considerable cutting to do. In such cases the extra anglers should do their own marking and recording, lest inopportune interruptions endanger the accuracy of the regular sounding work.

7. Extra observers should get the most important cuts on the regular positions, and others on even minutes or half-minutes. It is well to get the "mark" from some person other than the recorder. Precede the announcement of such angles by the word "cut", and follow it by naming the objects left to right, the time, and the object cut upon. When there are several extra observers, it is well to collect the cuts on a memorandum pad, from which the recorder may transcribe them to the sounding book, without danger to the sounding record due to inopportune interruptions.

8. It is a rule in all scientific work **never to erase any part of the record of an observation**, once it is made, but to draw a line through the part sought to be corrected and to write the supposed correction near by, with the reason for making it.

For example, suppose that the plotter, having heard the angles of position 17 above as 3—42 and 110—00 instead of 100—10, which place the position far off the line, finds that 5—42 and 110—00 will place the launch at *Q*, on the line, and notifies the recorder that he has used those angles. The latter should not erase the suspected angle, but leave it for the smooth plotter to examine. On examining the spacing of soundings as well as the alinement, the smooth plotter, in this case, would doubtless decide that the angle as originally recorded was correct.

In this connection, it may be stated that a good angleman always takes a second look at his angle after announcing it; and unless he is pressed for time, leaves the angle on the sextant until the position has been plotted.

A practice boat sheet will be needed for at least 2 days during the latter part of the training period. It may be small and show but few signals, but the latter should be accurately located. Meridians, parallels, pencil sounding lines, and a scale for the interconversion of knots and minutes should be shown on it. No compass rose should be furnished, but the plotter should be trained in the more accurate method of using the meridians and the Court's protractor for measuring and laying down courses. If no shoal is handy, an assumed shoal may be placed on the boat sheet for practice in approaching and developing.

DUTIES OF THE PLOTTER

In regular sounding, the plotter, under the direction of the boat officer, watches the speed to see that it is kept standard, selects signals for fixes, plots fixes, and notifies the boat officer how many seconds off the line the boat is at each position, or what change of course is necessary to bring her back to line. With the naming of a new fix at the next position, he furnishes the observers approximate bearings and angles for finding the signals on the horizon, if they are hard to pick up. When he believes, from the failure of a fix to plot properly, that the angles are incorrect, he calls for a second reading. If the angles are found to be correct, they are marked *O. K.*; otherwise they are corrected, without erasure, and marked "read again."

The plotter also corrects, for the benefit of the recorder, errors in naming signals. He plots, in pencil, sufficient soundings to keep track of the distances from shore where fathom lines are crossed, and especially shoal soundings that may require further investigation. To avoid the necessity of changing speed between positions, he gives warning, on the boat's approaching shoals or land, that it will be necessary to "slow on the next position."

SELECTION OF THREE-POINT FIXES

1. *By trial.* Let *L*, Figure 37, be the angle between stations *A* and *B* on a boat sheet, *R* the angle between *C* and *D*, and *X* the unknown position of the launch. Set off *L* and *R* on a three-arm protractor.

If a pencil point is held in the protractor center and the protractor is maneuvered with a continuous motion in such a way as to keep the edges of the left-hand and center arms passing through *A* and *B*, the pencil point will describe an arc *ABX* which may be conveniently called the *location arc* of the angle *L*. Similarly, if the protractor is maneuvered to keep the edges of the center and right-hand arm passing through *C* and *D*, respectively, the pencil point will generate the location arc of the angle *R*. If the position is possible and determinate, *X* will lie at the intersection (either of two) of the two location arcs.

The quality of the fix may be judged by the angle of intersection of the location arcs. An intersection of 90° is ideal; of 90° to 45°, good; of 45° to 20°, fair; of less than 20°, intolerable.

This form of the four-point fix may occur because of the failure of the observers to use the same signal for center, necessitating the separate plotting of the two loci.

When *B* and *C* are in fact the same signal, the location reduces to a three-point fix on *A*, *C*, and *D*, with the location depending on two location arcs, as before. In the operation of plotting, the center of the three-arm protractor follows small portions of the location arcs in succession until it succeeds in falling on both arcs. If the plotter notes the directions of these partial arcs, he will be able to tell whether the fix is strong or weak.

In boat work the plotter should not, at the outset, place the center of the protractor exactly in the expected position, nor on the line, but rather should adopt the attitude of being compelled by the angles to move the protractor finally to one and only one position; otherwise, weak fixes may escape detection. Some of these will be detected readily by their tendency to swing across the line. Others will be revealed only by an uncertainty and lack of uniformity in the distance intervals between successive positions.

Gross errors in observing angles may result in **impossible fixes**. The same effect may result from instruments out of adjustment, if the error is large. With smaller instrumental errors, or with stations a little out of position, the positions can probably be plotted, but the line run by the boat will show a tendency to curve. The tendency will be greatly magnified if the fix is weak or if the error is in a small angle.

If in a three-point fix on A , C , and D , the sum of the angles L and R is equal to the supplement S of the angle ACD , then in attempting to plot the position the protractor center will swing along the arc AD of the **main circle** passing through A , C , and D ; that is, the location arcs will coincide with the main circle and with each other, and so fail to give an intersection. In a mathematical sense, this can happen only when L is half the size of the central angle in the main circle subtended by arc AC , and R is half the size of the central angle subtended by arc CD .

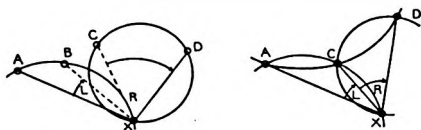


FIGURE 37.—Fixes, four-point and three-point.

Since only one such set of angles is possible, the chance of encountering an absolutely indeterminate fix is small. In a graphical sense, however, the effect of unavoidable errors of observers, sextants, protractors, and boat sheets is so magnified that fixes in which $L+R$ is nearly equal to S are practically indeterminate; while in some cases

fixes in which $L+R$ differs from S by as much as 20° are intolerably weak.

It is convenient to call all such weak fixes **swingers**.

2. *By looking at the sheet.*—Fixes may be classified as **convex** or **concave** according to the curvature of the main circle at the central signal.

A convex fix is always determinate. It is strong if the sum of the angles exceeds 30° , and may be considered fair even if the sum is only 20° . Long runs can sometimes be made on a single convex fix.

"Range and one angle" is a special case of this fix. It is effective when running on the range until the angle falls below 20° . When running across the range, however, the same signals often give a bad fix within a few positions, due to the fix becoming concave, with a distant center.

"Three signals in a straight line" constitutes a special case of the convex fix. Though this fix cannot give a swinger in the mathematical sense, for which reason it is often incorrectly used, fixes in which $L+R=20^\circ$ or less (S being 0° in this case) must be considered swingers in the plotting sense. The fix is particularly atrocious when one of the angles is very small.

A concave fix is strong when $L+R$ exceeds 180° . The ship, working with a small-scale sheet and distant signals, can often make effective use of this fix. Launches, however, working with small sheets and nearby signals, do well to avoid it as much as possible, because it is continually on the point of turning into a swinger, which is a special kind of concave fix. When inside the circle with $L+R$ less than 180° , keep the sum $L+R$ more than 20° above S .

3. *By noting the variation of angles.*—A certain set of signals originally giving a good fix may be continued safely for successive fixes as long as $L+R$ and either L or R show a considerable variation between positions, provided that the sum does not drop below 30° and neither angle becomes inconveniently large.

4. *By estimating the comparative distances to signals.*—With skill acquired in estimating comparative distances, the boat officer, whose duties of superintendence forbid constant reference to the sheet, may select good fixes by using the field itself as an imaginary sheet. He is also in a position to judge which of several available fixes is most practicable, considering visibility, background, and the best direction for reflected light. On a sheet well signaled out, there will seldom be need of analyzing fixes if the following rough rules are observed:

- (a) Use the nearest signal for center.
- (b) Use a wide spread of the protractor arms.

SCALE AND DEVELOPMENT OF HYDROGRAPHY

The general hydrographic development of an area is that necessary to furnish an excess of soundings for general or coastal charts, and to insure the discovery of all existing special features, such as channels, reefs, banks, and shoals.

When the bottom is believed to be regular, of materials having a flat angle of repose, the usual method is first to apply a general development, and then from a study of the results to add special local developments. Irregular rocky or coral bottom necessitates, regardless of the scale of the chart, a general development close enough to detect visually every irregularity between lines. With water of average clearness the maximum safe distance between lines ranges from 250 to 300 yards, according to the height of eye.

When the 100-fathom line is close to shore, a scale of 1:50,000 is suitable for the working sheets, and a development of the hydrography by parallel lines one-half mile apart, normal to the coast, is sufficient to provide soundings for coast charts. The 5-fathom, 10-fathom, and the 100-fathom lines should be developed, the latter being the most important. To judge whether the 100-fathom line has been sufficiently developed, first discard all "no bottom" soundings and then find whether the 100-fathom line can be drawn without uncertainty by means of the remaining soundings. On each line there should be at least two soundings between 60 and 100 fathoms; and if the deepest sounding on the line chances to fall short of 100 fathoms, the end soundings on adjacent lines should show bottom soundings of more than 100 fathoms.

The land forms exhibited by ridges and rivers may be expected to repeat themselves on the sea bottom, calling for extra soundings opposite these features beyond the particular 100-fathom line that embraces them. As examples of the persistence of land forms, San Antonio Knoll (Cuba) may be considered an outpost of Sierra Acostas, while Xagua Bank is clearly related to Sierra Cañada (Isle of Pines).

On a steep-to coast where the 100-fathom line gives insufficient warning of the proximity of land, further seaward development, by wire or pressure tube or sonic methods, is desirable.

On such a coast, though harbors may be infrequent, there may be inlets and temporary anchorages very useful to small coasting vessels. They therefore merit special development on a large scale, with the expectation of publication as insets on the coast chart.

For a flat coast, where the bottom is sand or mud and is fairly regular, a general development on a scale of 1:30,000 by parallel lines 300 yards apart is suitable for the less important parts. Alternate lines may be omitted over extensive but unimportant 2-fathom areas and also between the 15- and 100-fathom lines. For the sake of uniformity in scale the scale of 1:30,000 should usually be retained even if the bottom is irregular, but the interval between lines should be reduced to 200 yards or less.

Without special instructions, ordinary channels and harbors will be surveyed on a scale of 1:15,000, but sometimes a smaller scale is desirable to include conspicuous inland landmarks.

The following is believed to be good average practice for harbors:

Depth	Scale	Speed	Time Interval	Space Interval	Distance between lines
<i>Fathoms</i>	<i>Ratio</i>	<i>Knots</i>	<i>Minutes</i>	<i>Feet</i>	<i>Feet</i>
2 to 5.....	1: 15, 000	4	½.....	135	450
5 to 7.....			¾.....	203	-----
7 to 10.....			1 + stop.....	405	-----
2 to 5.....	1: 10, 000	4	½.....	135	250
5 to 7.....			¾.....	203	-----
7 to 10.....			1 + stop.....	405	-----
2 to 5.....	1: 5, 000	3	½.....	101	150
5 to 7.....			¾.....	152	-----
7 to 10.....			1.....	304	-----

The lower speeds mentioned would ordinarily cause the engines of a gasoline launch to overheat. To avoid this difficulty the plan devised by the U. S. S. *Argonne* may be used. It consists of a plank 6 feet long held across the stern by two strap irons leading to the bow chocks and connected across the deck by a rod run through eyes in the irons.

In addition to the systems outlined above, special systems are required for completing the development of shoals, reefs, channels, and anchorages.

SHOALS, REEFS, AND BANKS

Risings of the sea floor to a degree menacing navigation are called *shoals*, if of gentle slope and not rocky; or *reefs*, if abrupt and rocky. Larger areas that may be traversed safely are called *banks*. On survey sheets these hydrographic forms are outlined by plain or characteristic *fathom lines*, or *fathom curves* (Fig. 38).

With gradual changes of depth, the outlines of a shoal or bank may be obtained by interpolating between the soundings on standard interval lines, provided that the sounding vessel may pass over every part; but extra lines may have to be run between the regular lines where the latter are nearly parallel to the fathom curves. In order to secure the advantage of sounding over the whole shoal or bank, it may be advisable to sound at times of high water—a method often adopted in obtaining by interpolation the high water and low water shores of alluvial rivers. In any case, extra lines may be necessary to ascertain another fact of prime importance—the least water on the shoal, and the location thereof.

When part of the shoal is not navigable, the 1-fathom outline may still be obtained by the method of interpolated soundings, if the changes in depth are gradual, the conventional 1-fathom curve being a smooth curve joining the reduced 1-fathom soundings farthest from the center of the shoal on the ends of successive sounding lines. The same principle applies to other fathom lines. With abrupt changes of depth, however, it becomes necessary to “run” the fathom line in question quite closely in the case of a coral edge or visible reef line, less so in navigable depths. This simply means the location by position angles of a sufficient number of extra points to insure a faithful delineation of the outlines.

In the delineation of coral or rock edges or reef lines, where the boat, for safety, must take a circuitous route from one salient point to the next, intermediate soundings are discontinued and the work becomes purely location surveying. At each important point the boat is stopped, and in addition to the usual fix angles and soundings, cuts are taken to conspicuous rocks awash or above water, while the sketching of the part of the shoal passed since the last position is facilitated by taking tangents to bights, to patches of rocks, etc. Cuts to important features are recorded for the benefit of the smooth plotting, but tangents and bearings taken merely to facilitate the sketching, which should be done on the boat sheet before the boat moves on, need not be recorded.

The following **field symbols** for rocks and shoals are modified from the corresponding chart symbols in the direction of simplicity. On the field sheets there is more freedom in delineation, the main purpose being to convey information emphatically and unmistakably, beauty of delineation being a minor consideration. On boat sheets colored pencil or ink lines will serve for the generality of fathom lines, but coral and rock edges should be shown by scalloped lines, imitating the conventional signs, but with heavier lines. In addition to the crosses and asterisks used to denote rocks precisely located, notes should be written on the sheet concerning depths on rocks, heights of rocks, emergence, shape, and extent of shoals, their color, and so on, not forgetting to specify the stage of tide.

In the case of fathom lines along the shore, or around an island or an extensive shoal, simple fathom lines will suffice, for it is perfectly evident on which side the shoal water lies. This is not true of small shoals, isolated rocks, and occasional shoal soundings in deep water. Nor is it true of deeps, even fairly extensive ones, in large areas of shoal water. Here it is necessary to supplement the simple fathom line by something to show on which side the shoal water lies. If the simple fathom line is a continuous colored line, it may be shaded on the shoal side. If the shoal is quite small it may be cross-hatched. If the simple fathom line is a row of heavy dots, lighter rows of dots may be added on the shoal side—inside the 3-fathom curve, two rows; inside the 2-fathom curve (if used), one row; and inside the 1-fathom curve, light dots widely spaced over the whole shoal. In addition it is well to write the shoalest sounding obtained on every bank and shoal in heavy lines, causing it to stand out from the surrounding soundings. Care should be taken, in sanding, to avoid obscuring soundings, bearing in mind the subsequent photographing of sheets to a reduced scale.

Meaning

Rock above water, precise location.
 Rock awash, precise location.
 Submerged rock, precise location; sounding.
 Rocks awash, general location.
 Submerged rocks, inside 2-fathom curve.

Coral 1-fathom edges, coral debris inside.
 Ledge rock 1-fathom edges.
 Sand bar, 1- and 2-fathom curves, submerged rocks.
 NOTE.—The submerged rocks symbol is rarely used outside of the 2-fathom curve.

Tangents to reef lines taken at triangulation towers are valuable to give the general location of reefs, but are usually incomplete and lacking in detail, since it is only in bad triangulation weather that reefs are disclosed at a distance. Those so discovered should be shown in pencil on the boat sheets when issued, with the understanding that there may be others. For the discovery of all reefs and shoals and for their complete development, reliance must be placed on the boat locations. When aerial photography is available as a supplementary aid, the **shapes of reefs** and rocky shoals as determined by the boats may be improved by examining the prints.

The color of a shoal is important. When all shoals in a region have the same general characteristics, as for example in color, in nature of bottom, in form, in general trend, etc., the fact may be covered by a note on the boat sheet. For example: "All shoals and reefs on this sheet are of brown, living coral, high on the seaward side and strewn with debris, and extend generally in a northwest to southeast direction."

When a reef line or coral edge forms the bank of a narrow channel, which perhaps is destined to be shown in larger scale in an inset on the general chart, the only satisfactory development is by dragging. With a **short drag**, not over 500 feet long, this is a one-boat job. Set the drag at the critical depth of the channel plus allowance for stage of tide. Anchor one end of the drag at the edge of the channel and with all of the drag except that end in the boat, go downstream, paying out the drag but keeping it taut. When the drag is fully extended, swing it about the anchored end as a pivot, going upstream, until every part of it brings up against the edge of the bank. Anchor the second end of the drag, cast off and take a position and a sounding at each buoy of the drag. Pick up the first end and swing the drag, kept taut, around the second end as a pivot; etc.

There should be no stray line, or a minimum of stray line, at the anchored end of the drag. The stray line required for the maneuvering of the launch can be attached to the towing end of the drag.

For the development of a bank, the standard interval grid of sounding lines may or may not suffice, according to circumstances. If the bank is contiguous to a channel, all the salients where buoys are to be placed should receive a thorough local development with the idea of finding the tip of the bank and all outlying spots of depth not exceeding that of the buoyage system. For important channels this is accomplished most expeditiously as well as most thoroughly by dragging inward from deep water toward the tip of the bank.

The development of a bank or shoal far at sea, out of control of signals, presents many problems. That of finding an initial latitude and longitude will naturally be described in the specifications of the survey. With a single initial point known or assumed, the next question is that of the method of development.

If the shoal is small, development by radial lines from the origin is often satisfactory, the lines being short. The lines may be run by the ship on dead reckoning; or by a launch, getting

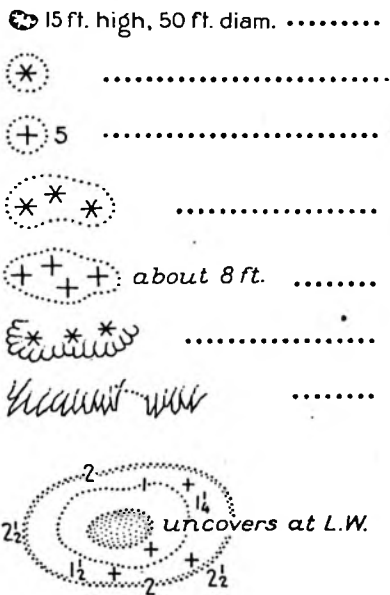


FIGURE 38.—Field symbols.

bearings on signal at intervals from the ship at anchor, and obtaining distances by masthead angles of the ship. With a mast 100 feet or more in height the method is fairly effective so long as the masthead angles, using a 10-second sextant with a telescope, do not fall below $0^{\circ}30'$.

On flat banks the distances between stations may be measured by means of **taut wire**.

If the shoal or bank is extensive, with the initial position determined astronomically with a sextant, the necessity of additional hydrographic signals arises. Whether it is best to locate these by independent astronomical observations, or by run of the ship, or by a combination of astronomical observations and repeated runs of the ship depends on the accuracy of the observations, winds, and currents, and chiefly on the extent of the bank. For a survey above the grade of the roughest kind of a reconnaissance, it might be estimated that no astronomical base, thus determined, is usable unless it is at least 100 miles long. With two astronomical stations, one at each extremity of the bank, intermediate signals are best located by run of the ship between the astronomical stations.

CHANNELS

The theory of location work in sounding, by three-point fixes, rests on the assumption that the sea bottom slopes gradually, so that displacements due to errors of instruments, observers, and plotters have little effect on the lay of the fathom curves. This theory breaks down in the case of channels, especially when the sounding lines of the general development lie across the channel in the direction of rapid changes of depth.

If it is assumed that, in general development work on an ordinary scale, the displacement of a sounding located by a three-point fix from a moving vessel is as likely as not to be over 15 feet and may be as much as 30 feet, while the limit of tolerance in good channel work might be placed at 5 feet, it will be seen that what is fair sounding for general areas is only reconnaissance for channels. Therefore, the general development soundings should be considered as provisory, and the shoaler ones, if conflicting with those obtained by special development, should be discarded.

The special intensive development of a channel may be either by sounding or by dragging at various depths until the critical depth is found. The latter may be defined as the greatest depth that could be carried through the channel with a fair width, if the channel were buoyed.

1. *By sounding alone*.—(a) The sounding lines should be laid parallel to the axis of the channel.

(b) The interval between lines should be reduced.

(c) The interval between positions should be reduced to 3 minutes or less.

(d) The speed of the vessel should be reduced.

(e) At critical points or at suspected spots a temporary buoy should be dropped, and with the launch stationary or drifting, the bottom in the vicinity should be felt out with the lead to find the shoalest sounding.

(f) All dangerous spots should be located by special position angles rather than by interpolation between adjacent positions.

(g) The method of location by intersections, described under "Rivers and Harbors", page 73, is more accurate than that by three-point fixes, but is not always practicable.

2. *By dragging and sounding*.—This method is more expeditious than that by sounding alone, and has the merit of insuring the safety of the channel.

THE WIRE DRAG

The length of wire drag is limited by the power of the launches to be employed and the dimensions of the channel. Two 60-foot tenders are commonly employed, and these can handle a drag 4,000 feet in length. For greater lengths a third launch may be used at the middle of the drag.

The essential parts of a 2,000-foot drag are as follows: Forty 50-foot sections of no. 7 American gage galvanized telephone wire, for the "bottom wire."

Eighteen spherical iron sinkers weighing 15 to 20 pounds, provided with 2 rings or staples. To the bottom ring of each the ends of adjacent 100-foot lengths of the bottom wire are shackled and swiveled.

Three spherical iron sinkers weighing 150 pounds having a lower ring and an upper shaft 1 foot long terminating in a ring. These sinkers are used for the ends and the middle of the drag where the towlines are attached, and are designed to minimize the lift of the drag.

Eighteen small nun buoys and three large ones to support the sinkers and drag at 100-foot intervals. Each should be fitted with a reel and ratchet mounted at the top of the buoy on a suitable framework, for adjusting the length of the stirrup to the stage of tide. The stirrup chain, coming up from the sinker, passes under a pulley suspended by a shackle and a swivel from the bottom of the buoy, then over a pulley riveted to the reel frame near the bilge flange of the buoy. Each buoy is also provided with a vane about 6 inches wide, designed to prevent the buoy from spinning and weaving from side to side.

Of course improvised buoys may be used, as for example gasoline drums for end buoys and can buoys for intermediates. The latter have a central pipe through which the graduated chain at the upper end of a stirrup may be threaded, convenient also for inserting a small flagstaff, used on long drags on center and end buoys. But the use of such makeshifts is expensive in the long run, for the drag has to be stopped to adjust the stirrups, and in the meantime the towing vessels, unless they anchor, are dragged inward by the weight of the tow, thus losing position and effective width of drag.

Twenty-one light chain stirrups for suspending the bottom wire and the sinkers from the buoys. They are to be graduated in place and marked at 1-foot intervals near the flange of the buoy, which serves as a reference mark.

Twenty cedar floats about 18 inches long and 6 inches in diameter, with a ring and a harness snap at one end. In use they are attached to the bottom wire at alternate 50-foot section joints midway between the sinkers, and are intended to take up the sag of the wire.

With deep submergence, cedar floats become waterlogged quickly. It would be better to use small steel floats with interior stiffeners designed to withstand heavy pressure. In any case the spacing and buoyancy of the floats should be such as to give the wire they support a neutral buoyancy. This may be determined by computation or by actual test, and in case of waterlogging the tests should be frequent.

Three vertical towing bridle, of rope. One end of each bridle when in use is attached to the upper ring of the large sinker, and the other to the bottom ring of the large buoy. The parts of the bridle should be of about equal length, about 60 feet, to keep the buoy and its sinker in the same vertical plane.

Two rope towlines about 80 feet long, one for each launch. The forward end of each is formed into a bridle to facilitate quick shifts from the point of attachment amidships of the starboard side to that amidships of the port side, or vice versa. Except in the act of shifting, both lines of the bridle are carried on the same side, one under tension and the other slack. A short messenger line is used to keep the line out of the propeller during shifts.

One tow line for the middle launch, if used.

Four spring balances, two for each end launch. They are intended to register the tension in the towing bridle, which is usually about 400 pounds. One is attached to the starboard side and the other to the port side, about amidships, where the engineer can watch them.

One large reel with two handles for reeling in the drag.

Swivels for the following places:

One for each 100-foot joint of the bottom wire.

Two for each stirrup.

One for each line of each towing bridle.

One for each spring balance.

Seven sister hooks for attaching the towlines, which should be of invariable length.

The dragging party should not be obliged to lose good dragging weather repairing a drag broken the day before, but should pass in the broken drag each night and receive an extra reel and bottom wire in good repair for the next day's work. A great many extra swivels, shackles and floats, and some extra sinkers and buoys will be needed.

PUTTING OUT THE DRAG

One of the launches takes the reel with the joined sections of the bottom wire wound around it; and the sinkers, shackles, swivels, and buoys with stirrups attached. The reel is mounted solidly in the stern sheets and arranged to pay out the bottom wire aft.

The launches proceed to a spot in deep water near the place of beginning dragging, and that launch having the drag goes to the upstream position, with reference to the current and anchors. She throws the buoys overboard, retaining the lower ends of the stirrups. Taking one of the large sinkers, the end of the bottom wire is shackled on, then one line of the vertical bridle, then the lower end of the stirrup of the end buoy. The other end of the bridle is shackled to the bottom ring of the buoy and the bridle is passed to the other end launch. The sinker is let down, and the launch after making fast her towline backs away slowly, down the current, drawing out the drag and unwinding the reel.

As the first 50-foot joint comes off the reel, the wooden float is snapped on. When the first 100-foot section comes off the reel, the towing launch is stopped momentarily, by signal, until the sinker and the stirrup of the first small buoy have been shackled on, after which the

launch backs away as before. In this way the whole drag is laid out, down current and always under enough tension to keep the bottom wire from sagging and catching on the bottom. When the middle buoy is reached, the middle launch comes up and receives its towing bridle; or this may be thrown overboard with a small buoy, and picked up by the launch shortly before dragging begins. When all the drag is laid out, the anchor is hoisted and the drag is towed to the initial position for the first strip to be swept.

COMMUNICATION BETWEEN LAUNCHES

On long drags a signalman with binoculars and flags will be needed on each launch. The flags describing the most common operations may be attached to staffs about 8 feet high and stood in sockets, each to be displayed until a change occurs. Flags for the following messages will be found convenient.

- | | |
|---------------------------------------|--|
| 1. Am on line; going ahead; go ahead. | 4. Stop (trouble, drag aground, drag parted, etc.) |
| 2. Am turning right; turn right. | 5. Am anchored; swing to next strip. |
| 3. Am turning left; turn left. | |

PATROL LAUNCH

An extra launch, carrying the officer in charge of the whole operation of dragging, is required to follow the drag, to watch the buoys, to adjust the lengths of the stirrups from time to time for the stage of the tide, to measure the lift of the drag, to clear and repair the drag after fouling, and to sound around the buoys whenever the drag touches bottom or encounters an obstruction. The former condition is disclosed by one or more buoys falling flat or leaning; the latter by the line of the buoys taking the form of a V. This launch should have fair power and a good height of eye. The duty is arduous. In a small launch it is even dangerous when there is a swell. For this service 50- and 60-foot motor launches have been recommended, after unsatisfactory trials of 30-foot launches.



FIGURE 39.—Sixty-foot drag boat with twin screws.

SOUNDINGS AND POSITIONS

All the launches are equipped for sounding. In water of near the critical depth soundings are taken at the standard intervals, notwithstanding the reduced speed, usually about 2 knots; but in deep water the interval is usually 2 minutes.

The usual position interval, except in long straight reaches in deep water, is 2 minutes. In rounding sharp bends in the channel the interval may be shortened to 1 minute.

When all is going well, the follow-up launch runs regular lines behind the drag. But when the drag hangs up or when shoal soundings are found, each separate sounding is located by angles. When the drag outlines a shoal, positions and soundings are taken at each buoy.

LONG DRAGS

The drawing illustrates the arrangement of parts for any length of drag, but this particular drag, used in 1934, was designed for dragging great depths, and consequently the parts are exceptionally heavy. On straight courses 1,000 feet of towline was used, and on turns this was increased to 1,500 feet. Time signals and other communications were maintained by radio.

In using long drags and in dragging at great depths there is less necessity for sounding from the towing vessels than there is with short drags in moderate depths.

ADJUSTING THE DEPTH OF THE DRAG

The initial mark on each stirrup chain should be attached to the chain when the drag is in the water, stretched out by the current in a straight line from the anchored launch. Adjacent marks 1 foot apart may be added afterwards, measuring from the initial mark. A braided wire provided with a sinker and a large hook and graduated near the top is convenient for the purpose. A foot or more, the actual amount to be determined by measurement with the drag under way, should be added to allow for the lift of the drag in actual use. Applying the measure and rotating the reel at the top of the buoy until the measured distance from water level to the bottom wire close to the weight is the required depth, plus the allowance for lift, attach a leather tag to the stirrup chain opposite the flange of the buoy. This is the initial mark.

After the attachment of the necessary number of foot marks to all the stirrup chains, the resulting scales will indicate the depth of the bottom wire below the actual water level. In order to suspend the bottom wire at any required depth below mean low-water datum while dragging is going on, the officer in charge will be obliged to keep in communication with a staff gage observer stationed near the work, and the compensations for stage of tide will have to be made from time to time by adjusting the stirrups.

Knowing the set of the drag, the effective depth may be measured at any time, and the lift computed, by suspending in front of the drag an iron rod about 5 feet long and coated with fresh white paint, and waiting for the bottom wire to come along and make its own depth mark. The rod may be suspended on a lead line, replacing the lead, or on any standard graduated line.

The transmission of tidal information to the officer in charge by signals or otherwise is often a difficult matter. For nearby work, flags may be used; for distant work, radio. When the transmission of signals is impracticable, predicted tides are sometimes used, at the risk of having to do part of the work a second time, if the corrections from predicted tides prove to be insufficient.

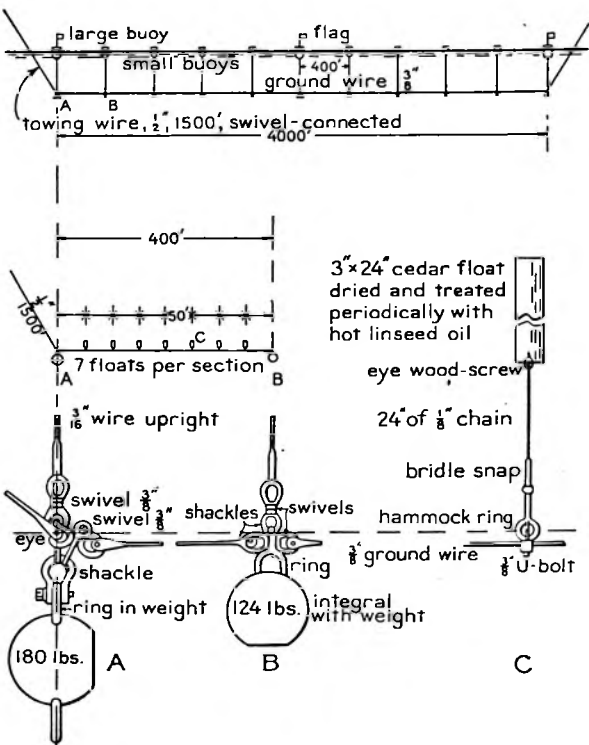


FIGURE 10.—4,000-foot, 10-section wire drag.

THE DRAG BOAT SHEETS

After the general development of a channel and before the special development by dragging, new boat sheets are issued to the boats, on the same scale as the old, but with such added features taken from the old sheets as may be useful in dragging, such as fathom lines near the depth of the drag to indicate the edges of the channel and to serve as danger lines; the edges of shoals and

banks; and all shoal soundings. This old information is furnished in red or green ink, with the expectation that it will be superseded or modified by the better new information to be obtained by dragging, the latter to be written in black ink.

The area expected to be dragged in full-width strips is laid out, with a hard chisel-point pencil, in strips four-fifths of the width of the drag between end buoys. The parallel lines bounding the strips are designated A, B, C, etc. A scale of feet, graduated to 50 or 100 feet, should be furnished in the title.

LOCATION OF END BUOYS

The bounding lines of the dragged strips are the paths of the end buoys, not of the launches. At each position, therefore, a third angle or a bearing must be observed in order to locate the end buoy. Since the length of the stray line is a short distance, and since the path of the end buoy

is much more regular than that of the launch, the angle or bearing may be obtained to the nearest degree and may be observed immediately before or after the position angles.

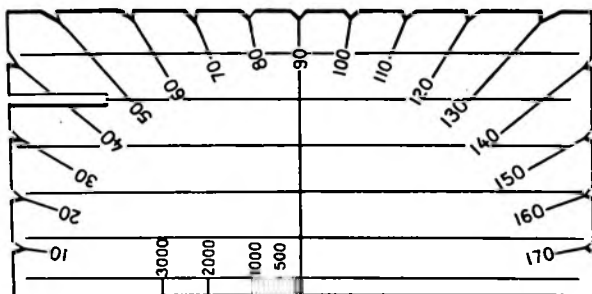


FIGURE 41.—End buoy protractor.

END BUOY PROTRACTOR

A transparent celluloid scale and protractor like that shown in the cut will prove to be convenient for plotting the end buoys. The lines are scribed with a needle and rubbed across with a dry soiled penwiper. Orientation

is obtained by eye by noting the parallelism of one of the longitudinal lines with an adjacent meridian.

In case of neglect to observe and record directions to end buoys, the plotter must plot them in the most unfavorable position, at the full length of the stray line inside the path of the launch.

RECORDER'S NOTES

Since an end launch is not free to move in the direction headed, magnetic headings need not be recorded. But at the beginning of each strip, the general direction of the strip about to be run should be noted.

The recorder should keep full and explicit notes of everything being done or attempted in his own launch and also what he sees going on in the other launches. The smooth plotter, having to plot the notes of all the launches as a simultaneous performance, will be in need of notes that refer to their cooperative work.

Agreement between the watches of recorders in the launches should be kept true by occasional signalled comparisons on exact minutes.

The smooth plotter is expected to insist on clear notes for his daily plotting of the preceding day's work. For this he must have all the notes of that day, including the record books of the drag launches and the follow-up launch, as well as the tidal corrections. Accordingly, in order that dragging and smooth plotting may go forward simultaneously, at least two distinct sets of record books must be provided.

The boat officers are expected to examine the notes each day, before signing them, to see that no ambiguity is left in them, particularly with respect to the following essential facts:

- The exact length of stray line.
- The total depth of drag.
- The allowance for lift.
- The allowance for stage of tide.
- The effective depth of the drag below the adopted datum.
- The width of the drag, or the length of the bottom wire.
- The set of the current during the running of each strip.

The officer in charge of the dragging operations should take charge of the tidal observations as well, and should see that the signaled tidal corrections are properly entered in the sounding books.

OVERLAPS AND "HOLIDAYS"

Overlaps of adjacent strips should be ample. When the plotted boundaries of adjacent strips come out so nearly coincident that there is doubt whether the whole area has been actually swept, it is the smooth plotter's duty to insure safety by insisting upon an extra strip to cover the part in question.

Boat officers will do well to take the following precautions against leaving holidays:

1. Do not use more stray line than is necessary.
2. On the rough boat sheets where plotting necessarily is hasty, deduct the full length of stray line for the boat plotting, not neglecting, however, to obtain bearings to end buoys for the benefit of the smooth plotting.

THE SHORT DRAG

The short drag, 500 to 800 feet long, usually operated by a second party, is useful in covering small gaps and in rectifying irregular outlines of the main dragged area; also in outlining banks and shoals bordering a channel, either for the purpose of developing the full width of the channel or for finding salient points for buoys. The effective width of a channel may sometimes be increased by this means from 10 to 30 percent.

SMOOTH PLOTTING

Of necessity the drag work must be kept smooth-plotted up to date. Not only must the outlines of the dragged area be plotted, but the soundings must be reduced for stage of tide and plotted in ink. When the plotting is likely to be complicated, as in the narrows of a channel, the plotting may first be done on several plotting sheets (copies of the smooth projection) afterward transferring the important features of each to the smooth sheet.

The tracks of the end buoys are usually shown in colored ink, red and green for alternate strips. When the whole area has been dragged, the outlines of areas dragged to various depths are tinted, using different colors for different depths. When any part of an area has been swept to different depths, it receives the color corresponding to the greatest depth.

In this connection, the needs of the office should be kept in mind. The soundings should be written in colors that photograph well, as black for the present work and yellow for previous work, if any. On the other hand, position numbers and overlaid tints should be in cobalt blue, in order that they may disappear in the photographs. Flat green and pink washes are permissible, but orange, brown, and especially yellow tints are to be avoided.

INTERSECTION METHOD FOR RIVERS AND HARBORS

For river and harbor work of the greatest accuracy, soundings—one sounding in every two, three, or four, according to the accuracy demanded—are located by intersections of simultaneous cuts from three transits on shore, on signal from the sounding boat running at reduced speed on set ranges. Ordinarily soundings are taken every 15 seconds with the located soundings falling on the even minutes.

The use of three transits usually insures that at least two of the cuts at every position will cross at a good angle, and provides for occasional misses by one transitman due to intervening objects. At the worst, when angles are missed, a sounding may be located by the intersection of a single cut with the range line.

In the method developed by the United States Army Engineers for navigable rivers under improvement, a 40-foot pole graduated to feet and tenths is used, and simultaneous cuts are taken on the pole as it comes to the vertical position with the advance of the boat in the operation of sounding. At this instant the signal flag, waving in warning, is dropped.

The ranges may be set in advance by a special party or as needed by the sounding party itself. In the latter case, before running each range, the "backsight" consisting of a tall "back flag" (a pike or banner) and a short "front flag", is set so as to point toward the "foresight" for

that range on the opposite shore. Before leaving shore the "foresight" for the next range is set, the distance to it being paced or measured. As it is seldom practicable to pace in a direction perpendicular to the ranges, it will be found convenient to carry a small wide-interval table of oblique distances

$$d = 100 \csc x,$$

which will give the distance d to be measured on a line x degrees from the desired perpendicular.

The direction of ranges, normal to a straight shore or radiating around sharp bends, may be determined by paced intervals, or by using a pocket compass, or by swinging sextant angles from a distant object. The lines are so laid, as to direction and spacing, as to develop the channel of a river or the most important part of a harbor fully and evenly.

The signals for cuts are given by flags, a white flag on three successive minutes and a red flag on the fourth, the only exception occurring at the ends of lines where two white flags (WW) are given instead of one (W).

Compass courses give a less regular development than ranges, but are more suitable for long lines.

FORM OF NOTES

Recorder's notes

Range	Time	Position	Sounding	Remarks
			<i>Feet</i>	
10	9:14 a. m.	1WW	7.2	At east shore, high-water line, beach.
	15½		8.5	Tracy on Mull's Dike No. 1.
			10.0	Byron on Rat I., 1922.
	16	2W	10.2	Tide gage at Stone Dock.
			11.0	
			18.3	Dredged channel begins.
			18.8	
	17	3W	18.6	
			18.0	
			18.4	
			18.5	
	18	4R	19.0	
	(*)			
	25	11W	6.3	
			5.1	
			3.8	
	26	12WW	4.7	West shore, face of dike.

* The notes between 9:18 and 9:25 have been omitted.

Transitman's notes

Range	Time	Position	Angle	Remarks
			° ' "	
10	9:14 a. m.	1WW	246 32	East shore, sandy beach.
	16	2W	267 43	
	17	3W	275 20	
	18	4R	280 58	
	(*)			
	25	11W	345 36	
	26	12WW	346 50	West dike, front. Checked zeros, O. K.

* The notes between 9:18 and 9:25 have been omitted.

At the end of the day the recorder reads a summary of his notes to the transitmen, thus: "Range 10, E. shore, 9:14 to 9:26, 12 flags, W. dike"; and the notes of all observers are harmonized as to time and position numbers. The three books are cross-referenced by number.

It is customary to keep two sets of books for use on alternate days. The recorder's notes chronicle all essential events, such as changes of station of transitmen and tide observer. For use with a Crozet protractor, the transit angles in pairs are reduced to the same base for the convenience of the draftsman.

Soundings are sometimes taken during periods of high water over alluvial or tidal flats bare of vegetation in order to find, by interpolation, the positions of high and low water lines. Part of the soundings thus obtained may be over areas higher than the datum. These are regarded as **negative soundings**. They are charted by using underlined figures.

DAILY TIDAL REDUCTIONS

In channel and harbor work, the approximate reductions to soundings for stage of tide should be made available to the boat officers at the close of the day's work, so that the reductions may be applied as the soundings are inked in on the boat sheets, for only reduced soundings can be expected to check. When the range of tide is not more than 1 foot, however, this precaution is unnecessary.

INKING IN SOUNDINGS, SHOALS, AND FATHOM CURVES

At the close of the day's work the boat officer will have all his positions plotted, numbered, and connected by straight lines. There will remain the task of inking in the soundings. Only waterproof inks are used. Next, the fathom lines and shoal lines of previous work are extended into the new work, in ink or in pencil. The certainty with which these can be drawn is an excellent test of the adequacy of the development.

It is most essential that this work be kept up to date by all boat officers, not only to enable each to correct deficiencies and to clear up possible ambiguities in the notes, but to furnish soundings for the ship's sheet in process of compilation from the work of all parties.

CHECK LINES

Check lines, as their name implies, are lines run across all the parallel lines of a sheet to check the soundings thereon. It would be contrary to the purpose of check lines to run them before running the main system of lines of soundings designed to be checked. If the reduced soundings on the main lines have been inked in, the officer charged with the duty of running check lines, on his own sheet or across the sheets of several boat officers, should be kept continuously informed of the tidal reductions while he is checking, in order that he may have reduced soundings to compare with those on the sheets.

The speed on check lines should be somewhat less than that on the original main lines.

When discrepancies occur that appear to be systematic rather than accidental, such as to suggest that the tidal effects on the day when the original soundings were taken might have been different in that locality than at the tide gage, an excess of soundings should be taken in the locality in order to find a correction to apply to that day's work. When a jump occurs in the original soundings on adjacent lines of two boat sheets, it is in order to ascertain whether the jump represents the reality or whether it is the result of error in one of the sheets.

INSPECTION OF BOAT SHEETS

When a boat officer has run all the regular lines on his sheet (usually there are several such sheets turned in at the same time), it is time to inspect the sheet to find what further development may be necessary. There may be wide spaces here and there between the lines. Some deep-water soundings may be too far apart, due to excessive speed. The edges of banks may be scantily defined. There may be no evidence of searches for possible shoals in the vicinity of sudden changes in depth. The soundings near the edges of the sheet may be discordant with those on adjacent sheets.

At this point it becomes the duty of an inspection officer to examine each sheet by itself and by comparison with others, and to add blue pencil lines where necessary to indicate further development desired; whereupon the boat officers concerned execute the further development necessary.

This plan is good as far as it goes. But the examination for obvious defects only of a set of sheets will not always suffice to maintain the proper standard in sounding. It is office inspection

merely, when what is needed is both office and field inspection. To obtain this, without interfering with the economy possible when all sounding units complete an area on the same day and move to the next area on the next day, the following procedure is suggested:

(1) That when the regular lines have been run but not the check lines, the sheets shall be inspected and blue lines added as may be necessary. There will then remain to be done:

- (a) Check lines.
- (b) Further development of shoals and search for new ones.
- (c) Gaps, or "holidays" to be filled.

(2) That all the boat officers concerned be constituted as an **inspection board** to divide the remaining work among themselves as may seem best, the running of check lines to be considered as field inspection, to be carried out with a thoroughness sufficient to derive lead line corrections, when necessary, to any day's work crossed by check lines.

(3) That the inspection of the work of various officers by the board of inspection be considered the joint effort of all toward certifying the reliability of the work.

Among *sources of discrepancies* between check lines and lines of the main grid, and between contiguous lines of adjacent sheets, the following may be mentioned:



FIGURE 42.—Sounding crew at their stations.

1. The tide gage may be in too sheltered a place, not subject to the full influences of wind, as well as of tide, experienced in the area actually sounded.

2. The tide gage may be too far from the work, considering the configuration of the land. When the coast is flanked at a considerable distance by a barrier of islands and shoals with deep water outside and shallows inside, the regional tides coming in at the entrance may be retarded to such an extent at a point far from the entrance (100 miles, say) that when it is high water outside the preceding low water prevails inside. In the direction of this retardation, tide gages are necessary at frequent intervals. Near gaps in the barrier the diverse tidal influences produce anomalous effects

which can be gaged only by additional local tide staffs. Similar effects may hold in channels.

3. The dipping of fathom curves near land or shoals may indicate a violation of the principle that extra positions are necessary, for correct location of soundings, at every change in speed.

4. The dipping of fathom curves on alternate lines of soundings run on the same day is an indication of the tendency of leadsmen, in sounding over flat bottom, to continue calling out the same sounding long after the depth has changed 1 foot.

5. Regularity of fathom curves near the tips of banks or shoals may indicate that the sounding interval is too great.

6. When lead lines are graduated to be read at the rail instead of at the surface of the water, adjacent lines run, with the wind first on one beam then on the other, may show a discrepancy of 2 feet or more; or an area sounded when the ship is deeply laden may appear to be deeper later when she is crossing the same area while lightly laden.

Figure 42 illustrates the centralization of activities possible with the use of motor sounding launches in fairly quiet water. The order of operators, left to right, is: sextant observers, recorders, officer in charge (speaking to the coxswain through the voice tube), plotter, leadsmen, and engineer.

The following notes on the Lietz, the Tanner, the Weule, and the Sigsbee sounding machines are abstracted from reports on their use by U. S. naval vessels.

LIETZ ROTARY BRAKE ELECTRIC SOUNDING MACHINE

The installation referred to consists of two of these machines, on either side of the main deck, abreast the bridge, each with a sounding boom and a fair-leader.

The essential parts of this machine are:

- A cast metal reel.
- A telescopic shaft.
- A rotary brake.
- A revolution counter.
- An electric motor.
- Stranded sounding wire.

The cast metal reel is mounted so as to form a telescopic shaft with the driving shaft. The reel shaft and the driving shaft are carried in bearings fitted in the frame. The reel is free to turn in its own bearing, and in a telescopic bearing formed between the reel shaft and the driving shaft.

The rotary brake consists of a split ring keyed and fastened with lock bolts to the driving shaft; two transmission pins; and levers. The entire brake mechanism turns with the driving shaft.

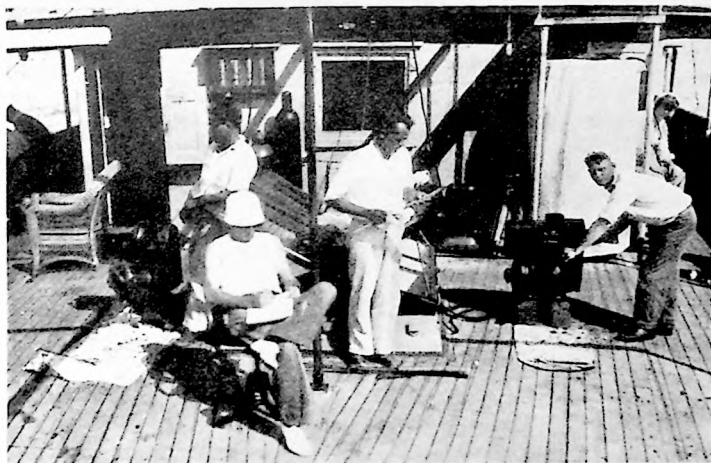


FIGURE 43. Typical installation of sounding machine

THE TANNER IMPROVED NAVIGATIONAL SOUNDING MACHINE

This is a hand sounding machine, suitable for tenders or steamers. It is used mainly for sounding from the 10-fathom curve to the 100-fathom curve. The essential parts consist of a metal frame, drum and sounding wire, the shaft, the cranks, the compressor arms, the brake lever, the register, and the correction tables.

The cranks for reeling in the wire are hinged to loose sleeves on the shaft. When thrown out of action they swing down each side of the column and are thrust into friction scores, for the double purpose of holding them in place and exerting a slight friction on the shaft, thus partially counteracting the friction of the drum when the wire is running out. The cranks are provided with automatic locking bolts.

The correction table, attached to the top of the machine, shows the number of fathoms of wire out corresponding to readings of the dial of the register, provided that in the previous reeling in the wire has been evenly distributed over the drum.

The brake is composed of two flexible compressor arms of brass, with the lower ends bolted

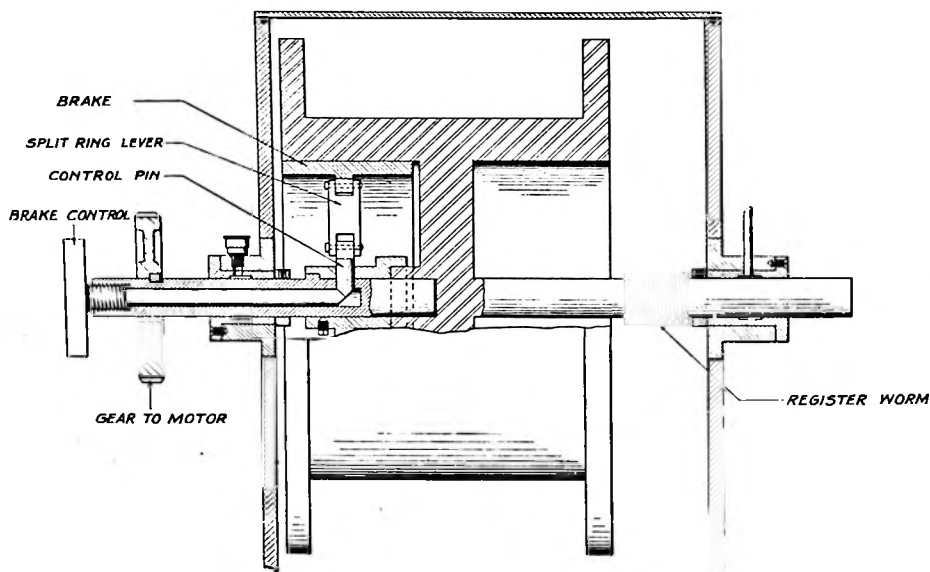


FIGURE 44—Reel and brake mechanism.

to the inner surfaces of the frame discs. Each arm carries a screw compressor bolt at its upper end, one right-handed, the other left-handed, working in the threaded T at the base of the brake lever.

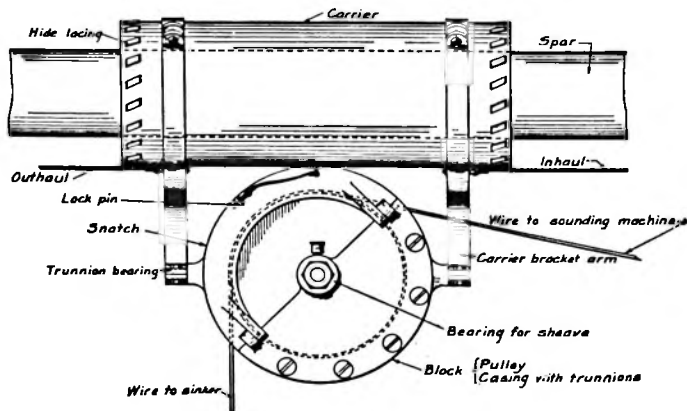


FIGURE 45.—Sounding sheave.

The oak friction blocks are carried on the inner surfaces of the compressor arms opposite the V-shaped flanges of the drum. Throwing the brake lever to the left brakes the drum. Throwing it to the right releases the drum.

The sounding wire is of American commercial manufacture, furnished in 300-fathom lengths. It is composed of seven strands of galvanized steel wire.

The machine is usually installed on blocks on the stern of a submarine chaser, or in the stern sheets of a steam launch, with a boom and sheave rigged aft to act as a fair-leader for the sounding wire.

The sinker is similar to a coasting lead, having an iron rod projecting from its upper end, terminating in an eye. It is about 34 inches long and weighs 18 pounds.

Three men, and their reliefs, are required to operate the machine—one to attend the brake and two to work the cranks.

To prepare to sound, see that the lead is secured to the end of the sounding wire and hung outside of the fair-leader sheave; that the brake is on; that the cranks are out of action with their handles thrust into the friction scores; and that the dial hand is set to zero.

In Navy survey practice, the vessel is stopped, obviating the use of pressure tubes and obtaining the sounding by a direct reading of the amount of wire out.

When the vessel is stationary, the brakeman eases the sinker down to near the water, then allows the wire to run freely, checking it only enough to prevent slack turns, and later in anticipation of the lead striking bottom, which it must not do with too great a force, on penalty of loss of the lead. When the lead reaches bottom, or when any designated amount of wire is out, he applies the brake and reads the dial. The other two men then throw the cranks into action and reel in. Finally, when there is no longer danger of the lead catching on the bottom, the vessel goes ahead to the next position.

Each sounding is located by position angles.

THE WEULE ELECTRIC SOUNDING MACHINE

The accompanying cut shows the Weule electric sounding machine installed on the stern of the ship, with the lead suspended by the sounding wire running over the fair-leader sheaves, secured to a small permanent outrigger on the rail. The machine is mounted in a channel iron frame. In the lower part of the frame is the 1-horsepower, 120-volt, direct-current motor, fully enclosed. The case marked "1" encloses the motor controller, which is operated by a small lever that functions as a rheostat, governing the speed of the motor and reel. The crank, seen over the top of the machine, operates the reel brake, which is asbestos-lined and internal-expanding. The pilot light, on top and at the left, enables the operator to read at night the amount of wire out, as shown on the indicator below it. There is a socket at the right-hand end of the machine for attaching a hand crank.

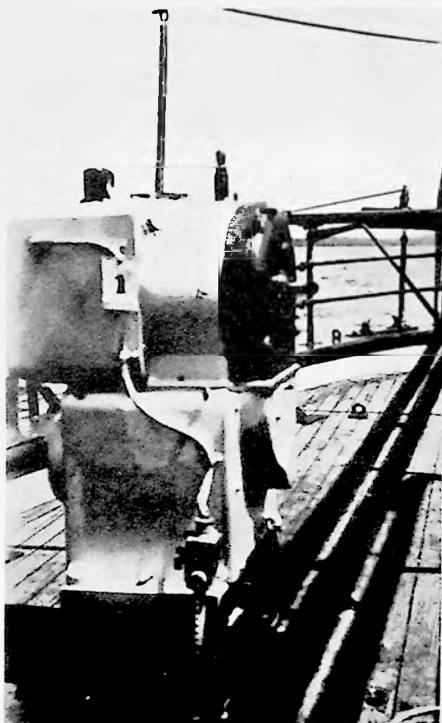


FIGURE 16.—Weule electric sounding machine

THE SIGSBEE DEEP-SEA SOUNDING MACHINE

For a complete description of the Sigsbee sounding machine the following works may be consulted. They will sometimes be quoted verbatim:

- (a) Deep Sea Exploration, Comdr. Z. L. Tanner, United States Navy.
- (b) Deep Sea Sounding and Dredging, Lt.-Comdr. Charles D. Sigsbee, United States Navy.

No attempt is made here to describe the taking of temperatures and water samples at various depths, although the figures herein show some of the apparatus used for that purpose. The description following pertains only to obtaining soundings and samples of bottom. The works cited above should be consulted if it is desired to obtain temperatures and water samples, and for further details.

Soundings and bottom samples at great depths can be taken with the Sigsbee sounding machine. This machine consists, in general, of apparatus to let go and take in 5,000 to 6,000 fathoms of 28- to 30-mil piano wire; a Sigsbee sounding rod; and a cast-iron sinker of about 60 pounds, which is detached upon striking bottom. In depths of 1,000 fathoms and over it is necessary to detach the sinker, otherwise the wire will almost surely break when reeling in. In depths of less than 1,000 fathoms a lighter sinker can be employed and hauled back.



FIGURE 47.

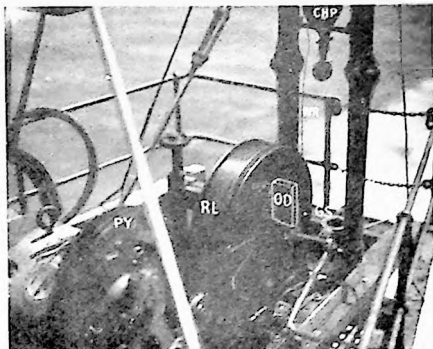


FIGURE 48.

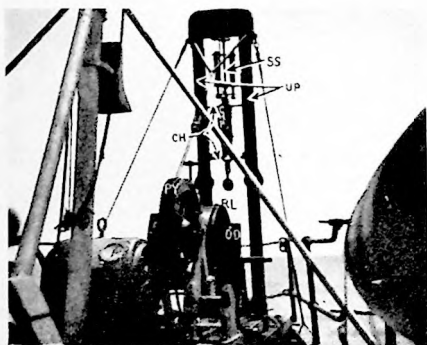


FIGURE 49.

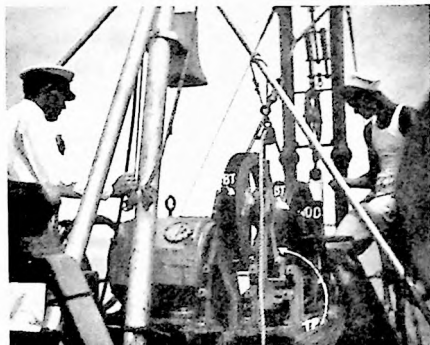


FIGURE 50.

The reeling apparatus consists of a controller (CT, Fig. 47), which operates a series-wound reversible motor, the shaft of which is geared by a stepped-down spur wheel gear to the V-shaped groove pulley (PY, Figs. 47, 48, 49).

When reeling in, the pulley (PY) is belted to the reel (RL, Figs. 48, 49) by means of a belt (BT, Fig. 50) made of 18-thread ratline stuff, which passes around the V-shaped grooves on pulley (PY) and reel (RL), and under the spring tension pulley (TP, Figs. 50, 51) which keeps the belt tight. When letting go, this belt is slipped off clear, and a friction line (FL, Fig. 51) is rigged, as will be explained later.

The reel (RL) is very heavy, being especially made to withstand the compression caused by reeling in turn after turn of wire under great tension. The original Sigsbee machine had a special

strain pulley to take up this tension and deliver the wire to the reel at reduced tension (see p. 72 of *Deep Sea Sounding and Dredging by Sigsbee*), but the machine used by the U. S. S. *Hannibal*

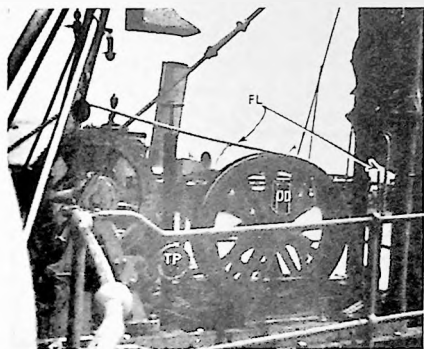


FIGURE 50.



FIGURE 52.



FIGURE 53. Installation on stern of ship.

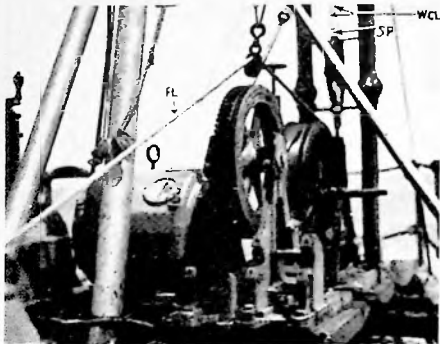


FIGURE 54.

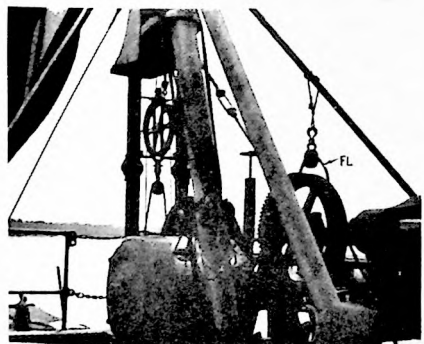


FIGURE 55.



FIGURE 56.

had not; the wire was reeled in directly on the heavy reel. The circumference of the reel is exactly equal to 1 fathom, but since many turns are piled on, the length of a turn gradually increases,

so that a correction curve must be made and employed to indicate true depth. Each reel has a worm gear on its axis engaging the odometer (OD, Figs. 48 to 51), which indicates the number of turns out. The odometer may be disengaged and reset at will. The axle on the other side of the reel is fitted with a ratchet wheel (RW, Fig. 52). By means of the pawl (PL, Fig. 52) the reel may be locked to prevent unreeling. The ratchet and pawl must never be used to stop the reel when it is revolving, nor should it be engaged when the motor is reversed.

The wire (WR, Fig. 48) leads from the reel over the crosshead pulley (CHP, Figs. 48, 49), thence down through the guide sleeves (GS, Fig. 48) and the guiding pulley (GP, Fig. 53) into the water. The crosshead pulley is fitted with a guard to prevent the wire from jumping off. A lignum vitae clamp (Fig. 57) can be fitted into the guide sleeves (GS), and is used to clamp the wire at the guide sleeves whenever it becomes necessary to clear the wire.

The crosshead pulley (CHP) is held by the crosshead (CH, Figs. 48, 49, 52), sliding on guides up and down between the two uprights (UP). The upper end of the crosshead is fitted with a small pulley (SP, Fig. 54) through which is passed the woven cotton line (WCL, Fig. 54). Each end of the line leads over a pulley wheel at the top of each upright and is made fast to

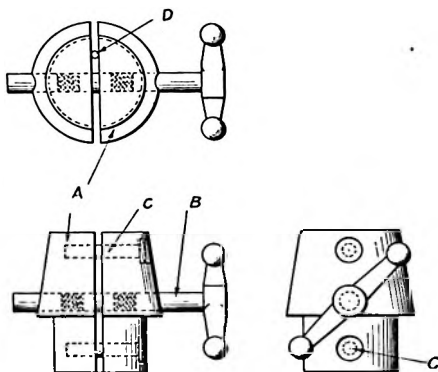


FIGURE 57.—Details of wire clamp.
A, Lignum vitae jaws; B, spindles with right- and left-hand screws;
C, guide bolts, brass; D, sounding wire.

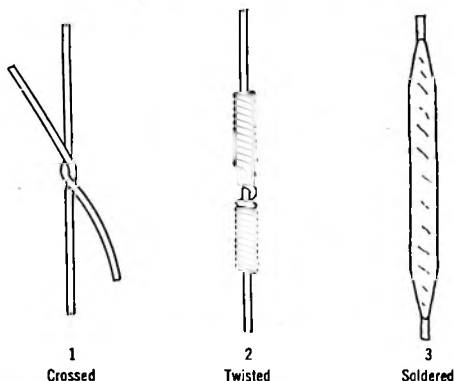


FIGURE 58.—Method of splicing piano wire.

two powerful tension springs, one inside each of the two uprights. These springs are called the **accumulator springs**.

The crosshead thus rides on the bight of the cotton line, and the tension on the two accumulator springs is equalized. For letting go, the tension of the accumulator springs is sufficient, and is used in connection with the friction line (FL, Fig. 51), as will be explained later. When reeling in, the tension of these springs is not sufficient, the crosshead pulley being pulled so low that the wire cannot be guided onto the reel. To overcome this, a special compression spring (SS, Fig. 49) able to support about 400 pounds (200 pounds over pulley) is strapped around the top of the horizontal piece between the two uprights. The lower end of this spring is fastened to the top of the crosshead when reeling in. This keeps the crosshead well up and permits guiding the wire onto the reel, which of course should be done in such a manner as to keep the turns uniform. The special compression spring is unrigged from the crosshead when letting go, and is triced aside to a clear position.

The **friction line** (FL) is made of 18-thread manila. It is made fast to the base of the machine near the left upright (looking from the motor end), whence it leads up through the pulley (PZ, Fig. 56) on the bottom end of the crosshead, thence down through a block (BL, Fig. 56) strapped to the base near the right upright, thence up and around about 60° of the groove on the reel, thence through a block so placed as to give a fair lead over the groove, thence to any convenient place for belaying. In paying out wire, when strain comes on the wire, the crosshead is pulled down against the tension of the accumulator springs, thus easing

the friction line and letting the reel run more freely. When slack comes on the wire, the full upward pull of the accumulator springs on the crosshead is applied to the friction line, thus slowing or stopping the reel.

The wire used is known as piano wire, the usual size being 28 mils. It can be obtained in great lengths from commercial firms. Its tensile strength is about 200 pounds. Lengths of this wire are spliced together to make the length desired. The reel will hold about 6,000 fathoms.

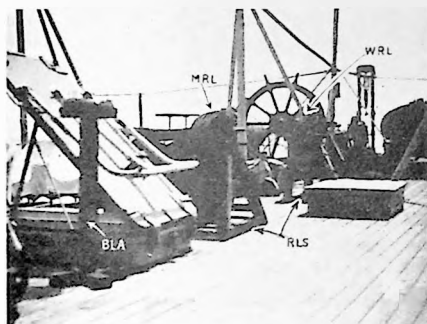


FIGURE 59.

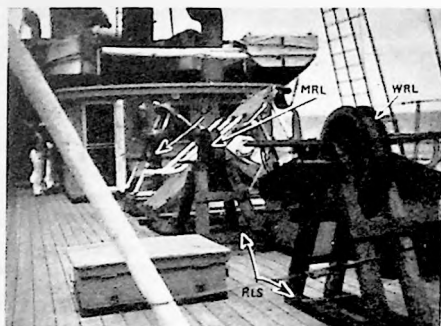


FIGURE 60.

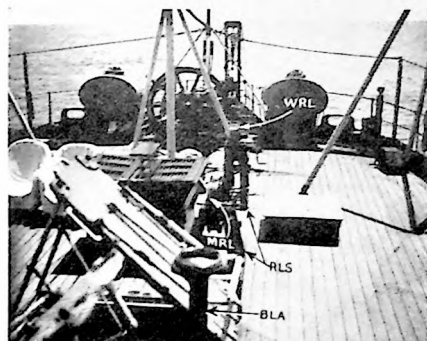


FIGURE 61.



FIGURE 62.

Splices are made as the wire is being reeled onto the working reel. The method of making the splice is illustrated in Figure 58.

Calibrating the wire.—To calibrate the wire, two reel supports (RLS, Figs. 59 to 61) and a blade (BLA) are constructed, the whole arrangement being placed as shown. The wire is led from the coils placed on the blade three times around the measuring reel (MRL), thence to the working reel (WRL). The crank is fitted to the working reel. Two odometers can be rigged—one for the working reel and one for the measuring reel, if they are available. If not, the odometer may be rigged on the working reel, and the turns of the measuring reel may be shown by a revolution indicator. The *Hannibal* used the arrangement shown in Figures 62 and 64. The point of the revolution indicator was fitted into a wooden block the other side of which was scored out square to fit over the square end of the axle.

When all is ready, the crank is turned. A man stationed at the brake applies sufficient friction drag to the periphery of the revolving disk to keep the wire taut. A man stationed at the measuring reel calls out "Mark" at every 100 revolutions (100 fathoms), and a man stationed at the working reel reads the odometer at each "Mark." These readings being recorded simultaneously, a correction table is constructed.



FIGURE 63.

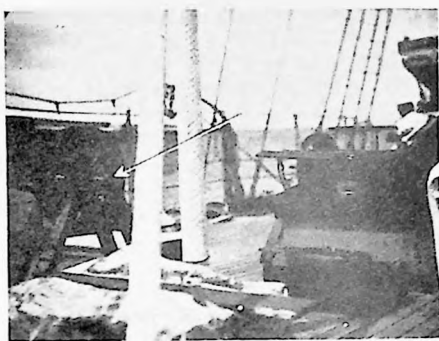


FIGURE 64.

Correction table (sample)

Measuring reel (fathoms)	Working reel (turns)	Correction (fathoms)	Measuring reel (fathoms)	Working reel (turns)	Correction (fathoms)	Measuring reel (fathoms)	Working reel (turns)	Correction (fathoms)
100	100	0	1,800	1,766	34	3,500	3,371	129
200	200	0	1,900	1,862	38	3,600	3,464	136
300	300	0	2,000	1,958	42	3,700	3,556	144
400	400	0	2,100	2,054	46	3,800	3,648	152
500	499	1	2,200	2,149	51	3,900	3,740	160
600	598	2	2,300	2,244	56	4,000	3,832	168
700	697	3	2,400	2,339	61	4,100	3,924	176
800	795	5	2,500	2,434	66	4,200	4,016	184
900	893	7	2,600	2,529	71	4,300	4,108	192
1,000	991	9	2,700	2,623	77	4,400	4,200	200
1,100	1,089	11	2,800	2,717	83	4,500	4,291	209
1,200	1,187	13	2,900	2,811	89	4,600	4,382	218
1,300	1,284	16	3,000	2,905	95	4,700	4,473	227
1,400	1,381	19	3,100	2,999	101	4,800	4,564	236
1,500	1,478	22	3,200	3,092	108	4,900	4,654	246
1,600	1,574	26	3,300	3,185	115	5,000	4,744	256
1,700	1,670	30	3,400	3,278	122	5,100	4,834	266

This correction table is used as follows: Suppose after sounding, the odometer shows 3,123 turns off drum.
4,834 turns on (working) reel equal..... 5,100 fathoms

3,123 turns off, odometer reading

1,711 turns left (correction 32 fathoms) equals..... 1,743 fathoms

Depth equals..... 3,357 fathoms

The end of the wire is spliced to the **stray line**, which consists of from 3 to 5 fathoms of 9-thread manila. The method of splicing the wire to the manila is shown in Figure 65.

The other end of the stray line is spliced into an eye splice about 3 inches long, which is slipped through the ring and down over the end of the sounding rod to secure the latter. Strips of sheet lead totaling about 5 pounds should be wrapped snugly around the stray line near the eye splice to assist in detaching the sinker.

SOUNDING ROD

The details of the sounding rod are shown in Figure 66.

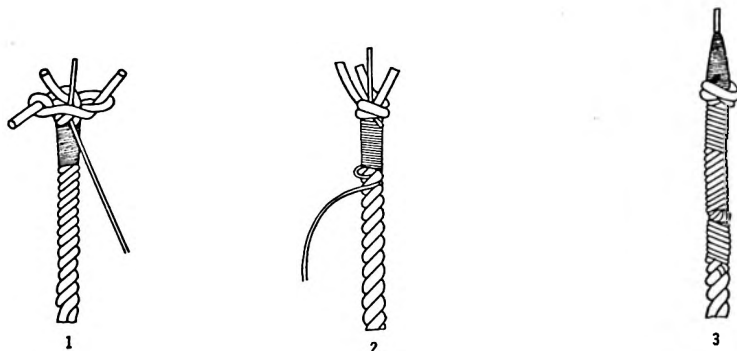
To use the sounding rod to sound and bring up a specimen of the bottom, attach the stray line, which should intervene between the wire and the sounding rod, to the swivel ring (K, Fig.

66), pass the rod through the hole in the sinker, hook the wire bail (R) over the lip of the tumbler (M), lock the pawl and tumbler, and suspend the rod from the swivel ring (K), when it will promptly assume a vertical position. The cone (I) remains unseated during its descent by contact with the shot, and the valve (F) is raised by the upward pressure of the water, which then circulates freely through the cylinder, reducing its resistance and increasing its rate of descent.

When the sinker strikes the bottom, the tension of the line is relieved and it becomes more or less slackened; the pawl assumes a horizontal position by its own weight, releasing the tumbler, which is thrown out of action by the spring (N), assisted by the excess of weight at the lip; and thus the sinker is released.

The combined weight of shot, sounding rod, and small lead, about 70 pounds, descending at the rate of 8 or 10 feet per second, forces the rod well into ordinary soils, lifts the valve (F), and fills the cylinder to a greater or less extent with a bottom sample. The reverse motion, when the ascent begins, promptly closes the valve and protects the contents from wash to which it would otherwise be subjected. The sample is readily removed by disconnecting the cylinder at the screw joint (B), which also facilitates the cleansing of the cup.

The 60-pound spherical shot of cast iron is the **standard sinker** in use on board United States vessels for deep-sea work. It is about 8 inches in diameter and has a 2¼-inch hole through



1
Single wire knot, with the wire tacked through it

2
The knot jammed. The wire wound against the lay of the rope, and tacked again for reverse turns.

3
Completed. The wire wound twice against the lay of the rope and once with the lay.

FIGURE 65. Method of splicing wire into stray line.

its center. Small eyes or lugs are cast upon opposite sides of its upper exterior surface, to which the ends of the wire bail are secured. It is possible that in exceptionally deep water, where the weight of the wire largely exceeds that of the standard sinker, or when from any cause the wire cannot be maintained in a vertical position, a 75- or even an 80-pound weight could be used to advantage. It must be considered, however, that the wear and tear and liability to accident increases with the size of the sinker.

The wire bails now used with **detachable sinkers** are of annealed iron wire, no. 8, American wire gage, and may be conveniently fitted by first cutting them from the coil in uniform lengths and then bending them over a form to insure a free and uniform seat of the lip of the tumbler.

Sinkers should not be bailed until they are required for use. When the requisite number have been placed in the racks, proceed first to examine the lugs and see them in good condition, then pass the sounding rod back and forth through the hole, and when satisfied that it moves freely, let it remain in the sinker with its weight resting on the base of the hollow cone; pass the ends of the bail through the lugs, put the bight over the tumbler (which for convenience has been placed in action and secured by a seizing of twine); then bend the ends up and take a couple of turns around their standing parts, leaving sufficient slack to unhook the bail without displacing the tumbler. This procedure will insure the proper length of bail, suspending the shot high

enough to lift the hollow cone off its seat without bringing the cone's apex in contact with the shoulder at the upper end of the stem of the sounding rod. Though it is not absolutely necessary to hold the cone off its seat during the descent, by doing so the rate of sinking of the shot is increased somewhat.

Caution should be observed, when fitting the bails, that no projecting scraps of iron are left in the hole or at either end of it, and that all scale is removed from its walls, for it expands

and becomes softened when it is wet. At the same time the material of the sinker shrinks under low temperatures near the bottom, both tending to crack and disengage the scale, which is liable to jam the sounding rod irretrievably.

Calibration of crosshead.—

Before sounding, the accumulator springs inside the uprights supporting the crosshead are calibrated. This is done by hanging known weights over the stern by the stray line leading over the crosshead pulley. A mark is placed on the right upright on the long scale opposite the crosshead for each weight used. Other points of the scale should be interpolated. The special compression spring for reeling in is calibrated in the same manner, and the short scale on the left upright is marked accordingly.

Position of ship.—In taking soundings the ship should be hove to in such a manner as to give minimum drift from wind and sea. Ordinarily this is stern to wind. If there is much drift, sounding becomes difficult. With much drift it is almost impossible to tell when the sinker strikes bottom. The ideal situation is obtained when the wire tends right up and down. Rudder and propellers can be used with caution to maintain position.

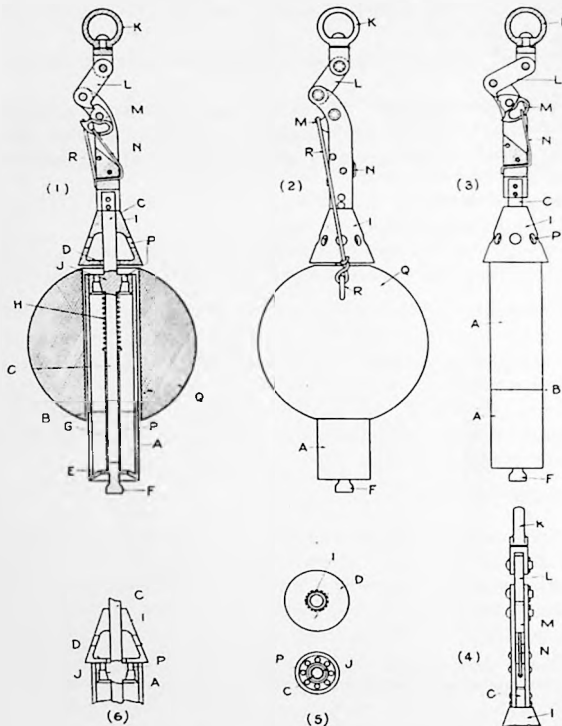


FIGURE 66.—Sigsbee sounding rod.

1, Longitudinal sectional elevation; 2, side view, sinker suspended from detachable crosshead by wire bail; 3, plan of cylinder and sectional elevation of detachable crosshead; 4, back view of detachable crosshead; 5, perforated plate J and cylindrical ring D; 6, detail of hollow cone I, cylindrical ring D, holes P, and cylinder A. A, cylinder; B, screw joint; C, upper and lower guide stem; D, cylindrical ring; E, valve seat; F, poppet valve; G, valve stem; H, spiral valve spring; I, hollow cone; J, perforated plate; K, swivel; L, pawl; M, tumbler; N, spring; P, holes for escape of water; Q, sinker; R, iron wire bail.

If there is much drift, a spar drift buoy should be dropped over when sounding is started, and its distance taken when the sinker strikes bottom. This procedure permits the vertical depth to be computed.

Procedure.—To take a sounding—

(a) Rig the machine for letting go. Engage ratchet and pawl so that the reel will not pay out. Hold the friction line in hand. Set the odometer at zero. See all clear and properly rove. Hang sounding rod and sinker over the stern.

(b) Grasping the outside of reel, heave in a few inches manually, until the pawl clears the ratchet and can be thrown back. Pull strongly on the friction line.

(c) Let go of the reel, easing the friction line sufficiently so that weight descends slowly to the water's edge.

(d) Release the pull on the friction line slowly until the crosshead rises to about 75 pounds. Belay end of the friction line. The sinker will descend. Mark the time.

(e) Call out each 100 turns and note the time. The rate of descent varies for different depths and conditions, but should average about 100 fathoms per minute. When the sinker strikes bottom, the reel will slow down to very slow speed or will stop entirely, depending upon the ship's drift.

(f) When the sinker strikes bottom, throw off the friction line, well clear, and put on the belt. Back off a few turns with the motor, insuring sufficient slack to trip the sinker. Watch carefully for kinks.

(g) Raise crosshead and fasten it to the special compression spring, the machine is now rigged for heaving in.

(h) Heave in slowly, avoiding undue strain on the wire. The rate of ascent should be approximately the same as the rate of descent. The scale on the left upright for the special compression spring should be closely watched, and the strain should never be allowed to exceed 100 pounds.

ACCIDENTS

The friction line parts.—Stop the reel by seizing it with the hands about the flanges of the drum, but not by throwing the pawl into the ratchet wheel. Against the latter it is necessary to caution untrained men.

A ridge appears in the coil of wire when paying out.—This results when the wire has been carelessly wound in reeling back from a previous sounding. If the ride begins to slip (as it probably will) and to slack some of the turns, stop the reel. Clamp the wire at the guide sleeve, using the lignum vitae clamp. Cut the wire near the reel. Wind the wire from the working reel to a spare reel until the ridge is reduced; then wind back to the working reel, cutting out all defective parts. When a ridge once begins to show slack turns, a persistence in paying out will cause the wire on the reel to kink in many places.

ECHO SOUNDING

In echo sounding, depths are measured indirectly by noting the time interval required for sound waves to go from a source of sound near the surface to the bottom and back again. In the various stages of development of echo sounding apparatus the sound has been produced by bombs, dynamite cartridges, shotguns, rifles, cannon, and hammers on the hull, and the echo has been caught by ear or by microphones. In 1921 an ultra-sonic method depending on high-frequency vibrations was developed, and this in turn has since then been improved to the point where it is possible to obtain accurate echo soundings in as little as one fathom of water. Simultaneously methods of recording soundings graphically were developed. Figure 67 illustrates a record of this kind.

For full accounts of the development of sonic methods, including descriptions of instruments and methods, reviews, and bibliography of professional papers and of reports of oceanographic expeditions, see successive numbers of the Hydrographic Review since 1923, also Special Publications 3, 4, and 14 of the International Hydrographic Bureau. For a brief theoretical discussion of the velocity of sound in sea water see Part III of the tables known as H. D. 282, published by the British Admiralty in 1927. For a discussion of errors and difficulties encountered in wire and sonic sounding see the current preliminary reports of the Snellius Expedition.

As may be discovered by examining H. D. 282, the velocity of sound in sea water of temperature 0°C ., of density 1.028, and of salinity 34.85 parts in 1,000, is about 1,445 meters per second near the surface of the sea. It increases about 3.3 meters per second for every 100 fathoms of increase of depth, and 1.3 meters per second for every part in 1,000 of increase of salinity. This increase of velocity with depth, however, is opposed by a decrease due to the accompanying falling temperatures. The net result, in most oceanic regions, is a velocity decreasing with depth in the upper layers, where temperature changes rapidly, but increasing with depth in the lower layers, where temperature changes slowly.

In calibrating a sonic apparatus it would be impracticable if not impossible to provide for a varying velocity. Therefore single nominal velocities are used, approaching velocities in average

depths. The **calibration velocities** in most common use are 800 fathoms or 1,463 meters per second, used by American ships, and 814 fathoms or 1,490 meters per second, used by Netherlands and German ships. A recent type of shallow-water apparatus designed for depths up to 20 fathoms is calibrated for 820 fathoms or 1,499.6 meters per second. The precise velocity calibration figure is of small importance, since it may be reduced to any other velocity by simple proportion. As illustrating this, and also the high salinity of the Red Sea, witness the following note on a recent chart:

Depths in the Red Sea obtained with echo apparatus calibrated on the basis of a velocity of sound of 4,800 feet per second should be increased by 5% to agree with the charted depths.

In **charting echo soundings** two schools of thought have arisen. The first desires a general agreement among nations to correct the observed echo depths for the varying velocities of sound in sea water at different depths, densities, salinities, and temperatures, in accordance with tables computed from the known physical properties of water. The second desires that the **crude soundings** be charted, at least in depths over 100 fathoms and in the general ocean routes, reasoning that the crude soundings are recoverable, without troublesome corrections, by all mariners using an echo apparatus, whatever may be its calibration velocity, provided that the original calibration velocity is stated on the chart.

The sonic soundings usually taken by a survey ship in the course of a season fall into three classes, namely, soundings taken on or near the hydrographic shelf of the survey area; dynamic soundings extending outward from the shelf or in other designated areas; and ocean track soundings taken during trips to and from the home port.

The **shelf soundings** are intimately connected with the shallow-water soundings of launches and tenders, and in fact should cover the same area with a generous overlap, for the double purpose of checking the hydrography and of making a **table of corrections** to sonic soundings by comparison with wire soundings, accompanied, at a sufficient number of stations, by observations of salinity and temperature. At certain depths, depending on the weather, wire soundings may cease to be reliable, because of the drift of the ship, and may have to be rejected. The deficit of dependable wire soundings in deep water, say up to 1,000 fathoms, may perhaps be remedied by special deep-water soundings taken in calm weather. With a dependable measuring sheave of large diameter, calibrated, check soundings with wire are thought to be desirable even beyond 1,000 fathoms. The comparisons up to 100 fathoms should be made at about 25-fathom intervals, and beyond that depth at every 200 fathoms. They should be considered as an essential part of the daily routine of sonic sounding.

For testing and correcting echo soundings up to 50 fathoms, the *Nokomis*, in 1935, used a special test lead line marked at every fathom and carrying a 50-pound lead. During sounding, every morning and every afternoon, as nearly as possible where the depths were 20, 30, and 45 to 50 fathoms, the ship was brought dead in the water, and echo soundings were observed continuously until the lead line came to the up-and-down position. The corrections were recorded in the soundbook, and corrections to the echo soundings were deducted. They ranged between +2.5 and +3.0 fathoms, depending on the draft of the ship. With the corrections applied the echo soundings checked exactly, or to the nearest fathom, with the hand-lead soundings of the launches in their depth limits of 20 to 30 fathoms.

After the completion of the sonic work it is well to run an over-all check line as far seaward as good fixes can be obtained from the triangulation system. During the deep-water tests **bottom specimens**, which would otherwise not be obtained in an all-sonic area, may be obtained by arming the lead.

As to the other two classes of sonic soundings mentioned, partly or wholly outside of the survey area proper, it is assumed that special instructions will be given for the dynamic soundings, often taken in collaboration with other organizations; and that the ocean track soundings will be submitted uncorrected, leaving the office to correct them, if desired, either by the tested behavior of the apparatus on the survey grounds, or by laboratory tables, or by such regional tables as are available. The carefully made tests and dynamic soundings taken during the season go toward supplementing and correcting such tables.

In the **interpretation of echoes or echographs** it must be considered that when a train of echoes under water is received the direction of the echo point from the receiving point is uncertain. The uncertainty may be reduced by directing the emission of waves vertically downward in a narrow cone, in which case a sharp declivity or a pinnacle will be indicated by a weakening or

a scattering or a total loss of echoes. But other factors may enter, analogous to those which change the direction of the propagation of sound in a fog. The echo point, or source of the most distinct echo in a train, is assumed to be the nearest reflecting surface, for ordinarily that would be the echo corresponding to the least loss of energy in the outgoing and returning waves. In great depths, however, where there is a choice of paths of different densities and a considerable variety in the materials of the reflecting surfaces, it may happen that the most distinct echo has not arrived by the shortest path.

The echo point is not a mathematical point, nor merely a plane surface. Rather, it is a series of laminae all of which contribute to the character of the echo. It is even possible to

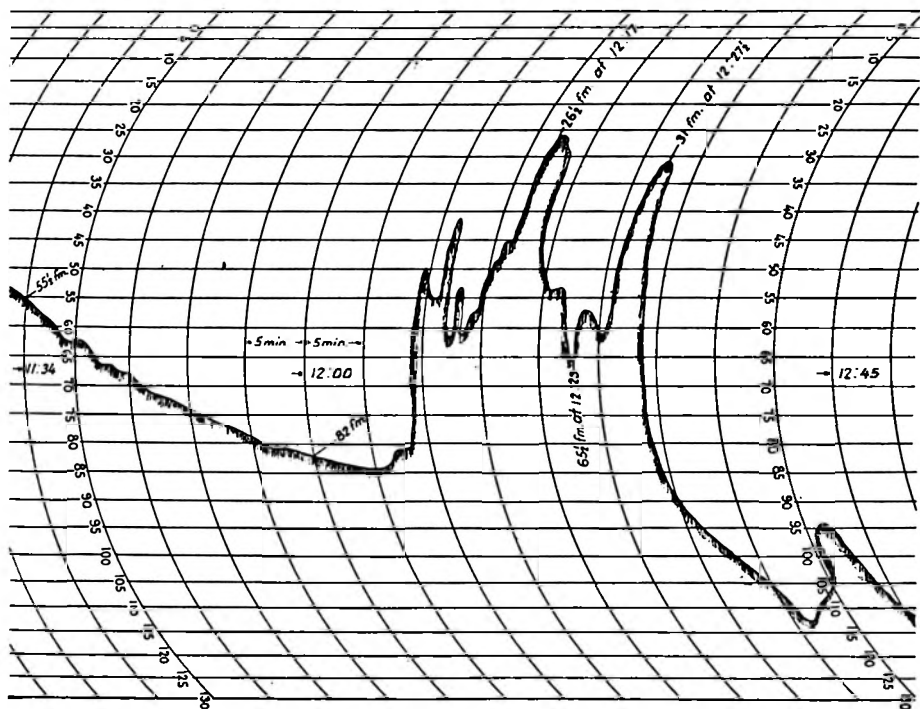


FIGURE 67.—Echo-sounding graph.

judge from the graphical records of echoes what is the general nature of both surface and underlying materials of the bottom.

In deep water, with sound waves going out in all directions, it is possible that an echo received may have come from one side or the other of the ship, or from a point up-slope in any direction, or from the face of a terrace or cliff, or from any sizable irregularity of the bottom. Another factor to be considered is the impedance or absorption of echoes by the hull of the ship, or by water passing under the ship, or by air bubbles in the wake. With a receiver in the forward part of the ship, if a line of soundings were run up-slope and then re-run down-slope, the latter might be expected to register greater than the former.

The **slope of the sea bottom** in great depths rarely exceeds 20° . In gentle slopes the displacement of the echo point from the receiving point, and also the reduction in apparent depth due to shortening of distance, are of small consequence. Corrections of this kind, if applied, would displace not merely the observed positions of individual soundings, but fathom curves. A 20° slope would displace a 30-fathom curve about 60 feet, and a 1,000-fathom curve about 2,000 feet,

in a direction up-slope. The only way by which the angle of slope and its direction could be determined would be by scaling cross sections on the completed sheet—a troublesome and unsatisfactory process.

A period of training is necessary for the successful operation of echo apparatus, especially the type of apparatus for deep water that requires the operator to mark by eye the arrival of an echo caught by ear. There is an appreciable reaction here consisting of two parts, one due to the personal equation of the observer, the other due to the distinct or indistinct character of the echo, in general a function of the depth. Differing personal errors are most likely to become manifest at the times of changing watches.

Figure 67, which is two-thirds original size, shows a portion of a continuous graphic record of an echo-sounding apparatus. These graphs are particularly valuable for obtaining least depths where the bottom is sharply irregular, as the profile of the figure indicates.

CHAPTER V

TIDES AND CURRENTS

TIDE STATIONS

The determination of the constants of nature that explain and interrelate local and regional tidal phenomena has an importance transcending the immediate and obvious use of the observations for reducing soundings to a datum plane.

The fundamental yearly and seasonal values can be obtained only by the continuous employment of a principal **base station** for at least a year, and preferably for the whole period of the survey. Otherwise, if observations are discontinued between survey seasons, periods which usually correspond to months of unfavorable weather and to changes in the prevailing direction of the trade winds, the omission of essential elements may lead to an erroneous determination of the plane of reference and to misinformation relating to currents.

With a base station in continuous control, local values in harbors, in outlying channels, and off stretches of the coast, may be obtained conveniently during the period of sounding in the vicinity through observations covering a single lunar month at **local stations**; and the results of such short period observations may be generalized and corrected by referring them to the base station.

The nature of the control and coordination of local values by a base station may be illustrated as follows:

Suppose that in the course of a season's survey, on a north coast in the belt of trade winds, local observations are taken at *L* in November, during a period of strong northeast trades and at *M* in March with the contrary trades prevailing; and that a base station *B* is in operation during the whole period. The automatic records at the base station show that the mean of the November low waters is higher than the yearly average, while the mean of the March low waters is lower than the average. From these facts, and from a comparison of ranges as well, the mean low waters obtained from short period observations at the local stations may be corrected and coordinated. For the chart and for sailing directions, it is desirable to relate all tidal results at local stations to those at the base station, and to furnish a summary of meteorological facts that cause seasonal variations from the mean, with their effects in various localities, especially in channels and approaches.

For satisfactory results, both base and local stations require a type of tide indicator that gives a continuous automatic graphical record.

A third kind of tide station, which may be called a **daily station** because it is moved from day to day and kept within sight of the work, is used in connection with sounding, sweeping, dredging, and rock boring in rivers and channels. It usually consists of a graduated "staff" or board watched by an observer or observed through field glasses. The zero of the first staff is compared with the zero of the staff at the nearest local station by simultaneous observations, and thereafter elevations are transferred to other staffs by water level.

STATION SITES

Other things being equal, it is well to establish the base station in a central port or harbor and in a site accessible from both land and water, subject to the full effect of the tides without undue exposure to wave action, and in the case of marine surveys, outside the influence of rivers. For local stations, proximity to the anchorage and a depth of water sufficient to permit landing at the tide house by launches at all stages of the tide, are considerations. At daily stations, the staff is usually spiked to the pile of a beacon or to a pole driven into the mud in the lee of a shoal.

AUTOMATIC TIDE GAGES

The automatic **printing** tide gage, Figure 68, is a device for printing corresponding times and heights on a ribbon wound on a drum and advanced by clockwork. At regular intervals, usually every 15 minutes, the height mechanism, actuated by the float, stamps the figures of the height of tide on the ribbon.

The automatic **graphical** recording gage, Figures 69 and 70, developed by the United States Coast and Geodetic Survey, traces a continuous curve called a **marigram**, of which the abscissae

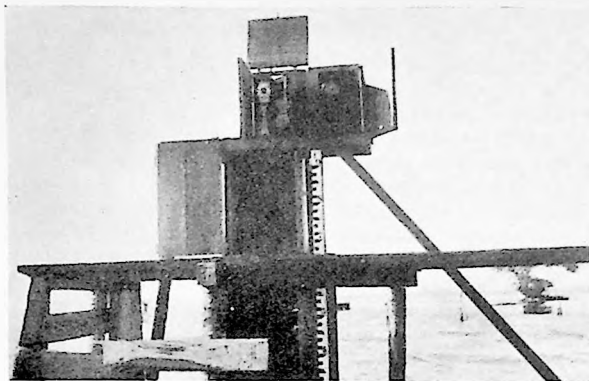


FIGURE 68.—Automatic printing tide gage.

are proportional to the lapse of time and the ordinates are proportional to the height of the tide. The paper is carried forward at a uniform rate by clockwork over a drum on which rests a pencil

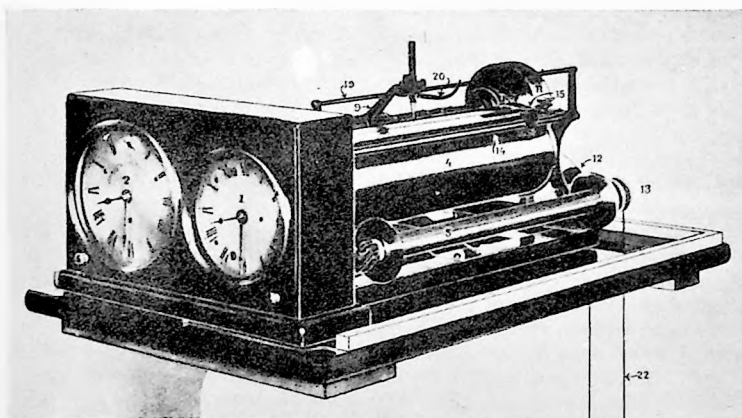


FIGURE 69.—Automatic tide gage, front and clock end.

in a carriage moved across the paper by a worm gear rotated, through reduction gears, by a float rising and falling with the tide.

The following descriptive matter is abridged from Special Publication No. 26 (second edition) of the Coast and Geodetic Survey.

In the cut, clock no. 1, on the right, is the **motor clock**, and clock no. 2 is the time clock. The motor clock drives the main cylinder, which is 12 inches in circumference, at the rate of 1 revolution in 12 hours, thus drawing the paper forward at the rate of 1 inch per hour. To prevent

slipping of the paper, the cylinder is armed near each end with steel pins. The motor clock has two mainsprings, both connected with the driving apparatus. In case of one breaking, it is sometimes possible to operate the machine with the remaining one, pending repairs.

The **time clock**, not an essential unit of the apparatus, marks the hours on the record. One spring runs the clock, while the other operates a device that trips the recording pencil, making a short horizontal mark on the record each hour.

The clocks will run 8 days on one winding, but it is better to wind them twice a week. They may be corrected and regulated in the ordinary way, except that, to avoid injury to the hour-marking device, the minute hand of the time clock must not be turned backward between 11:50 and 12:05.

Rollers numbered 3, 4, and 5, in the cuts are, respectively, the **supply roller**, the **main cylinder**, and the **receiving roller**. Paper $13\frac{3}{4}$ inches wide and 66 feet long, enough for a lunar month, is supplied in rolls. The machine is loaded by removing a flange from the end of the supply roller, replacing the flange and the roller, feeding the paper over the main cylinder, inserting the free end in a slit in the receiving roller, and giving the latter a turn or two, keeping the paper

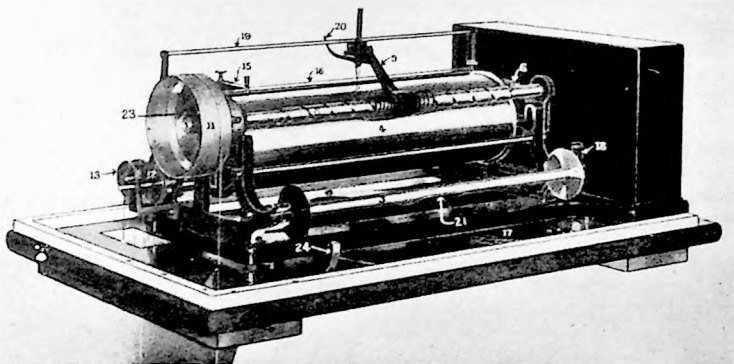


FIGURE 70.—Automatic tide gage, rear and float end

smooth and taut. This roller, also, has a removable flange, permitting the record roll to be slipped off at the end of the month.

The axle of the hour hand of the motor clock extends through the back of the case, and carries a toothed carrier wheel (6, Fig. 71). The main cylinder has a hinged carrier arm attached to its axis. **Connection between the clock and the cylinder** is made by throwing this arm over until it fits between two teeth of the carrier wheel.

The **tension weight** is the smaller of the two weights furnished with the machine. It is attached to a cord (22) wound around the tension pulley (13). The latter is provided with a pawl and ratchet for winding up the tension weight from time to time. The action of the tension weight winds the tide roll onto the receiving roller, keeps the paper on this side of the machine taut, and assists the motor clock in turning the main cylinder.

The **tension spring** (21) presses against the supply roll of paper and keeps the paper on that side of the machine taut. As the paper is held by a set of sharpened pins, an excess of tension on either side of the cylinder is likely to cause the paper to tear, in damp weather.

The **float** is a copper cylinder $8\frac{1}{2}$ inches in diameter and 3 inches high, and is weighted to float one-third above water. It is connected with the float pulley by no. 23 American gage phosphor-bronze wire. The float rises and falls with the tide in a float box to which the water has access through a small opening.

A set of four interchangeable **float pulleys** (11), with circumferences of 6 inches, 9 inches, 12 inches, and 16 inches, is furnished with each machine to adapt it to various ranges of tide. They are about an inch wide and have threads cut in the faces to prevent the float wire, one

end of which is attached near the edge of the pulley, from winding upon itself. There are from 18 to 24 turns of the thread on each pulley. For the removal or adjustment of the pulley, there are two clamp nuts (23), which are set by means of a special wrench (24). This pulley and the counterpoise pulley are attached to the worm, causing it to turn as the tide rises and falls.

The **counterpoise pulley** (10), which is threaded like the float pulley to which it is clamped, carries a wire or cord to which the counterpoise weight is attached.

The **counterpoise weight** is the larger of the two weights furnished with the tide gage. Attached by a wire or cord to the counterpoise pulley, it serves to take up slack in the float wire and rewinds the latter as the tide rises.

The function of the **sliding grooved pulley** (12) is to carry the counterpoise cord away from the float wire and to keep the wire, as it winds or unwinds, opposite the proper thread on the counterpoise pulley. When the counterpoise cord is led directly to a fixed pulley in the ceiling of the tide house, the sliding pulley is unnecessary.

The **pencil arm** (9) carries the pencil at one end and a bearing at the other in which is a nut engaging the pencil screw. The motion of the nut and arm is toward the clocks for a rising tide and away from the clocks for a falling tide. If a very high or a very low tide moves the arm to either end, the nut runs off the thread, thus preventing jamming. When the tide begins to reverse, springs on either side of the pencil arm force the nut back on to the thread.

The **pencil screw** (8) is made of phosphor-bronze about $\frac{3}{8}$ inch in diameter, and has a square thread with a 1-inch pitch. For stations having a large range of tide, a pencil screw with a one-half inch pitch is frequently used. The threads at the ends of the pencil screw are turned down to prevent the pencil arm from jamming.

The **datum pencil holder** (15) may be clamped in any position on the datum pencil rod (14), preferably near the middle of the latter.

The following table indicates the proper pulleys and pencil screw for various scales of height:

Extreme range	Scale ratio	Float pulley circumference	Pencil screw pitch
		<i>Inches</i>	<i>Inch</i>
Less than 6 feet.....	1:6	6	1
6 to 9 feet.....	1:9	9	1
9 to 12 feet.....	1:12	12	1
12 to 16 feet.....	1:16	16	1
16 to 18 feet.....	1:18	9	$\frac{1}{2}$
18 to 24 feet.....	1:24	12	$\frac{1}{2}$
24 to 32 feet.....	1:32	16	$\frac{1}{2}$

When setting up or inspecting the gage, rough comparisons between the curve and the staff may be made by means of a fork (17) sliding along a metal scale (16) corresponding to one of the foregoing ratios. Heights thus transferred must not be recorded on the tide roll, lest they be mistaken for true readings of the staff.

The **hour-marking device** is shown in Figure 71. On the hour, the time clock moves a lever (8) connected with a tripping rod (19) supported above the metal scale on rocker arms, and engaging a hook on the pencil arm, which is jointed. The effect is to cause the pencil to make a short stroke parallel to the edges of the paper. A spring returns the pencil to position.

TIDE HOUSES

The essential units of an automatic tide gage station are an elongated box or tube in which the float is free to move up and down with the tide, a platform above the tube for supporting the automatic gage, a staff gage, and a water-tight padlocked box to cover the instrument. A tide house may be required for additional shelter in stormy weather. The Coast Survey design, Figure 72, is suitable for wooden structures where there is a moderate sea and for various

ranges. On exposed coasts where there are heavy seas and a large range of tide, iron structures may be used. Figure 75 shows a double installation by the U. S. S. *Niagara* in the Gulf of Panama. The picture was taken at about half tide.

The water gains access through small holes in the sides, if the tube stands in mud, or through a hole in the bottom if the tube is raised. The former type is preferred for strength and the latter for ease in clearing the hole when fouled. A tube made of rough lumber sometimes requires no openings except the cracks at the edges. Openings that have been made must sometimes be choked down with strong coarse cloth.

Figures 73 to 77 show installations of automatic continuous-recording tide gages suitable for various ranges of tide.

SETTING UP THE GAGE

The gage is usually set directly over the top of the float tube, and adjusted with the pencil near the middle of the main cylinder at half tide, with the float pulley and the counterpoise pulley each about half filled with wire or cord (varnished fish-line). The datum-line pencil is preferably set near the middle of the main cylinder, in order to minimize errors due to absorption of moisture by the paper.

One or more fixed pulleys are placed overhead in such positions as to carry the counterpoise to one side of the house, out of the way. The cord from the counterpoise pulley is led upward over these pulleys, then down through a movable pulley to which the counterpoise is attached, then upward to a fastening in the ceiling. The line will be wound upon the counterpoise pulley by the descent of the float to the water.

The number of feet, L , of wire required for a float pulley C feet in circumference having T threads half of which are to be filled, is $L = \frac{1}{2}TC + L'$, L' being the length of wire needed between the pulley and mean sea level.

Unclamp the float pulley by barely loosening the nuts. Free one end of the wire on the spool, and while preventing the turns of wire from springing off the spool, pass the end through a small drill hole near the edge of the pulley and secure it. Wind the wire off the spool and on the pulley, the whole length L , or as much of it as possible. Attach the free end to the float, and with the counterpoise cord entering its pulley, lower the float gradually to the water. Adjust the gage to insure that the float may pass all the way up and down the tube without touching the sides, and secure it in this position.

Adjust the pencil arm to be near the middle of the paper at half tide, using the metal scale, if necessary, to allow for the difference between the actual height of the water and that at half tide. If the arm is far out of position, a first approximate adjustment should be made by running the arm to the end of the pencil screw nearest the clocks, thus disengaging it from the thread. This is done by rotating the float pulley. The pencil arm should be held at this end and the float pulley turned either backward or forward until it lacks three turns of being filled with the float wire. The pencil arm is then released and the float permitted to descend to the water. This should bring the pencil arm within an inch of its proper position. To bring it closer, press the counterpoise pulley with one finger to prevent the weight from turning it, a finger of the same hand resting on the last coil of wire on the float pulley to hold the wire in the grooves. With the other hand, slightly unclamp the two set screws within the float pulley. Revolve the counterpoise pulley until the pencil is in the desired place. Finally reclamp the float pulley.

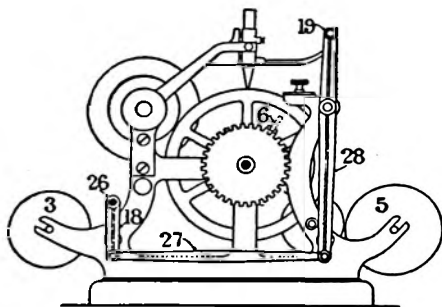


FIGURE 71.—Hour-marking device.

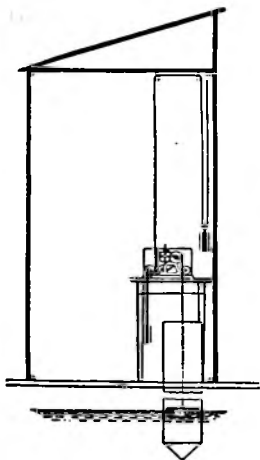


FIGURE 72.—Sectional view of tide house.



FIGURE 73.—Tide house suitable for great ranges of tide.



FIGURE 74.—Tide house for small ranges.

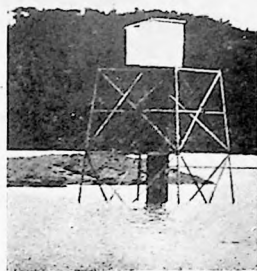


FIGURE 75.—Two gages installed at an exposed station.



FIGURE 76.—Assembly of tide house on forward deck of ship.



FIGURE 77.—Transport of tide house by use of a catamaran



FIGURE 78.—Temporary tide staff to be read from a distance with binoculars

The tension weight is attached by a cord to the pulley at one end of the receiving roller. As with the counterpoise cord, it is desirable to lead this cord over a pulley in the ceiling to provide space for the falling of the weight. The motion of the latter may be halved by using a movable pulley.

STARTING THE GAGE

Before loading the supply roller, record on the inside of the end of the paper the name of the station, the date, the meridian of time used, and the name of the observer. The supply roller is the one that has the solid rod. Remove it, take off the removable flange, slip on the paper, replace the flange, and return the roller to its bearings with the free end of the paper leading from below inward toward the machine. Disconnecting the clock, pass the paper over the main cylinder and insert the end in the slot of the receiving-roller. Secure the paper by giving the roller a few turns in the direction required to receive the paper at the top of the roller.

Wind up the torsion weight, wind and set both clocks, and connect the motor clock with the main cylinder.

Clamp the datum-line pencil holder near the middle of the main cylinder. Adjust the datum-line pencil and the recording pencil. Usually 2H pencils are best for the purpose, but softer pencils may be better in damp weather. They should be short pointed and not too sharp.

The worm screw must be clean at the outset, and must be kept clean by wiping it with gasoline from time to time.

The hour-marking device should be adjusted so that the hook just clears the rod.

OPERATING TROUBLES

The best way to head off operating troubles is to watch the gage for several hours when it is first set up and to visit it frequently during the first high and low waters. If there is excessive vibration of the pencil with a choppy sea, the water openings in the tube should be choked down. A little vibration, however, is favorable, for an absence of vibration may be due to a lack of sensitivity which may result in a failure of the apparatus to register the full range of tide.

If the screw is dirty, the pencil will show a tendency to lift on a falling tide and to dig in on a rising tide.

If daily visits to the gage are contemplated, and if the hour-marking device fails to work perfectly, or if the clocks do not run together, both keeping good time, it will pay to forego the doubtful advantage of this device, and to mark the hours between daily notations by using a diagonal scale somewhat longer than that of the machine, together with a scale 1 hour in length divided into 5-minute spaces. A paper scale about 38 inches long divided into 36 equal parts is convenient for the purpose. By this method, the observer merely marks the correct time on the record each day, and ignores the errors of the time clock.

The following notes come from the experience of the U. S. S. *Hannibal* in a region of small ranges, where tide houses were unnecessary.

The Coast Survey type of gage will not function properly unless the base of the recording apparatus is level. Use a carpenter's level when constructing the base.

Varnished trout line for the counterpoise weight is more satisfactory than the wire furnished with the machine.

To reduce wave action, it has been found advantageous at some stations to use, instead of the larger float, a float of equal displacement but of smaller diameter.

Particular care must be taken to provide clearance between the box cover and the end of the float pulley.

An excess of shot in the counterpoise will prevent proper submergence of the float and will result in erratic action of the gage.

To reduce the travel of the tension weight, the counterpoise has been substituted for the tension weight, and two sheaves 1 inch in diameter have been placed in a brass housing on the new tension weight.

DAILY VISITS TO THE TIDE GAGE

The purpose of daily visits is to keep the machine in good working order and to provide data for measuring the record when completed. More frequent visits are advisable when the

machine is first set up, in order to catch as many high and low water stands as possible, for it is by measuring the ranges on the record and comparing the best of them with the corresponding staff ranges that the actual working scale of the machine is obtained. For various reasons, this working scale, which is the true scale to be applied in tabulating heights, is seldom exactly equal to the nominal scale for which the machine is set.

The true scale and the height of the datum line cannot be found with precision, nor are they needed, until the completed record has been removed. It is important, however, to record the observations in a form designed to facilitate measurements. As well as not, the observations may be recorded on an even 5 minutes of the correct time, and the time line, made by rotating the screw by hand, may be made several inches long, facilitating time measurements between two such lines a day or more apart. On one side of the line, record the date, the correct time, the staff reading, and the state of the weather. On the other side, record the clock readings and other notes, thus:

June 6, 1924, 8:15 a. m.; S. G., 2.52; light S. E. breeze
(T. C., 8:20; M. C., 8:18)

When the record is continuous, the tabulator measures between the time lines of successive days, ignoring the time clock and motor clock notations. If the record is interrupted, it is then necessary to use the marks made by the time clock, or the pin pricks made by the cylinder driven by the motor clock, as reference points, applying the correction to true time and the rate for the day.

June 6, 1924, 8:25 a. m.
(T. C., 8:25; M. C., 8:25; corrected)

If the motor clock is disconnected for any purpose, a new initial time line is required.

With practice, it is possible to read the staff within 0.02 foot even in a choppy sea. Readings less precise than this, being of no value for the control of the automatic gage, should be noted as approximate.

A regular routine of daily inspection of water openings, float, screw, paper, etc., of winding up the tension weight and the clocks on certain days of the week, and of reading the staff and recording the observations should be adopted, and a description of it posted for the use of various observers.

WATER STAGE REGISTERS

Water stage registers of the continuous-recording type, as made by American manufacturers, are as a rule somewhat smaller than the automatic tide gages previously described, but offer superior advantages in certain respects. They operate on stainless steel ball or roller bearings; are enclosed in weatherproof casings; and record times and heights, to a great variety of scales, on cross-sectioned paper, thus facilitating readings. Detailed descriptions of many types may be found in the specifications and proposals for supplies issued by the Treasury Department, Washington, D. C.

TIDE STAFFS, TIDE NAILS, AND BENCH MARKS

A **tide staff**, or tide board, is an essential part of every tide station. It is simply a board bearing a graduated scale, usually feet and tenths, sometimes feet alone, fastened vertically in a substantial manner to a pile near the tide house, or to the supports of the house itself, with the zero several feet above the bottom and below water level at the lowest stage of the tide. If two boards with zeros at the same level are used, on opposite sides of the tide house and facing in different directions, there will be a greater chance of always finding the face of one of them in the lee. To partially break up the eddies around the edges of the boards, strips of gunny sacking somewhat wider than the boards and with frayed borders may be nailed to the backs. To damp wave action in exposed places, it may be necessary to attach to the face of a board, temporarily or permanently, a glass tube having the lower end covered with coarse cloth and bearing within a thin disc of wood, red or black, as a marker. To avoid capillary action the diameter of the marker should be considerably less than that of the tube.

To provide for the restoration of the board at the same level at any future time, a spike (galvanized iron or zinc) called a **tide nail** is driven all the way up to the head into the pile

opposite a foot mark, and the number of the mark selected is carved in the pile, thus: V for a 5-foot tide nail.

To provide against settling or tilting or destruction of the tide house supports, which would change the level of the zero of the staff, it is well to establish a reference object near at hand, such as a boiler tube driven well down until its top is at the same elevation as one of the foot marks on the staff.

In addition, wherever possible, especially at base stations, a reference mark called a **bench mark**, permanent in nature, easily described and identified, and well known to the inhabitants of the place, should be established, and its elevation should be doubly defined, first as a certain number of feet above the zero of the staff, and finally, when the mean low water point on the staff has been found, as a certain number of feet above mean low water. The bench mark is not necessarily close to the tide staff. If possible, the required difference of elevation should be obtained with a spirit level, and in selecting a site for a base station, the existence of a suitable site for a bench mark should be regarded as an essential condition. Less important stations may be inaccessible from land, making impossible the use of a level for part of the distance to the bench mark. The elevation may be carried to shore by water level when the water is quiet, and thence by level or transit to the bench mark. If a transit is used for the water reach, it will be necessary to make the proper allowance for curvature and refraction. The water level is usually more reliable.

Even the bench marks that appear to give the greatest promise of permanency may be destroyed in the course of the growth of towns. It is desirable, therefore, to establish numerous reference elevations to increase the chances of some of them surviving. The concrete corner post foundations of towers that stand near shore afford an opportunity to establish, at no extra expense and with little trouble, a system of well distributed elevations. To refer them to the nearest local station, it is necessary to mark the time when the elevations above sea level are measured.

Every season's record should include a list of bench marks numbered serially and fully described.

TRANSFER OF ELEVATIONS BY WATER LEVEL

It is not intended to discuss here the transfer of elevations in rivers, where the problem is complicated by the slope of the surface of the water and by the difference in elevation of the surface near opposite banks. Similar conditions in channels are avoided by waiting for slack water.

If certain precautions are taken, an elevation may be transferred by water level with surprising accuracy. The water must be comparatively smooth. There must be no appreciable current. The tidal conditions at the two places must be similar in both phase and range. The elevations must be observed simultaneously, and the true water level at each place must be found a short distance offshore. At one place, it may be found by observing the water level on a tide staff. At the other, it is best to drive a stake in the water by repeated light taps of a sledge until the top is level with the surface, and then to use the top of the stake as a turning point to pass to the object whose elevation is required.

In dragging, dredging, or other channel work, where the director of operations must know the stage of the tide at all times, a series of tide staffs will be necessary. At slack water, a day or two before beginning the work, set the first staff in a central position with the water level at some half-foot mark (to avoid signaling feet). Leave an observer there with a white flag and a black flag, the white for signaling tenths of the reading and the black for hundredths, one raising, waving, and lowering for each unit. Zero units may be denoted by the appropriate flag held aloft for a minute without waving. Proceed to the remaining stations and set the staffs in accordance with the signals received. To derive the common level of the zeros, compare any staff with that at the nearest local station.

A tide nail should be established at each station. One of unusual shape and of noncorrodible material is best.

COMPARISON OF STAFFS AND GAGES

When two places have the same tides, considering both times and ranges, their tide staffs may be compared by a single simultaneous reading. This condition seldom holds for any great

distance. Usually it is necessary to make simultaneous observations for a period long enough to embrace at least four consecutive high and low waters, an equal number of each. The basis of comparison is **mean sea level**, which is assumed to have the same elevation at the two places.

By comparison of staffs, soundings and elevations over a large area may be referred to MLW at the base station, provided that the range is nearly constant over the whole area. If

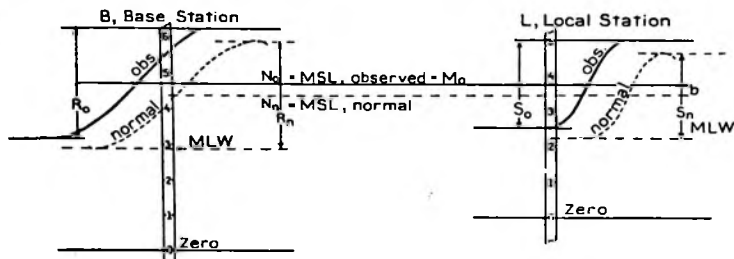


FIGURE 79.—Comparison of tide staffs.

the range at the base station is somewhat greater than elsewhere, the error committed in the reduction of soundings will be on the side of safety.

The following example shows the manner of using mean sea level to refer elevations at *L* and *M* to mean low water at *B*.

	Station B		Station L		Station M	
	On gage at B	Above MLW at B	On gage at L	Above MLW at B	On gage at M	Above MLW at B
MLW.....	+1.95	0.00				
Zero.....	0.00	-1.95	0.00	-2.19	0.00	-2.84
MSL.....	(+3.45)	+1.50	(+3.69)	+1.50	(+4.34)	+1.50
BM.....	+10.22	+8.27	+4.70	+2.51	+5.41	+2.57

The comparative readings are those enclosed in parentheses.

Since 1.50 above MLW at *B* expresses mean sea level at all stations:

To reduce column 1 to column 2, subtract 3.45 - 1.50, or 1.95.

To reduce column 3 to column 4, subtract 3.69 - 1.50, or 2.19.

To reduce column 5 to column 6, subtract 4.34 - 1.50, or 2.84.

Premising that two stations are subject to similar meteorological and tidal conditions, both general and during a short period of simultaneous observations, by using mean sea level as a basis it is still possible to compare their tide staffs for establishing a new mean low water plane, though there may be considerable difference in times and ranges.

In Figure 79 it is assumed that mean sea level, both normal (long-period) and temporary (short-period) is the same sea surface at both stations, and that the difference between the two mean sea levels, found to be *b* feet at *B*, is also *b* feet at *L*.

Further, if the observational mean range at *L* is found to be *m* times the observational mean range at *B*, it may be assumed that the normal mean range at *L* is *m* times the known normal mean range at *B*. Therefore, using the notation of the figure, the reading *M*₀ of the temporary mean sea level on the gage at *L* represents the same absolute elevation as the corresponding reading *N*₀ on the gage at *B*. Applying the variations from the normal, or *b*, we find the midpoint of the normal range at *L*. Applying to this result $\frac{1}{2} S_n$, which equals $\frac{1}{2} R_n$, multiplied by the factor *m*, we have finally

$$\text{MLW at } L = M_0 - b - \frac{1}{2} m R_n.$$

DIURNAL INEQUALITIES

The common type of tides is **semi-diurnal**, two high waters, HHW and LHW, alternating with two low waters, LLW and HLW, one high water and one low water corresponding to each transit of the moon, upper and lower. The amount by which the mean of the higher waters for one or more lunar months exceeds the mean of all the high waters is called the **diurnal high water inequality**, DHQ. The amount by which the mean of the lower low waters falls short of the mean of all the low waters is called the **diurnal low water inequality**, DLQ.

As DHQ and DLQ increase, the tides tend toward the double-headed type, and finally to the **diurnal** type, in which there are only two tides a day.

LUNITIDAL INTERVALS

The interval of time from a transit of the moon, either upper or lower, over the local meridian, to the time of the next high water is a high water interval, HWI; and to the time of the next low water, a low water interval, LWI. If the interval is so short as to give rise to occasional negative results while maintaining the correspondence of transit and tides, the transit occurring about 12 hours earlier may be used instead of the nearest transit, and 12 hours 25 minutes may be subtracted from the mean. Only one high water and one low water, however, may be referred to the same transit.

CORRECTION TO FICTITIOUS INTERVALS

Since the lunitidal intervals are intervals of time between two events taking place on the local meridian, namely the transits of the moon and the arrivals of high and low waters, the



FIGURE 80.—Meridians.

times in which these events are expressed, one in Greenwich or Washington time, the other possibly in zone time, must be reduced to the same meridian, preferably the local meridian. The desired results being means, it is not necessary to reduce the time separately to local time. Instead, fictitious intervals are derived and their mean is then corrected in a single operation.

If the times of high and low waters are expressed in the standard time of a zone, Z^h , the correction, z , to be applied to the minuend in any expression for a fictitious lunitidal interval is, in hours,

$$z = Z - \lambda.$$

Since the times of transit of the moon (the subtrahends in the expressions for lunitidal intervals), are given in Greenwich time or else in Washington time, and since the average rate of retardation of the moon is about 50 minutes per day, or $3\frac{1}{2}$ percent of the difference of longitude between any two meridians, the corrections from fictitious times of transit to local times, g applicable to Greenwich transits and w to Washington transits, are:

$$g = 0.035\lambda \text{ and } w = 0.035(\lambda - 5.1377).$$

Hence the total correction in hours to fictitious intervals is for Greenwich transits,

$$z - g = (Z - \lambda) - 0.035\lambda, \quad (1)$$

and for Washington transits,

$$z - w = (Z - \lambda) - 0.035(\lambda - 5.1377). \quad (2)$$

Z and λ are positive for west longitudes, negative for east longitudes.

TIDE RECORDS

Without special instructions, data pertaining to the harmonic analysis of the tides will not be required of field parties, beyond the customary tabulation of hourly heights from marigrams, and the computations of the establishment of the port at principal stations. When the machine used is of the printing type, the recorded 15-minute heights, tabulated in columns in preparation for plotting the tidal curves, should be kept as part of the permanent record.

Tide records consist of a graphical record, either made by the machine or plotted, and a substantially bound record and computation book in addition to the regular tide books, if any, in which readings of staff gages are written.

Whenever a roll is put on the machines, and again when it is taken off, classification and identification notes analogous to the following should be written on the end of the roll in ink or indelible pencil, thus insuring that *both ends* of the roll are labeled:

U. S. S. *Niagara*, 1926, Capt., U. S. N., commanding.
Castilletes, Gulf of Venezuela.
Tide Roll No. 3, July 2-29. 75th meridian time.
Observers (or principal observer)

Printed records should be numbered and submitted in a strong envelope or in a tin can, with a complete descriptive label.

Usually, a single book will suffice for computations. The clothbound memorandum book about 5½ inches by 8½ inches furnished to survey parties, having 32 lines to a page and no headings, is suitable for the purpose. This may be ruled in columns corresponding to those of form N. H. O. No. 92, with opposite pages serving for each lunar month. The top headings may be written once for all on the first left-hand page and on the last right-hand page, and will serve for the whole book if about half an inch is cut out of the book, at the top, between the first and last pages, so as to reveal the headings. The side arguments, which are the days of the month, must be written in. Except in the case of permanent stations, it is of no consequence whatever whether the first recorded day is the first day of the calendar month or any other day. All of the observations should be tabulated, and from all of them, as many consecutive lunar months as possible should be selected for tabulation.

If pages of the form are used instead of a book, they should be bound into a book. On account of losing loose pages in the field, or in the office, their use is not recommended except for preliminary tabulations, permanence being considered more desirable than temporary convenience.

Each section of the tabular computations should refer to the corresponding roll by number.

TABULATING HIGH AND LOW WATERS

The tabulation of tidal observations should go forward in the field as opportunity affords.

The first problem is to determine the working scale of the machine and the height of the datum line. From selected high and low waters and other observed comparisons of the staff and curve, compute, on the tide roll itself, the average ratio of the paper ranges to the staff ranges, and construct a scale corresponding to that of the staff, on a piece of paper cut from the roll. With this scale measure from the curve, at the selected points of comparison, vertically to the datum line, and so find the average height of the latter referred to the staff. This value may not hold for the whole roll, and should be tested at intervals, writing above the line the height that applies to each section.

Next, draw fine light lines parallel to the datum line and cutting off the tips of the curves at high and low waters; with a thin thread of light showing at intervals between the line and the curve. The length of the secant line represents the duration of the stand. Select a point of the curve midway of the stand and through it draw a short horizontal line and a vertical line or ordinate to the datum line. If portions of the curve are faint, they may be reinforced by a pencil line. If portions are missing where high or low waters would be expected, supply them by estimation, using four adjacent tides as a unit of comparison. Scale times and heights and note them on the roll.

In tides that tend to become double-headed, approaching the diurnal type, it is sometimes impossible to assume a stand defined by a secant line. In brief, the stands degenerate to single points or peaks, which must be estimated by eye. Except in extreme cases, however, it is possible to consider the first and second high points, outside of which the curve appears to break, as the high water points.

Two time scales will be found convenient—one, an hour in length, divided into tenths or into 5-minute spaces, according to the system used—the other 36 to 48 hours long with each hour space somewhat longer than that of the record. The latter may be made of heavy mounted paper. When it is laid obliquely along the record with the proper divisions on the vertical correct time lines of the daily observations, the abscissae of high and low waters may be read where the edge of the scale crosses the ordinates.

Freshets and high winds may cause a low water to be higher than the preceding high water, though there will generally be some slight indication on the curve of the beginning of high or low water. Failing this, the doubtful positions must be supplied by estimation. By shifting the beginnings and endings of lunar months, it is sometimes possible to dispense with these doubtful periods. Abnormal high or low waters, however, should be noted, and the causes thereof.

Having written the times and heights of high and low waters in the appropriate columns, next take the Greenwich or Washington times of the moon's transits from the Ephemeris, distinguishing lower transits by employing parentheses. Subtract the time of each transit from the times of the next two tides, in a way to preserve the correspondence of transits and tides, and write the results in the columns of lunitidal intervals. Sum separately the high-water intervals and the low-water intervals for 29 days, and find the means. These are fictitious intervals, and must be corrected as previously described. When the meridians used differ from those of the illustration, consider Z and W as general symbols, in hours, for the meridians of times and transits, respectively. In particular, when Greenwich transits are used, take $W=0^h$.

Next, sum the high waters and the low waters and find the means. The half sum is mean tide level, and the difference is the uncorrected mean range.

To find the diurnal height inequalities, check off the HHW and the LLW for 27 days, omitting the 1st and 29th days. If the tides for any part of the month have become diurnal, it is not the major tides, HHW and LLW, that have vanished, but the minor tides. Sum, find the means, and compare the results with the means of all the high and low waters, respectively. These are the uncorrected height inequalities.

The moon's node, or the intersection of her orbit with that of the earth, travels along the ecliptic at a rate sufficient to make one complete round in about 18.6 years. To correct the mean range and the diurnal height inequalities obtained from a short period of observations to the corresponding cyclic values, the first must be multiplied by the factor F (Mn) and the second by the factor $1.02 F_1$. See the following tables. In the argument for the first table the uncorrected values of DHQ, DLQ, and Mn may be used.

To find mean low water, subtract half the corrected mean range from mean tide level. To check mean lower low water previously found, further diminish this result by the diurnal low-water inequality.

TIDES AND CURRENTS

Tabulation of heights

U. S. S. Niagara

Station: Castilletes, Venezuela

Highest tide, 5.4 feet, date July 2, 4, 5

Lowest tide, 1.8 feet, date July 24

Argument 2 (DHQ + DLQ) + Mn = 0.85 F(Mn) = 0.99 1.02 F₁ = 0.91

Latitude 11°51'01" N.

Longitude 71°19'20" = 4h7548

Meridian used + 75° = 5h = Z

Date, 1926	Moon's transit		Time of—		Lunitidal intervals		Height of—		Remarks
	WCT	HW	LW	HW	LW	HW	LW		
	Hour	Hour	Hour	Hours	Hours	Feet	Feet		
Forward				146.5	327.8	142.1	93.2		
July 18	6.2 18.6 (7.0)	11.4 23.8 12.0	5.0 16.5 5.2	(5.2) 5.2 (5.0)	11.2 (10.3) 10.6	3.9 4.6 4.0	3.0 3.2 2.8	1st quarter.	
July 19	19.4 (7.8)		16.5 7.0		(9.5) 11.6		3.3 2.5		
July 20	20.2 (8.6)	12.5 0.2	17.0 7.5	(4.7) 4.0	(9.2) 11.3	3.9 4.7	3.3 2.3		
July 21	21.1 (9.6)	13.5 1.0	18.4 8.0	(4.9) 3.9	(9.8) 10.9	4.0 4.9	3.2 2.2		
July 22	22.1 (10.6)	14.5 1.3	19.0 8.5	(4.9) 3.2	(9.4) 10.4	4.0 4.8	3.4 2.0		
July 23	23.1 (11.6)	15.5 2.0	20.0 10.0	(4.9) 2.9	(9.4) 10.9	4.0 4.8	3.3 1.8		
July 24		16.5 3.0	21.5 10.5	(4.9) 2.9	(9.9) 10.4	4.0 4.7	3.3 2.0	Full moon.	
July 25	(12.6)	17.5 4.0	22.4 11.5	(4.9) 2.8	(9.8) 10.3	4.3 4.9	3.3 2.5		
July 26	1.2 (13.6)	4.0 19.0	11.5 23.8	2.8 (5.4)	10.3 (10.2)	4.9 4.6	2.5 3.0		
July 27	2.1 (14.6)	5.5 20.0	13.0 24.0	3.4 (5.4)	10.9 11.4	4.8 4.8	2.2 4.8		
July 28	3.0 (15.5)	6.0 20.5	1.0 13.5	3.0 (5.0)	(10.4) 10.5	4.8 4.3	3.3 2.2	Meridian of— 5h0000 = Z, time zone.	
July 29	3.9 (16.4)	7.5 21.0	2.0 13.7	3.6 (4.6)	(10.5) 9.8	4.7 4.8	3.2 2.4	4.7548 - λ, tide station. 5.1377 = W, Washington	
July 30	4.8 (11.2)	9.0 22.0	3.0 14.5	4.2 (4.8)	(10.6) 9.7	4.7 5.0	2.9 2.7	0.2452 = Z - λ = z -0.0134 = .035(λ - W) = w	
July 31								+0.2586 = correction = z	
Sums, 29 days, July 2-30				56)254.8	56)585.3	56)254.9	56)162.5	HHW Sums 25)120.0	
Means				4.55	10.45	4.55	2.90	I.LW 26)63.6	
Correction to intervals				+0.26	+0.26			Means 4.80 4.55	
Corrected intervals				4.81	10.71	Mn	1.65	DHQ= 0.25 DLQ=0.45	
Duration of rise		3.72 hr.				MTL	3.72		
Mean rise interval		2.90 ft.				MLW	2.90		
Computed by G. M. Date, 1926.						Corrected Mn	1.65 × 0.99 = 1.63.		
						Corrected DHQ	0.25 × 0.91 = 0.23.		
						Corrected DLQ	0.45 × 0.91 = 0.41.		

TABLE 6.—Factor $F(Mn)$ for correcting short-period ranges

$\frac{2(DHQ+DLQ)}{Mn}$	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939
0.0	1.03	1.03	1.03	1.03	1.02	1.01	1.00	0.99	0.98	0.97
0.2	1.03	1.03	1.03	1.03	1.02	1.01	1.00	0.99	0.98	0.98
0.4	1.03	1.03	1.03	1.03	1.02	1.01	1.00	0.99	0.98	0.98
0.6	1.02	1.03	1.03	1.02	1.02	1.01	1.00	0.99	0.98	0.98
0.8	1.02	1.02	1.02	1.02	1.02	1.01	1.00	0.99	0.99	0.98
1.0	1.02	1.02	1.02	1.02	1.01	1.01	1.00	0.99	0.99	0.98
1.2	1.01	1.02	1.02	1.01	1.01	1.01	1.00	1.00	0.99	0.99
1.4	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00	0.99	0.99
1.6	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	0.99
1.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.0	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.01

$\frac{2(DHQ+DLQ)}{Mn}$	1940	1941	1942	1943	1944	1945	1946	1947	1948	1949
0.0	0.97	0.97	0.97	0.98	0.98	1.00	1.00	1.01	1.02	1.03
0.2	0.97	0.97	0.97	0.98	0.98	1.00	1.00	1.01	1.02	1.03
0.4	0.97	0.97	0.97	0.98	0.99	1.00	1.00	1.01	1.02	1.03
0.6	0.97	0.97	0.98	0.98	0.99	1.00	1.00	1.01	1.02	1.02
0.8	0.98	0.98	0.98	0.98	0.99	1.00	1.00	1.01	1.02	1.02
1.0	0.98	0.98	0.98	0.98	0.99	1.00	1.00	1.01	1.02	1.02
1.2	0.98	0.98	0.98	0.99	0.99	1.00	1.00	1.01	1.01	1.02
1.4	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.01	1.01	1.01
1.6	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.01
1.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.0	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	0.99	0.99

TABLE 7.—Factor $1.02F_1$ for correcting short-period inequalities

	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939
Jan. 16	0.81	0.80	0.80	0.80	0.81	0.83	0.86	0.91	0.94	0.98
Feb. 15	0.94	0.93	0.92	0.93	0.95	0.97	1.01	1.06	1.12	1.16
Mar. 17	1.08	1.06	1.06	1.07	1.09	1.13	1.19	1.26	1.34	1.43
Apr. 17	0.98	0.98	0.98	0.99	1.01	1.04	1.09	1.15	1.22	1.29
May 17	0.84	0.83	0.83	0.84	0.85	0.88	0.91	0.96	1.00	1.05
June 17	0.77	0.77	0.77	0.77	0.79	0.81	0.84	0.87	0.91	0.95
July 17	0.81	0.80	0.80	0.81	0.82	0.85	0.88	0.92	0.96	1.00
Aug. 17	0.93	0.92	0.93	0.94	0.96	1.00	1.04	1.09	1.15	1.21
Sept. 16	1.08	1.07	1.07	1.09	1.12	1.16	1.23	1.30	1.39	1.47
Oct. 16	0.99	0.98	0.99	1.01	1.03	1.07	1.12	1.18	1.26	1.32
Nov. 16	0.83	0.83	0.83	0.84	0.86	0.89	0.93	0.97	1.02	1.06
Dec. 16	0.77	0.77	0.77	0.78	0.80	0.82	0.85	0.89	0.93	0.96

	1940	1941	1942	1943	1944	1945	1946	1947	1948	1949
Jan. 16	1.02	1.03	1.03	1.00	0.96	0.92	0.88	0.85	0.82	0.81
Feb. 15	1.23	1.25	1.24	1.20	1.15	1.08	1.03	0.99	0.96	0.93
Mar. 17	1.49	1.52	1.49	1.44	1.36	1.28	1.20	1.14	1.10	1.07
Apr. 17	1.34	1.36	1.34	1.29	1.22	1.15	1.08	1.04	1.01	0.99
May 17	1.08	1.09	1.08	1.00	1.00	0.95	0.91	0.88	0.85	0.84
June 17	0.97	0.98	0.97	0.94	0.90	0.86	0.83	0.80	0.78	0.77
July 17	1.03	1.03	1.02	0.98	0.94	0.90	0.86	0.83	0.81	0.80
Aug. 17	1.25	1.26	1.23	1.18	1.12	1.06	1.02	0.98	0.95	0.93
Sept. 16	1.52	1.53	1.48	1.40	1.32	1.24	1.18	1.14	1.11	1.08
Oct. 16	1.36	1.36	1.33	1.26	1.19	1.13	1.07	1.03	1.00	0.99
Nov. 16	1.08	1.10	1.05	1.01	0.97	0.92	0.88	0.86	0.85	0.83
Dec. 16	0.98	0.98	0.95	0.92	0.89	0.84	0.82	0.79	0.78	0.76

DEFINITIONS AND REFERENCES

SPRINGS AND NEAPS

Twice every lunar month, when sun and moon act nearly in a line, there is, normally, an unusually high tide and on either side of it an unusually low tide. These are the **spring tides**. The small-range tides occurring when the moon is in quadrature are the **neap tides**.

1. *Intervals*.—The mean of the lunital intervals of the high waters at springs is called **high water full and change** (with reference to the phases of the moon), or the **vulgar establishment** (with reference to a port). Similarly, the mean of the lunital intervals of low waters at springs is called **low water full and change**.

As distinguished from the vulgar establishment, which refers to intervals of particular high waters, the mean interval of all the high waters in a lunar month (MHWI) is called the **corrected establishment**.

2. *Heights*.—The means of the heights of the high- and low-water spring tides, respectively, are called **high-water springs** and **low-water springs**.

3. *Planes of reference for heights and depths*.—Mean sea level is very generally used as the plane of reference for heights. In the Baltic Sea it is even used as a plane of reference for depths.

Special planes of reference, described on the chart, are sometimes used when the tidal conditions are unusual. In general, however, the following planes of reference are most used for depths (International Hydrographic Bureau Special Publication 22, 1928):

Mean low water.—United States (Atlantic Coast), Argentina, Norway, Sweden.

Mean lower low water.—United States (Pacific Coast).

Mean low water springs.—Great Britain, Italy, Germany, Denmark, Brazil, Chile.

Mean monthly lowest low water springs.—Netherlands.

Lowest low water springs.—Brazil, Portugal.

Indian spring low water.—Great Britain (India), Argentina, Japan.

Mean semiannual lowest low water.—Netherlands (East. Arch.).

Lowest low water.—France, Spain, Norway, Greece.

International low water.—Argentina (provisionally).

The last-named plane is 50 percent lower, reckoned from mean sea level, than low-water springs. Indian spring low water depends on component tides found by harmonic analysis.

FURTHER STUDY OF TIDAL PHENOMENA

The United States Navy's interest in tidal and current phenomena off foreign coasts that are as yet incompletely charted is chiefly concerned with considerations of safe navigation, and hardly justifies the time and expense entailed by exhaustive studies looking to the production of tide tables.

For an "Investigation of harmonic constants, prediction of tide and current, and their description by means of these constants" see International Hydrographic Bureau's Special Publication No. 12, May 1926. The various national systems reviewed differ chiefly in the methods of mechanical distribution of hourly ordinates to bring these into effect according to the periods of the component tides and according to the weights of the ranges. Further reference is made to:

Coast and Geodetic Survey Special Publication No. 98, 1924. *Manual of Harmonic Analysis and Prediction of Tides*, by Paul Schureman.

Service Géographique (France) Publication No. 870, 1905, *Observation, Etude, et Prediction des Marées*, by Rollet de l'Isle.

Great Trigonometrical Survey of India, Vol. XVI. *Details of Tidal Observations*.

Annalen der Hydrographie, Heft II, 1923, article by Prof. Sterneek.

Royal Netherlands Meteorological Institute, Professional Paper No. 8, 1910, *Elementaire theorie der getyden*, Getyconstanten in den Indischen Archipel, by Dr. Van der Stok.

Harmonische Analyse der Gezeiten des Meeres, 1924, by Dr. Rauschelbach.

Analysis and Prediction of Tidal Currents, by A. T. Doodson, 1928, Tidal Institute, University of Liverpool.

CONSTANT CURRENTS; CIRCULATION

The constant currents in a region are those which, on the whole, tend to maintain a certain direction, or circuit, and, if interrupted, to establish themselves again. The basic circulatory system often consists principally of a current and a countercurrent persisting generally over the

region except in channels, where they are alternately aided and opposed, to a greater or less extent, by tidal currents of the rectilinear type, constrained to flow in one of two opposite directions.

The causes of constant currents are numerous and obscure. On an open coast, such as the east coast of Florida, they may arise from a participation in larger circulatory systems. In a landlocked area, the impetus which maintains the water in circulation may be partly the influx of water of less than average density. Whether the impulse is from within or without, the direction is influenced by the orientation of the coast; by the shape of the basin; by the presence or absence of fiords; by evaporation over large shallow areas; and largely by the relative cross sections of inlets and spillways.

Outflow must be balanced by inflow. The current and countercurrent may flow in different channels, setting up a clockwise or a counterclockwise circulation, mainly flood current in some channels and ebb current in others; or they may flow in the same channel, one over the other. The first condition is easily manifest. The second, without subsurface measurements, may escape notice. The counterclockwise currents in the Gulf of St. Lawrence, the ebb-over-flood currents in the Dardanelles, and the flood-over-ebb currents in the large Strait of Bab-el-Mandeb, exemplify various types of constant currents.

Ocean currents, caused by differences in temperature, salinity, density, and barometric pressure, which cause variation in the distribution of weight; and by seasonal winds and the rotation of the earth, are generally of the constant type.

TIDAL CURRENTS

A current, in general, consists of three parts:

- (a) The basic, or constant current, setting in a particular direction, on which are superimposed the following:
- (b) The tidal stream, the resultant of periodic elements depending on the heads of water produced by the tides.
- (c) The barometric pressure elements, variable and seasonal in nature.

A tidal current, in common usage, is one that is predominantly tidal.

With respect to tidal currents, the times of events are usually referred to the times of high water, sometimes to the times of low water as well, and again to the times of the moon's transits.

The most important as well as the most definite events connected with tidal streams or currents are the times and velocities of the streams at their strongest. These are referred to as **strength of flood** and **strength of ebb**, respectively. Flood and ebb will be used here to mean those currents which, at their strength, accompany rising and falling water, respectively. When either term becomes ambiguous by reason of a long continuance of the current in question, it must be abandoned in favor of a description indicating the direction of the current.

Tidal streams, like the tides causing them, are periodic, and may be semidiurnal, diurnal, or mixed in type. A correspondence between types, however, is not always to be inferred. Quoting from I. H. B. Special Publication No. 12:

It may even happen that the characters of the two are opposed, as at Sembilangan, in the northern approach to Soerabaja, where the current is preponderantly semidiurnal while the tide is preponderantly diurnal; * * * or at Anjer, in Sunda Straits, where the tidal current is mixed with a preponderant diurnal character while the tide is semidiurnal.

When the current is largely tidal, however, with small constant and barometric components, and when it is of the same type as the tide, the theoretical relations deduced by Mr. Marmer in *Coastal Currents along the Pacific Coast of the United States*, are likely to be satisfied. They are summarized in the following terms, page 11: "The change in the velocity of the current from springs to neaps should be approximately proportional to the corresponding change in the range of the tide"; and "the diurnal inequality in the current at any place is approximately half of what it is in the tide."

The **direction of tidal streams** offshore changes continually and progressively, making a complete rotation every two tides. In each rotation, there are two periods of maximum velocity and two of minimum velocity. These phenomena are reproduced also in wide channels, but are modified by friction, the minimum velocities being greatly reduced; while in narrow channels,

owing to the restraint of the banks, the minimum velocities tend to disappear, producing the phenomena of slack waters and sudden reversals between the upstream and downstream directions. The changes in direction take place in the order upstream, away from shore, downstream, toward shore. It is obvious that the times of turning are not the same near the sides of the channel as in the middle, and that flood and ebb may occur simultaneously in the same channel. Furthermore, the configuration of the land may be such that one side of the channel is preempted by flood currents and the other side by ebb currents. Finally, these two conditions may concur, the streams meeting and crossing.

BAROMETRIC PRESSURE CURRENTS

Variations in barometric pressure over wide areas affect currents in two ways, by changing sea level, and by causing winds, which act on the surface of the water by friction. The sequence of events is about as follows:

With a low barometer and consequently a high sea level, the current begins to run down the slope of the pressure wave toward the area of high pressure, several hours before the arrival of the wind from that quarter, and continues to quicken during the first part of the blow. At

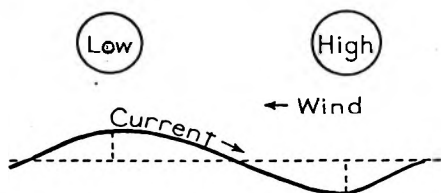


FIGURE 81.—Pressure current.

this stage, the wind has very little effect on the current, even though the seas may be high. Only the surface particles are affected. With time, however, the impetus is communicated to the particles below, to a depth of several fathoms, with cumulative effect. Finally, with the advance of the area of high pressure, the current will be checked and may be reversed; or the current may be reversed by a piling up of the water on shore and a dying down of the wind.

Long-continued winds and seasonal winds, such as the monsoons, may generate sizable currents, but the effect of ordinary winds is commonly overestimated. In a report on the investigation of currents in the Gulf of St. Lawrence, covering seven seasons, where it was the custom of the observing ship to hold her station as long as the anchor held, it is stated that "There was no evidence, after any of the gales, that the wind was able to reverse the direction of the tidal streams, or that it was able to check, to any noticeable extent, the dominant flow which prevailed at the time." The reach of the wind was in some cases 470 miles.

Winds may displace currents, toward or away from shore, or may prolong or shorten their duration. The horizontal effect of a constant wind combined with that of the rotation of the earth is a deflection to the right in the northern hemisphere and to the left in the southern hemisphere. The effect decreases with the depth, creating a spiral of velocity vectors representing surface, deep, and bottom currents. See *Über Horizontalzirkulation bei winderzeugten Meeresströmungen*, by V. Walfrid Ekman.

INFORMATION FOR CHARTS

The body of information with reference to tides, currents, and weather that can and ought to be collected, compiled, analyzed, generalized, and finally summarized in a form suitable for charts and sailing directions is essential to the full development of the value of a survey. In any survey extensive enough to result in a general chart and a number of large-scale harbor charts, the study of these matters may well be considered as a department, comparable in importance to triangulation, shoreline, topography, and sounding.

With reference to currents, the information desired may be inferred from the following outline, only partially applicable in most cases:

1. *In approaches to principal ports, on soundings.*—Current stations selected in cross sections. Observations by means of an automatic current meter at the principal central station, and by means of a free float or a current meter, for at least four days at spring tides and four days at neap tides. Observations to be taken every half hour, and the mean of adjacent values to be tabulated for every hour, arguments hours from the actual times of high waters at the base station, tabular values velocities in knots, and directions (except when the current is turning) in degrees. Direction of turning, right or left. Observations to be averaged to derive, separately at springs and neaps, the following:

(a) *Strength of flood*.—Time from high water, before —, after +; velocity in knots, average of four maxima, also maximum single observed velocity; direction.

(b) *Strength of ebb*.—Time from high water, velocity, average of four maxima, also maximum single observed velocity; direction.

(c) Times of beginning and ending of slack-water periods.

(d) Notes—depth of flotation, apparatus, weather, etc. .

2. *In principal channels, depth 6 fathoms or more*.—Same as in 1.

3. *In lesser channels, depth less than 6 fathoms*.—One station in the center of the narrows, and if the channel is long, similar stations at the entrances. Observations over four consecutive tides at spring tides, by means of a current meter or a free float, to derive the quantities at strength of flood and strength of ebb.

4. *General circulation stations, outside of channels*.—Purpose, to study the general circulation of waters in sounds and bays, using an anchored vessel and either a meter or surface and subsurface restrained floats. Undercurrents to be studied as may be necessary. Stations 10 to 30 miles apart. Weather effects important. Wind currents to be eliminated by using a subsurface meter or float as control, or by applying an approximate correction equal to $1\frac{1}{2}$ percent of the velocity of the wind.

Coastal stations are often needed to determine the set of currents along a coast. They should be established 25 to 50 miles apart, and as near the tracks of coasting vessels as is practicable.

CORD AND FLOAT METHOD OF MEASURING CURRENTS

When a heavy object in the water is first taken in tow with a strong but light line, there is a moment, at the instant when the line straightens and before the object moves, when the length of the line accurately measures the distance to the object. Under similar circumstances, in measuring the distance to a float, advantage may be taken of the inertia of the float, if it is heavy enough with respect to the line. This is the fundamental condition for satisfactory results in measuring a current by the cord and float method.

There must, therefore, be two lines to the float—a light, graduated, measuring line and a heavier line for handling the float in the strongest currents. The measuring line should be graduated in a catenary under a light tension and while wet, and the same tension should be used, by estimation, in making observations, with the line wholly or mostly out of the water.

The best shape for a float is one approximating a spar buoy. The depth of flotation should approximate the usual draft of vessels in the locality, but not less than three-fourths of the draft of the largest vessel likely to visit the place. For small channels, 10 feet is a good depth of flotation, and for deep waterways, 18 feet. The following design is suggested for an 18-foot float:

Frame a frustum of a pyramid 4 feet square at the bottom and 2 feet square at the top, using 4-inch by 4-inch by 19-foot corner posts, with a solid board or sheet metal bottom for receiving ballast, and a skeleton top designed to hold in place a $\frac{1}{2}$ -inch pipe flagstaff to extend 4 feet above water. Nail on canvas sides under light battens. Lead the hand line, about 70 fathoms of 9- or 12-thread manila, with a thimble in the end, from a ring stapled into the outer corner of one of the posts about halfway up. Graduate a light fishline of good quality, about 300 feet long, with division marks every 10 feet for 200 feet from the end to be held aboard ship. To the other end attach a swivel, a harness snap, and a $\frac{1}{4}$ -inch ring, the latter to fit loosely over the top of the flagstaff between collars. After the lumber is well water-soaked, add ballast until the tops of the posts are 1 foot above water level. Attach a small flag to the staff by day and a lantern by night. Other accessories include a compass, a sextant, a small spring balance, and a stop watch. Attach a permanent sling for hoisting the float out when not in use.

In making an observation, the hand line should run off a reel, elevated if necessary, in a way to restrain the float as little as possible. When the first mark of the measuring line reaches the deck mark, hold the coincidence for a moment until the advance of the float brings the tension up to standard, mark the time with the stop watch, and release the float. The process is repeated at the end of the run, which should last for 2 minutes unless the end of the line is reached before that time. Record the initial time, the time interval, the distance, and the direction.

If it takes the float more than a minute to draw out the first 25 feet of the graduated part of the measuring line, discontinue the observation, and record the time, the velocity as less than 0.25 knot, and the approximate direction. One observation per hour is usually considered sufficient. This is an inferior method, but frequently it is the only one available at night.

FREE FLOAT METHOD

The free float method has been used in United States Navy surveys since 1908. It is an excellent day method, in channels and in offshore tracks of ships, being, in fact, the only method available beyond anchorage depths.

The float consists of a light pole or a 2-inch by 3-inch scantling weighted with sheet lead at the lower end and bearing a $\frac{1}{2}$ -inch iron rod and a flag, handkerchief size, at the upper end. The scantling should float with its top about 1 foot above water level.

In channel work, one boat can tend three such floats, one for the middle of the channel and one for each side, at each cross section. The principal objects are to measure the strengths of flood or ebb, to observe the times of turning of the current, and in wide channels, to detect the presence of countercurrents. The floats are, therefore, disposed to pass the section in the middle of their runs, usually lasting 15 to 30 minutes, after which the process is repeated until the maximum currents are obtained.

In practice, each float is located at both ends of its run by three-point fixes. The line joining the positions is a vector the elements of which are velocity and direction. If a float gets too far from the section or too far from the desired path, it is placed in a better position and a new run is begun. If the middle float remains in the middle of the channel with undiminished velocity, it may be allowed to run far beyond the section, but in that case another float is placed in the middle of the channel near the section. Each vector is marked on the sheet with a direction arrow and the observation times are written beside the positions. In taking positions of the floats, the launch creeps up on the leeward side, the angles are snapped at the nearest position short of actually touching the float, and the launch is allowed to drift away before starting the engines.

The regular sounding books and either boat sheets or special **current sheets** are used.

In offshore work, it is well to use two floats—one in deep water as near the tracks of ships as practicable, the other near the 10- or 100-fathom line, especially on steep-to coasts, where there may be a countercurrent along the edge of the shelf.

It is only on calm days that small boats can safely tend floats in deep water. Larger vessels, however, while engaged in sounding, can conduct the observations with sufficient accuracy without interrupting their regular work, dropping the floats early in the day and cutting on them at intervals. Even out of sight of land this method can be used, with a precision depending on that of the ship's positions. It may be employed also in studying the circulation of waters in a bay or sound, by any vessel engaged in sounding. Incidentally, the movements of the float will assist the navigator in making allowance for current while running lines of sounding.

VELOCITY CURRENT METERS REQUIRING OBSERVERS

Various types of velocity current meters are described in Johnson's Surveying and in the catalogs of instrument makers. Most of them are designed to be raised and lowered by sliding on vertical poles extending to the bottom of the stream. They may be adapted for deeper water by providing an outrigger, and in lieu of a pole a cable held vertical by a heavy weight.

All of these meters lack a means of finding the direction of the current. The bifilar suspension direction indicator used by the Coast and Geodetic Survey in connection with the Price meter is the result of an attempt to supply this lack. It consists of one or more vanes, each composed of a $1\frac{1}{2}$ -inch pipe filled with lead and provided with a sheet-metal rudder at one end, suspended to the ends of an inverted V-frame, as many frames as vanes, rotating above water on ball bearings and carrying on the inner angle of the V a pointer moving over a pelorus.

VELOCITY AND DIRECTION METERS REQUIRING OBSERVERS

The **Rauschelbach meter**, manufactured by the Bamberg Works, Berlin, seeks to supply the same lack by a double suspension of an elongated frame carrying the instrument, from an anchored vessel, the heading of the latter providing a zero direction for the instrument. For the suspension two heavy insulated cables are used, each requiring a winch. The span of the frame, 1928 model, is 4 meters. The instrument and accessories weigh 890 kilograms, net.

The instrument is intended to be suspended from an anchored vessel, and for observing currents at any depth.

The record is made on board the ship in three colors of ink by 14 pens on a strip of paper 110 millimeters wide drawn forward by clockwork at the rate of 30 millimeters per minute. Each pen is actuated by its own electromagnet. The first pen traces the time record, a straight line with a jog every minute. The second pen traces a similar parallel line, with jogs corresponding to revolutions of the current vane. The next four pens, comprising the first group, trace parallel lines corresponding to the quadrants from 0° to 360° , and the particular quadrant in which the meter is pointed is indicated by dots 5 or 10 seconds apart along one of the lines. The next three pens, forming the second group, trace three lines, one of which is dotted, and indicates the particular third of the quadrant. Finally, the next group of five pens divides the third of a quadrant into five equal parts, the selected one of which is marked by dots. The direction is then read off to the nearest thirtieth of a quadrant. When the instrument oscillates, dots sometimes appear on adjacent lines.

The claim of a precision of 3° is open to question, even supposing that the ship is in irons and does not move. For, if the subsurface current moves across the direction of the keel line of the ship, the frame, suspended by flexible cables, will tend to depart from parallelism with the keel, thus vitiating the zero direction.

For a minute description and the solution of examples, see *Deutsche Seewarte—Beschreibung eines bifilar aufgehängten, an Bord elektrisch registrierenden Strommessers*, von Dr. H. Rauschelbach, 1929.

In the **Ekman current meter**, Figure 82, the **drift measuring unit** consists of an impeller actuating two revolution counter dials through suitable gearing. Dial A reads 0 to 400 and counts revolutions 0 to 4,000. Dial B is graduated from 0 to 10 with half divisions, and indicates revolutions 0 to 100. When the meter is ready to be lowered, the impeller is locked, the dials read 0, and the shutters C1 and C2 are closed. At the desired depth the 8-ounce plummet is released and the stop watch is started. The plummet slides down the lowering wire and trips lever D, opening the shutters and releasing the impeller. Exactly 15 minutes after the release of the plummet, a second one is dropped. It trips lever D again, and releases lever E, which actuates F, the function of which is to stop the impeller. The meter is now hauled up, the dials are read, and the number of revolutions per minute are obtained. Entering the calibration curve with this, the drift of the current is obtained in knots.

In the **set recording unit** (Figure 82, upper part), a rudder rigidly attached to the meter, heads the impeller directly into the current, or upstream. In this position of the frame the axis of a certain one of the 36 radial compartments of the pan M below—always the same one and painted red to distinguish it from the others—will lie in the upstream direction.

Above all the compartments, at their peripheries, there are holes for admitting brass pellets, one of which is dropped at every seventh revolution of the impeller from a shot tube situated centrally above a hole through the axis of A, and is conducted by a chute to that hole of the periphery which happens at the moment to be under the north point of a magnetic needle in cylinder G.

The training of the chute toward magnetic north is accomplished by its rigid attachment to the needle (or needles). Therefore the angle between upstream and magnetic north is reproduced in the meter by the angle between the red compartment and that into which the pellet falls.

To find this angle, a pan similar to M is matched to M, and is then inverted and placed in a circular frame graduated in 10-degree divisions. The graduation nearest the compartment containing the pellet gives the magnetic heading of the current meter, and so that of the current, **provided that the magnetic needles of the meter are not affected by the local attraction of the ship.** To avoid this, the use of a wooden launch without engines, moored at a considerable distance from the ship, would seem to be indicated.

The use of a lowering wire is unavoidable, for it is required as a carrier for the plummets. It leads through a notch in the lever D in a way to guide the plummets to the lever. But all other wire or magnetic material, as for attaching the 50-pound lead to the lower ring or for attaching preventer lines, should be avoided.

The procedure is as follows:

1. Install the desired impeller (choice of high or low reading).
2. Install the magnetic needle unit in G.
3. Install pan M1. It fits in only one way.
4. Close the shutters C1 and C2.
5. Place shot in the shot tube.
6. Cock the lever E, pulling it toward the rudder.
7. Lower the meter until the 50-pound lead, with its length of suspension line properly adjusted, is on the bottom. Check the desired depth of observation by the length of the lowering wire.
8. Fit the plummet to the lowering line. Drop the plummet and at the same instant start the stop watch.
9. At the end of the desired interval, for example 15 minutes, drop the second plummet. Hoist the meter.
10. Read and record the dials.
11. Remove pan M1. Fit the red compartment with that of pan M2, invert the latter, and place it in the frame. Read the magnetic set.

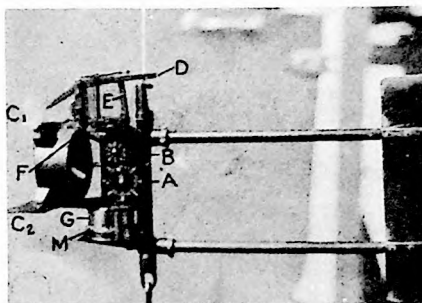
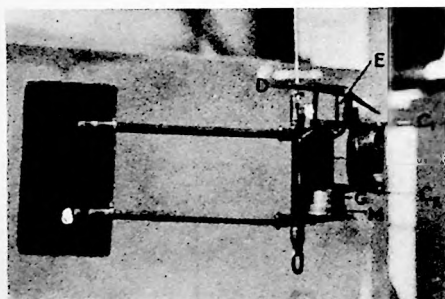
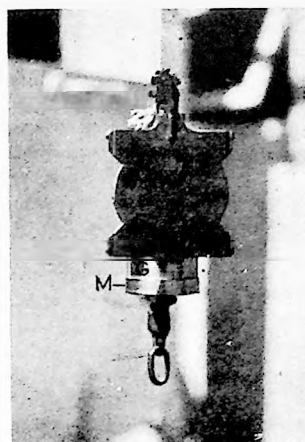
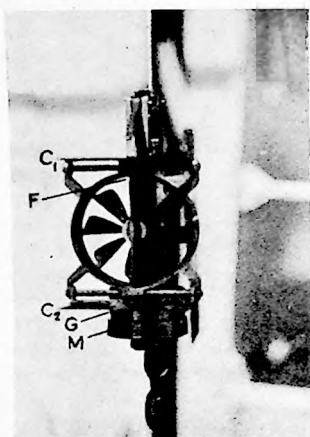


FIGURE 82.—Ekman current meter.

The Carruthers vertical log current meter measures the quantity and the direction of the flow of water past an anchored vessel during consecutive periods of 24 hours 50 minutes in each of eight or more sectors of 360° . It is a circulation meter rather than a velocity meter, but velocity may be obtained from the records by observing in addition the times of changing direction from one sector to another. It is a simple, sturdy apparatus designed to clear itself of seaweed and obstructions, and to give long service in the heaviest weather.

For each sector (12 are recommended for research vessels) there is a wooden slat fitting into fore-and-aft slots in the open top of a small wooden meter box, which is suspended by two beackets on a greased pole mounted so as to project athwart-ship at the waist of a ship. Each

slat carries a revolution counter actuated by a pin fastened in a nut at the top of a short spindle coming up through the bottom of the recording box. At the lower end of the spindle, below the box, there is an eye for attaching a length of rope for supporting a cup apparatus at any depth desired.

The cup apparatus consists of a cup-bearing rod, and below it, also suspended by rope in a vertical position, a weighing rod weighing about 58 pounds. The cup rod, about 4½ feet long, bears six heavy iron cups at the ends of slightly bent arms 14 inches long, spaced at 60° radially and at about 9 inches vertically.

The counters read up to 99,999 revolutions, and in the instrument described in the *Journal du Conseil* of August 1935, 270 revolutions represent a nautical mile. This results from a calibration, which is necessary with every meter.

The set is measured by streamlining a heavy float far astern and noting the direction of the float cord. To avoid using a light on the float at night, it is well to employ a dumb compass divided into degrees and sectors, and riding on a collar of a short vertical pole, to the center of the top of which the float cord is attached. The dumb compass being set to the ship's heading, either magnetic or true, or being oriented by the magnetic or true bearing of a distant object, the first part of the float cord, passing close above the face of the dumb compass, will indicate the sector of the current set and the number of the slat that should be used in the current box. With the passage of time, perhaps several hours, the cord will gradually move over into the next sector. Then it will be time to put in a new slat.

The measurement of the set, therefore, is free of the magnetic influence of the ship.

At the end of a tidal day the records from all the slats used and laid aside may be compiled in a form similar to the following:

Sector number	Mean direction	Miles in 24 hours 50 minutes		
		First part	Second part	Total
1	0°	7.6	6.8	14.4
2	45	0.0	0.2	0.2
and so on.				

In studying the circulation of water these results may be plotted as vectors, combined in any way desired, or related to the times of high water in a standard port.

The **Pettersson meter and tripod** is designed to measure currents and temperatures at the bottom of the ocean. Above the table of the tripod are the compass and the rudder. Below are the turbines and the register. The tripod has an inverted U-handle like the bale of a bucket pivoted below and weighted above, which is large enough to reach to the floor of the sea when the tension of the supporting cable is sufficiently slackened. The description given by Dr. Otto Pettersson in the papers of the Swedish Hydrographic Biological Commission, 1929, is as follows:

This tripod is lowered by a bronze wire until it reaches the bottom of the sea. When the wire is slackened, the handle of the apparatus turns on its pivots, whereupon the recording instruments are set in motion by the bottom current.

The velocity of that current is measured by a rotating wheel. The direction is given by a compass needle which carries a fine glass tube filled with radioactive substance. The imprint of this moving source of light appears on a photographic film enclosed between two transparent sheets of celluloid glued together.

If the wire can be kept slack during the time of observation, it is not necessary to anchor the ship. When the instrument is lifted from the bottom, the compass needle and the recording current wheel are stopped and the two thermometers turned.

This current meter, which gives the actual direction and velocity of the movement of the water 1 meter above the bottom of the sea, has been in use in every hydrographic expedition in the Skagerrak and Kattegat for the last 3 years.

VELOCITY AND DIRECTION METERS, AUTOMATIC

Dr. Pettersson's automatic current meter, made in 1906 and perfected in 1914, gives an automatic printed record of velocities and directions for 14 days. It is intended to be anchored on

station. The Ekman submarine hydrographic "station" (which anchors the buoy supporting the instrument below the level of wave action at any desired depth, by means of two wire ropes stretched fore and aft to buoyed anchors) is most suitable for anchoring the instrument. The meter is described in the quarterly journal of the Royal Meteorological Society for January 1915; in the Nautical Magazine for May 1917; and with some additions in Special Publication No. 124, 1926, of the Coast and Geodetic Survey.

The frame of the meter consists of a brass cylinder 10 centimeters in diameter and 60 centimeters high, inside dimensions, capable of withstanding a pressure of 800 fathoms of water; a rudder attached to the cylinder; and a skeleton turbine box below the cylinder. The whole is suspended from a bronze ring attached to the lid and rotating on ball bearings. When the lid bolts are tightened, the cylinder is hermetically sealed. The contents, in order from top to bottom, are:

- (a) The controls—an index lamp to show when the camera lamp is working; the clock start-stop; and the handle for setting the time of exposure.
- (b) Clockwork for driving the film and making contact every 30 minutes for the camera lamp. The film advances about 1 inch per hour.
- (c) A small camera loaded like an ordinary camera with 16.5 millimeter Ciné-Kodak film.
- (d) A compass needle and a disk marked with numbers every 10° .
- (e) An opaque velocity disk with graduated edge numbered 1 to 48 in transparent figures.
- (f) A camera lamp, under the edges of (d) and (e), which in turn are under the roll of film in (c).
- (g) Two dry cells to light the lamp.
- (h) An axis terminating above in the velocity disk and below in reducing gears leading to a permanent magnet close to the bottom of the cylinder. The compass needle is protected from the magnet field by soft iron disks just above the gears.

The turbine box contains:

- (i) A permanent magnet close to the bottom of the cylinder.
- (j) Reducing gears.
- (k) Current vanes.

The rotation of the vanes is transmitted, reduced, to the outside magnet; and the rotation of the latter is followed by the inside magnet, rotating, at a further reduction in speed, the axis and the velocity disk. When the clock contacts and lights the lamp, the numbers on the two disks, and also a fiber stretched across the opening, are photographed. The successive images of the latter give reference lines for interpolation. An interpolation to tenths gives a precision of about 1 centimeter per 5 seconds in velocity and 1° in direction.

The meter may be rated in the field, or in Washington by the Bureau of Standards. It will be found convenient to transform these results, usually given in feet per second per revolution of the vanes, into a table of knots corresponding to dial differences for 30 minutes. It requires 1,800 revolutions of the vanes to cause one revolution of the velocity dial, in the type described, and in any case the ratio of reduction may be obtained by counting the teeth in the gears.

The **Idrac automatic current meter**, the ordinary type, manufactured by Barbier, Benard, and Turenne, Paris, gives a continuous graphical record for 7 days, and is intended to be used in depths up to 50 fathoms, when anchored on station. A lighter model has been used in the Straits of Gibraltar in 600 meters, from a rowboat. A heavy type, for depths up to 1,500 meters, runs 4 hours.

The velocity record is obtained photographically on the upper part of a strip, 1.75 meters long, of ordinary cinematograph film unwinding at fixed speed in front of a slit through which a beam of light is projected at every N revolutions of a spindle rotated by cup vanes and reducing gear. Two degrees of sensitiveness are available, for high and low speeds. The slit photographs as a vertical line. By counting the number of lines in a unit of length and by applying the result to a speed curve furnished with the instrument, the velocity may be found.

On the lower half of the film, devoted to directions, three white points projected through a slit crossing a black compass card on which are painted in white an inner circle, an outer circle, and a spiral of Archimedes joining them at their north points, are photographed as dots in close succession. By joining the dots, three graphs are obtained, those coming from the circles being straight boundary lines, while that coming from the spiral gives a series of ordinates, measured

from the north line, proportional to the angle from north at which the current is flowing at any time. A collimating lens is employed to project the white points.

The determination of velocity is good, but, on account of the narrowness of the film, the determination of direction is feeble.

AUTOMATIC AND NON-AUTOMATIC CURRENT METERS

In the choice of current meters, the saving in time, labor, fuel, and boats, and the gain in respect to continuity and completeness of results, are prime considerations.

Inside the 100-fathom curve, in bays and sounds and on offshore banks, there is little doubt of the superiority of automatic meters anchored on station, for measuring surface, sub-surface, and bottom currents. The limiting condition is not depth, but the security of the anchorage of the buoys by means of which the instrument is recovered. With seas or a great range of tide, there is some danger of losing the apparatus.

In the next depth zone, say 100 to 250 fathoms, a ship engaged solely or principally in current observations would probably anchor and use a suspended meter of the most convenient type, considering other observations going forward at the same time, (e. g., density and temperature observations). For various depths, as every 25 fathoms, the Rauschelbach meter might be most convenient. On the other hand, a ship engaged principally in sounding might find free floats, surface and subsurface, more advantageous, as they could be dropped near the middle of each day's sounding area and cut in from time to time from nearby sounding positions with very little expenditure of time.

In water too deep for anchoring a ship, the only possible method of measuring currents (off the bottom) is by the use of free floats.

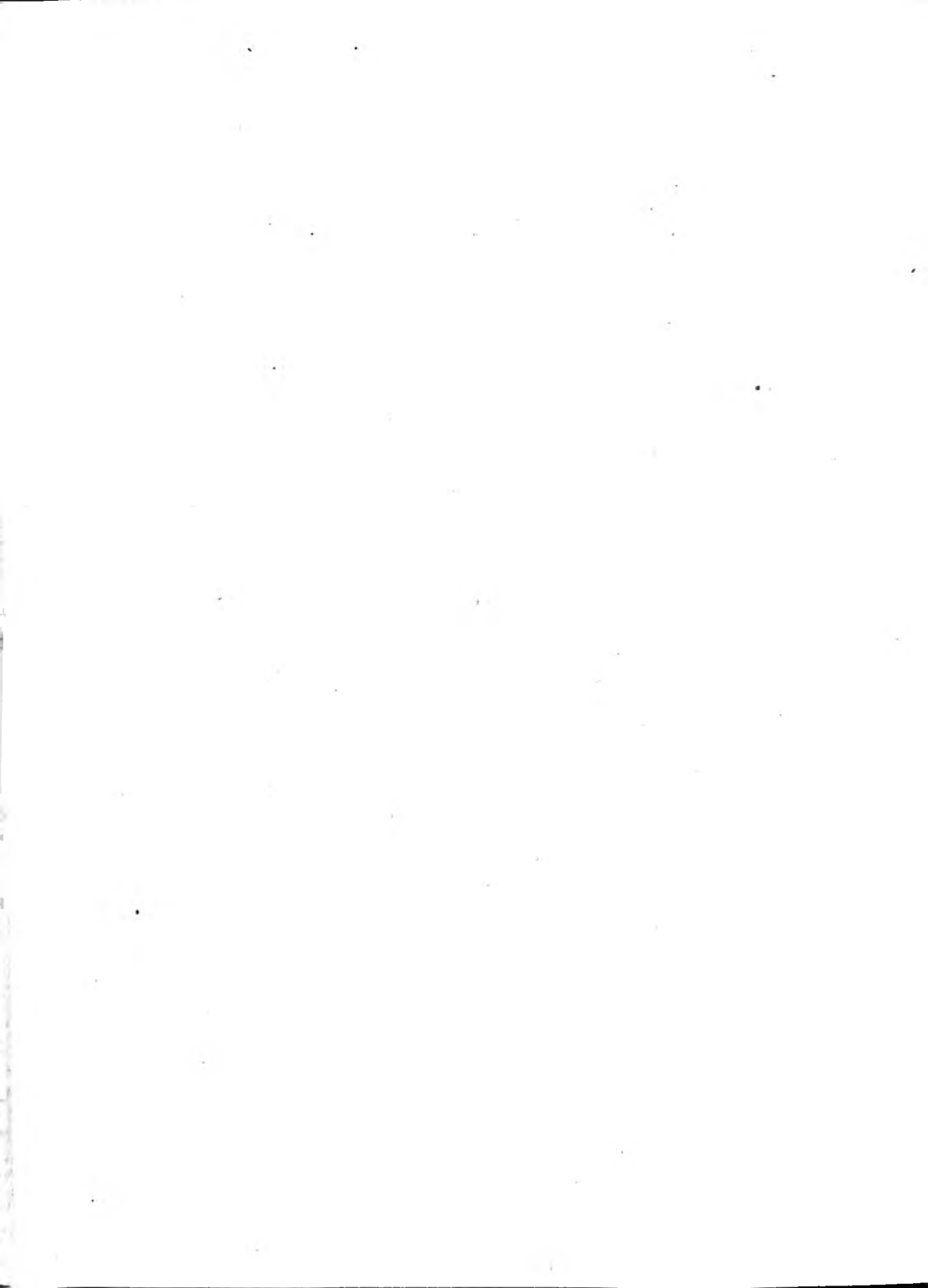
CURRENT OUTFIT

Expeditions engaged in general coast surveying including investigations of a scientific nature should devote part of each season to observations of the density and temperature of sea water, as these quantities affect the circulation of water and the distribution of marine plant and animal life. The observations may well be related to echo and wire sounding and current observations. In most cases, a week's work by one sounding unit would suffice to gather a body of significant facts comparable in value to the entire body of local inshore soundings.

The following outfit is proportioned to expeditions employing as many as eight sounding boats:

For current work inside the 100-fathom curve, three automatic current meters. Pettersson or Idrac, two in use and one spare.

For deep-water work, a Carruther's vertical log current meter, or a Pettersson automatic meter and tripod, and free floats made aboard ship as the need arises. For important and extensive ocean-current surveys, a Rauschelbach graphical recording meter. The latest type of this instrument is called the Askania current meter.



CHAPTER VI

BASE LINE MEASUREMENT

MARKING AND REFERENCING THE ENDS

It is often advisable to use towers at the ends of a base line to lift the line of sight above the heated stratum of air near the ground. In that case the distance between the heads of the towers becomes the **visible base line** that is carried forward to the triangulation net. The towers should be rigidly constructed on concrete foundations and should be exactly alike, if it is intended to avoid small but appreciable errors in length and azimuth due to the displacement of the heads by solar heat and wind pressure. An inequality in temperature of 20° between the opposite sides of an 80-foot steel tower will cause the tower to lean away from the sun about half an inch. In windy weather coverings or "decorations" may cause a displacement amounting to several inches. The towers should be allowed at least 3 days to attain equilibrium through the action of the wind, the sagging of the joints, and the equalizing of pressure on the footplates. Guys, if used, should be slack; and decorations should be limited to a few yards of black cloth tightly stretched about the narrow part of the tower under the platform.

To transfer the visible base line to the ground:

Center a pin in a plug in the pipe at the head of the tower. At or near the center of the tower turn an angle of 90° from the distant tower, and place a transit on or near this line at a distance equal to about twice the height of the tower, sufficient to avoid a change of focus.

With a second transit range in, within an inch or two, between the two towers. Sight with both transits on the pin at the head of the tower.

If the transits are perfectly adjusted, their lines of sight, when the telescopes are plunged, will generate vertical planes intersecting in the vertical of the pin. However, it is well to assume that the transit off the base line is not in perfect adjustment, and to establish a truly vertical plane for it by "double centering", using telescope direct and reversed; and also to establish a "backsight."

The intersecting lines of sight will serve to place the surface mark in position and also to locate a short plumb line for placing the subsurface mark.

For accuracy, the base-line monuments should be set simultaneously, or at least under similar conditions.

For the sure recovery of the subsurface marks after the destruction of the monuments, perpendicular distances should be measured with a tape to two or more ranges likely to be permanent, if any are available within several hundred feet.

CLEARING AND ESTABLISHING THE LINE

Clearing close to the ground favors a low suspension of the tape and the avoidance of accidents to it. If the clearing is made wide on one side of the line and narrow on the other, the tendency to walk and stand on the line of sight while transitmen are waiting for the line to be cleared will be discouraged, and fewer stakes will be disturbed by stumbling feet. In grazing land it may be necessary to drive extra stakes for fenders.

At the summits of rises, if the profile of the base line is undulating, and in any case at intervals of half a mile or more, transit points should be established on line, with considerable care, by ranging in between the towers when atmospheric conditions are good. Near these points triangular black **range banners** mounted on poles 12 to 15 feet high, braced along and across the line, may be set with the peaks exactly on range and with the feet far enough off the line to allow the tape to pass. By means of these the front chainman can set his stakes nearly on line at the first trial, and the transitman directing the alinement can keep within effective distance of the work without loss of time. The disturbing effect of heat waves will be eliminated to a large extent, and it will be found easier and more accurate as well to range in between the banners than between the distant towers.

For ranging in between two objects the transit must be free of collimation error. This may be removed at the first station, and for the purpose in hand may be considered to have been removed when—after orienting on the nearer object and transiting, with the vertical circle left of the telescope, and repeating the operation with the vertical circle right of the telescope—the line of sight cuts the center of the distant object in both cases. Ranging in may be done systematically by estimating the relative distances of the objects and using a pocket rule to measure the distance from the point of the plumb bob to the next set-up. For example, if the station is approximately one-fifth as far from one object as from the other and if the line of sight after transiting appears to miss the distant object by 3 feet, the transit is about one-sixth of 3 feet off the line.

STAKING OUT THE LINE

When there is good footing for stakes, they may be made of 1- by 4-inch, or 2- by 4-inch, or 3- by 3-inch material that does not split easily, in 30-inch lengths for a well cleared base on flat land, and in greater lengths for rolling or swampy land, according to need. It is important to square the tops, to chamfer the top edges to avoid brooming, and to taper the points symmetrically in order that the stakes may drive straight. For braces, $\frac{3}{4}$ - by 3-inch batten material is convenient. Every stake requires to be braced lightly across the line and substantially along the line. In mud that cracks while drying and in loose sand, it is well to brace each stake both forward and backward along the line.

The stakes may be set in advance of the regular measurement by a special party using a 100-meter piano-wire tape, thus saving the invar tape from excessive use and possible injury. It may be graduated by comparison with a distance between two stakes, the first and third of three sets by using the 50-meter invar tape in the intended manner, but with the piano-wire tape suspended only at the ends, and with a tension decreased to the point where the catenary of the latter is a little lower than the catenaries of the invar tape, thus insuring that the latter will hang clear in use. Marking sleeves are required at the ends and at the middle. The middle and forward sleeves should be graduated in centimeters to permit allowance for difference in temperature, amounting to about 1 centimeter for every 14° F. or for every 8° C. at the forward sleeve, and half that amount at the middle sleeve.

When the footing for stakes is bad, with no great irregularity in elevation, **portable stakes** may be devised, consisting, in general, of stakes mounted and braced four ways with angle iron, on wooden built-up bases 2 feet square or more. The bases are armed on the bottom with spikes or bolts, and are held down at the corners by driven hook-bolts. This form is useful on beaches on sand dunes, and in swamps. Three stakes are required—one for moving up while the other two are in use. On gravel and macadam roads, on rocks, and on ice the grip on the earth may be obtained by cone-pointed bolts threaded their entire length and adjustable in height through nuts in the center layer of the base; and by weighting the base.

In water, on rough rocky shores, and on very irregular ground, **portable tripods** and even scaffolding are sometimes used. Iron tripods are provided with ball-and-socket joints for adjusting a small marking table that carries a strip of copper for receiving marks opposite the tape ends.

The use of high stakes or other supports not well braced is a fruitful source of systematic errors due to—

A slight friction of the tape at the time of marking.

Laying a hand on the tape to steady the application of the tape to the stake.

Unequal distribution of weight about the stake.

Walking near the stake on quaking ground.

MEASUREMENT WITHOUT MARKERS, USING TELESCOPIC SIGHTS

In measuring along a graded road or railway, especially when the tape is laid flat, or in measuring over a flat plain where low stakes can be used, the telescopic sights of two transits may be used to define the ends of successive tape lengths. Precautions must be taken to avoid disturbing the transit that is holding the distance at the forward end. It is well to shade the transit. The transit should be within 10 feet of the end of the tape and nearly at right angles to the line. To

guard against losing the point, as soon as the sights have been placed on the forward end of the tape place a pin on the line of sight near the tape, without changing the focus of the telescope.

TAPE STRETCHING APPARATUS

When the tape is to be suspended, the rear stretcher staff is preferably a crowbar about 7 feet long. The tape is attached to it by a loose leather thong with one full turn about the crowbar, giving an easy adjustment for height when the tension is slackened. The crowbar is steadied during the operation of getting rear contact by a light line to a pulley carried on an 18-inch iron rod driven about 10 feet to the rear. (See Fig. 87.)

The front stretcher staff may be like the rear one, but without the backing up rod; or it may be an iron tube, pointed below, on which runs a loose friction collar carrying a frame for supporting a spring balance, the frame being pivoted and free to take a horizontal position to which it is held by an adjustable counterpoise; or it may be an iron-shod split staff carrying a

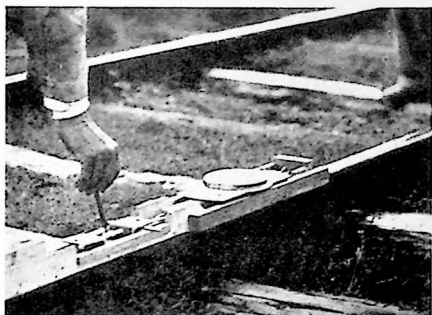


FIGURE 83.—Forward end, adjustment to standard tension.

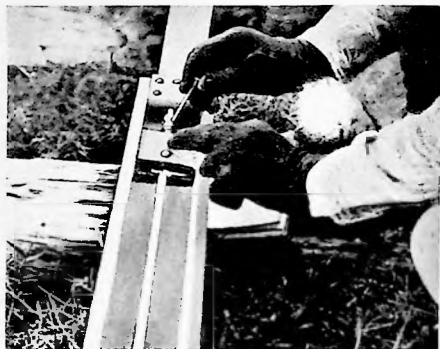


FIGURE 81.—Rear end, adjustment of zero to telescopic sight of transit.

ball-bearing pulley over which passes a cord connecting the forward end of the tape to a weight intended to give the tape the proper tension.

When the tape is laid "flat", the tension may be applied by hand, or by levers, or by slow motion screws.

MEASURING ON RAILROAD RAILS

In the laboratory the certified length of an invar tape is obtained indirectly, by computation from the standardized length of the tape suspended in catenaries, thereby eliminating friction. In the field the element of friction enters when the tape is supported throughout its length, and furthermore, the amount of error thus introduced is variable, due to moisture, rust, and roughness of the rails.

The error due to friction may be made the same for all tape lengths, and so may be estimated, by employing a wooden tape carriage; and the amount may be minimized by lifting the tape about an inch off the supports at intervals along the tape under full tension, and allowing it to fall back into position, first, near the rear end, then near the middle, then near the forward end.

Jerks on the tape, twistings, kinkings, and sudden changes in temperature due to laying a tape on a very hot or cold rail may be avoided by using a wooden carriage to transport it and to insulate it.

A solution of problems often encountered in the measurement of base lines on rails may be illustrated by the experience of the U. S. S. *Hannibal* in 1923 in measuring the base Batabano to Pozo Redondo.

A stadia survey showed that $4\frac{1}{2}$ miles of straight railroad tracks in good condition were available. At one end the tangent terminated at the shore and at the other in cultivated fields, so that the base-line towers could not be placed on the extensions of the tangents. They were placed near the tracks and on opposite sides thereof.

A carriage like that shown in Figures 83-85 was made in sections and assembled ashore. At the rear end two iron blocks, one fastened to the carriage and the other movable by means of a capstan screw, were placed in a recess of depth corresponding to the thickness of the distance pieces *D*. At the forward end a longer recess accommodated two similar slow motion blocks and the spring balance. The distance pieces *d* were made removable, on account of the fishplates at the junctions of the rails.

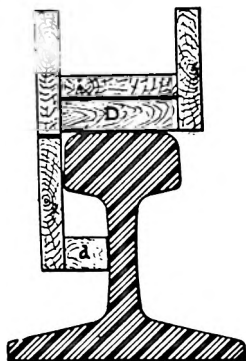


FIGURE 85.—Tape carrier.

Two transits were set up centrally over the east rail, no. 1 opposite the southern tower, and no. 2 far enough up the line to furnish a short base for fixing tower B at by triangulation. Also transit no. 3 was set up about 9 feet from no. 1 on the line of sight of the latter at 90° from the direction of the track. Transits nos. 1 and 2 were now removed, leaving no. 3 with its vertical cross-hair holding the intended initial end of the base line. No. 3 transitman immediately established a mark on the rail to represent the initial mark, and a pin in the ground near it—a precaution against accident taken at every set-up. In the meantime two men with a wire tape measured 50 meters up the track and opposite this point no. 1 was set up.

The carriage was now placed on the track, the tape was stretched therein with light tension, and the carriage was pushed along the track until the zero mark was within a centimeter of position. The following operations now went on practically simultaneously:

1. The front chainman took up the remaining tension and when it had steadied to standard kept it there by using the capstan bar.
2. The rear chainman, under the direction of transitman no. 3, adjusted with the slow motion the zero of the tape to the line of sight of the transit.
3. Three tapemen, A, B, and C, at intervals along the tape in order counting from the rear, lifted the tape about an inch and allowed it to fall back into position, first A, then B, then C. This was done several times and, finally, just before the measurement.
4. At the word that the rear mark was on, the tension full and steady, and the tape clear (that is, alined and freed from friction), transit no. 1 (in adjustment and all motions clamped) was sighted on the 50-meter mark; and as soon thereafter as possible, without changing the focus of the telescope, the station pin was set, as described above.
5. The recorder attended to taking the temperature of the tape and to recording the events at "station 1."
6. The wire tapemen established the approximate position of station 2, and opposite it transit no. 2 was set up.

The first application of the tape finished, the tension was relaxed and the tape disengaged from the hook to avoid strains while being carried forward; the rear chainman announced "station 0", and the front chainman and the recorder "station 1"; the tape and all operators except transit-

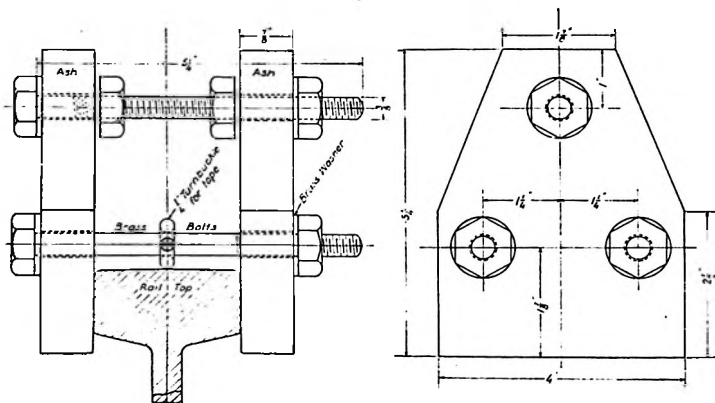


FIGURE 86.—Rail clamps

man no. 1 and the painter went forward, the latter stopping to mark the flange of the rail and the end of the nearest tie for the benefit of the level party; and the second application of the tape began.

At intervals of about a kilometer, stakes were established on both sides of the line, marked with knife cuts in copper tacks, to divide the base into sections any one of which might be remeasured in case of a suspected error.

The distance, 154 tape lengths, was measured in 1 day on the east rail, and on the next day in the opposite direction on the west rail from initial points taken at random; including triangulation connections with the towers and their center marks. The check was made on the length of the **working base**, as computed from the measurements made on the separate rails. The difference amounted to 1.75 centimeters. Another party ran levels over the line beginning at a bench mark connected with the tide gage.

RAIL CLAMPS

In a recent development of measurement on rails, the carrier, which is rather cumbersome and meant principally to protect the tape from the heat of the rails, was dispensed with by limiting the times of measurement to early mornings and rainy days, when the rails were cool. For this purpose clamps were fashioned of semi-oval pieces of ash wood (Fig. 86), fitting on either side of the head of the rail and joined by three bolts, two through the wide parts of the ovals and one through the narrow ends, all horizontal. Of the two, one vertically over the other, the lower rested on top of the rail, provided a fastening for the stray end of the tape, and was the means by which the railhead was pinched, the bolt above it acting as a fulcrum. To avoid fishplates connecting rails, the clamps were cut off at the bottom of the railhead. Clamps like this were used at both ends of the tape. Slow motion under tension was obtained by a turnbuckle in the stray line. Plusses and the use of transits were avoided by using white adhesive tape for markers, and marking tape ends as dots opposite the fiducial marks, employing a hard pencil with a fine round point.

PROBLEMS ENCOUNTERED WHEN THE TAPE IS SUSPENDED

The governing principle is that the tape must be in equilibrium between two forces only, gravity and the standard tension. Accuracy of measurement, therefore, is mainly a matter of eliminating disturbing forces.

WIND SHIELD

A light breeze, sufficient to give a circulation of air about the tape, is to that extent favorable for base measurement. Such an ideal condition seldom prevails.

The figure, showing the arrangements adopted by the U. S. S. *Eagle* in measuring a base at Port au Prince in 1912-13, illustrates in other ways good practice in suspending tapes. A wind shield 310 feet long was made of two thicknesses of 36-inch muslin held in place by battens 25 feet apart. The stakes were substantial and were braced in two directions. The rear stretcher consisted of a $\frac{3}{4}$ -inch round iron rod, up and down which a grommet attached to the end of the tape was free to move. A cod line guntackle with the fixed block attached by a loop to the backing-up rod, and the pulley to the stretcher rod, afforded a double purchase to the operator while watching the movement of the rear mark. (See U. S. Naval Institute Proceedings, Vol. 39, No. 4, p. 1511.)

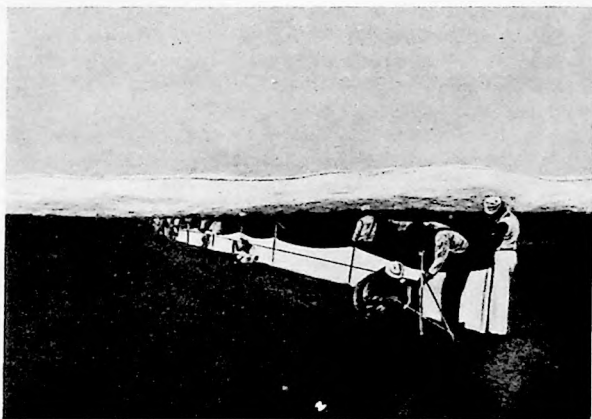


FIGURE 87.—Wind shield and rear backing-up rod.

Marking sleeves and patches are not permissible between end supports, the correction for sag being computed on a uniform cross-section. Points of intermediate support may be marked with paint.

Thermometers must not be suspended from the tape except as described in the certificate of standardization.

APPARATUS FOR SETTING INTERMEDIATE SUPPORTS

For each intermediate support in a tape length provide a **hanger** made of strap iron and designed to support the tape without friction at a distance h below a nail driven horizontally into the side of an intermediate stake. Also provide **two blocks** of 2-inch by 4-inch wood sawed to the height h , and into the end of one of them fasten a small **telescope** with its axis in the upper plane on the block. The intermediate stake, a temporary stake carried by the intermediate chainman, may be made of 2-inch by 3-inch material with holes bored every two inches to receive, with a snug fit, a wire nail painted white.



FIGURE 88.—Tape hanger.

The observer at the rear stake places his block on top of the stake and by sighting through the telescope at the top of the similar block held on the front stake is enabled to set the white nail for the intermediate support on grade between the tops of the blocks, which ensures that the tape will be supported on grade between the tops of the end stakes. The tape will take its own line when the stake is so adjusted that the tape touches neither side of the hanger. A light tap with the finger will bring the hanger to a vertical position, supporting the tape without friction. The limit of tolerance in the longitudinal position of an intermediate stake may be taken as about half an inch.

CHANGING FROM "TAPE FLAT" TO "TAPE SUSPENDED"

When it becomes necessary to raise or lower a mark at the end of a tape length, the transfer should be made with the aid of the telescopic sights of a transit. A plumb bob is a makeshift too crude for the purpose. Both marks should be left for the levelman. In some cases a double mark can be avoided by digging a small channel for the first part of the tape catenary.

OFFSETS

Figure 89 suggests several methods of passing an obstruction on a base line.

1. Fix Q by triangulation from B and C , two tape-end points of the base line. Solve triangle CBQ for BC ; or
2. Find R , a whole number of tape lengths from B , and measure DBR and DCR , D being a distant point of the base line. Solve triangle CRB for BC ; or
3. At B set off 90° and measure BE by tape or stadia. At D set off 90° and make $DF' = BE$. Measure EF' on the line EF' as carefully as if it were a part of the base. At F make $F'FC = 90^\circ$ and find C as the intersection of FC and BD , both sights telescopic; or
4. At B make $DBM = 60^\circ$ and measure BM an integral number of tape lengths. Make $M = 60^\circ$ and measure $MC = MB$.

Method 3 is the most generally convenient one, and the best, provided that the angles are turned from the distant foresight rather than from the short sides BE and EF . Number 4 is usually the poorest method. Numbers 1 and 2 are good when the receiving angles exceed 30° . By any method angles should be turned from a distant point, and the sights should be carried around telescopically, without recourse to the plumb bob.

For example, in method 1 it is essential to use a third transit to place the transits at P and B vertically over the ground points; and the point Q , when sighted from P and B , should be represented by a transit, not by a stake. For the triangle CBQ place a transit at C telescopically on the line BD ; and after observing the three angles find the ground point at C by means of a transit near and opposite the transit at C .

CORRECTION FOR ERROR OF SPRING BALANCE

When a spring balance, by design a weighing machine, is used to measure the horizontal tension in a tape, account must be taken of the decreased tension in the spring due to the removal of the unknown weight of the movable parts of the balance. The following sources of error, also, are or may be present.

Difference between temperature of use and that of standardization.

Variation of the force of gravity with latitude.

Displacement of the dial hand due to fatigue of the spring, or to sudden changes in tension, or to shock.

When submitting the tape and the balance for standardization it is well to request a statement of the variation of dial readings for a considerable range of temperature; also a statement

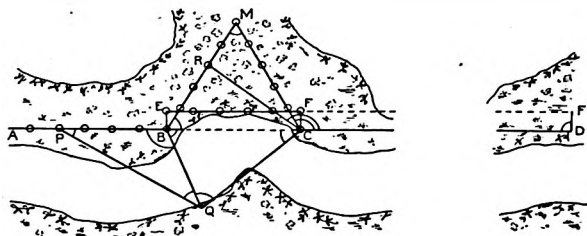


FIGURE 88.—Offsets from a base line.

of the reading of the balance (being in adjustment as a weighing machine) when used to stretch the tape with the standard tension.

A standard mass should be taken to the field for testing the balance during base measurement.

Let
and

W = weight of standard mass in Washington,

W' = local weight of the same in latitude ϕ .

$$W' = \frac{980.095 W}{\gamma_0},$$

in which

$$\gamma_0 = 978.039[1 + 0.005294 \sin^2 \phi - .000007 \sin^2(2\phi)].$$

The amount that must be added algebraically to the scale reading to indicate a tension of W' in the spring, or the **tension index correction** of the balance, may be obtained by using a so-called frictionless pulley, or less accurately by using a well-greased spar, as follows:

The balance being held horizontal and connected with the weight W' by a cord of weight w over the pulley supporting the weight $W' + w$ at rest, gradually increase the tension until the pulley begins to move, and note the reading. Again, with the weight at rest, gradually decrease the tension until the pulley begins to move, and note the reading.

Let T' be the mean of the readings and C the tension index correction. Then for a tension equal to $W' + w$, or for any tension within the elastic limit of the spring, and for the particular temperature of the test, $C = W' + w - T'$.

$W' - W$ is small but not always negligible. At the equator, if $W = 15$ kilograms, $W' - W = 31\frac{1}{2}$ grams.

A rough value of C may be found by using for a test weight the known weight of the balance itself. Let B be the weight of the balance, f the weight of the frame and fixed parts, s the weight of the spring, m the weight of the other movable parts, r the reading of the dial when the balance is suspended hook down unloaded, and R the reading when it is suspended hook up. Then since $f + s + m = B$ and C is the actual tension less the indicated tension, we have

hook down, weighing $m + s$, $m + s = r + C$

hook up, weighing $f + s$, $f + s = R + C$

$$\text{Therefore } C = \frac{1}{2}(f + s + m) - \frac{1}{2}(R + r) = \frac{1}{2}[B - (R + r)] + \frac{1}{2}s$$

By neglecting the unknown but relatively small term $\frac{1}{2}s$, this becomes

$$C = \frac{1}{2}[B - (R + r)].$$

The formula with the term $\frac{1}{2}s$ included applies to simple tongue balances that measure weight by elongation of a spring. This term disappears for balances that measure weight by compression of the spring.

CORRECTION FOR ABSOLUTE ERROR OF TAPE

The correction for absolute error is that required to change the nominal length of the tape, an integral number of feet or meters, to the actual length under the conditions of standardization.

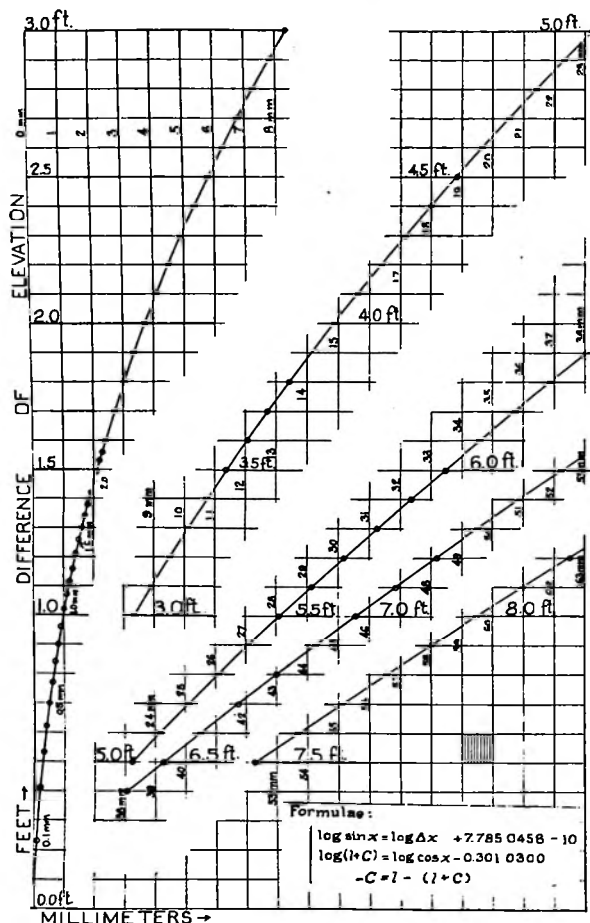
CORRECTIONS FOR INCLINATION
50-Meter Tape Lengths

FIGURE 90

Reduce all the differences to meters and proceed as in case 1. Or, provided that the grade does not exceed 6 percent, compute all the corrections for Δh feet and d feet, and multiply the sum of the corrections by the square of the conversion factor from feet to meters, or by the number whose logarithm is 8.968, 0316-10. Or, for 50-meter tapes use the accompanying graph. It is sufficiently accurate to obtain the separate corrections to the nearest tenth of a millimeter.

CORRECTION FOR SAG

If a length of metal, L , of uniform cross-section and of uniform weight w per linear unit is stretched by a moderate tension T between horizontal supports d units apart, the correction

This correction multiplied by the number of tape lengths gives the total correction for absolute error.

CORRECTION FOR INCLINATION

Let x be the angle of inclination to the horizontal of a straight line d units long; h the difference of elevation of the ends of the line; and $-C$ the correction for inclination.

Then $\sin x = \frac{\Delta h}{d}$
and $-C = d(1 - \cos x)$.

It is convenient to use for d the nominal length of the tape, which is integral and constant, rather than the actual distance between end divisions; and to apply to the sum of all the inclination corrections, when necessary, an additional correction based on the ratio of d to the average tape length with other corrections applied. Among the latter the set-forwards and set-backs need not be included, because they tend to sum to zero.

Case 1.—When Δh and d are expressed in like units.

The corrections for all whole tape lengths may be obtained, by easy mental computation, by using a 5-place table of natural sines and cosines, without finding the angle x , as follows:

Find $\sin x$ to five decimal places by dividing Δh by d .

In the table find $\cos x$ opposite $\sin x$.

Subtract $\cos x$ from 1 and multiply the result by d .

Case 2.—When Δh is expressed in feet and d in meters.

$C=d-l$ that must be applied to l to give d is composed of two elements, namely, the correction for sag, which is negative, and the correction for stretch, which is positive.

Suppose that the tape is measured in n equal catenaries each l units long, so that the total length of metal is $L=nl$; and that the friction of the supports has been minimized to the point where it may be assumed that the tension in each catenary is T . For the short lengths of metal used in ordinary base measurement the catenary equation may be replaced by the more simple parabola equation. The correction for sag, then, is equal to $\frac{1}{24}$ of the length of the tape multiplied by the square of the ratio of the weight of each catenary of the tape to the tension applied; or

$$C_s = -\frac{L}{24} \left(\frac{wl}{T} \right)^2,$$

C_s and L being expressed in like units, also the forces wl and T .

Illustrations.—Find the corrections for sag for a 50-meter tape weighing 25.5 grams per meter, under a tension of 15 kilograms, when supported horizontally, as follows:

- At the end divisions only.
- At the 0-, 25-, and 50-meter points.
- At the 0-, $12\frac{1}{2}$ -, 25-, $37\frac{1}{2}$ -, and 50-meter points.

	wl , kilograms	T , kilograms	L , meters	C_s , meters
(a).....	1. 275	15	50	—0. 015052
(b).....	0. 6375	15	50	—0. 003763
(c).....	0. 31875	15	50	—0. 000941

The sag varies inversely as the square of the number of catenaries.

CHANGE IN SAG RESULTING FROM A SMALL CHANGE IN TENSION

Regarding C and T in the sag formula as variables and the other quantities as constants, and differentiating,

$$\frac{dC_s}{dT} = -\frac{2C_s}{T},$$

whence for small finite changes

$$\Delta C_s = -\frac{2C_s}{T} \Delta T.$$

Illustration.—The certificate of the above-mentioned tape and spring balance stated that the distance between the 0- and 50-meter marks, with a tension of 15 kilograms applied to the tape supported horizontally, was

For supports at the 0-, 25-, and 50-meter marks, 49.99760 m.

For a continuous support the whole length, 50.00137; and that the balance set up horizontally and loaded with 15 kilograms indicated on the average, 14.759 kilograms.

The balance was used with an indicated tension of 14.8 kilograms. Find the change in the sag correction.

Applied tension, $T' = 15 \left(\frac{14.8 \text{ kg}}{14.759} \right) = 15.042 \text{ kg.}$

When $T = 15 \text{ kg}$, $C_s = 49.99760 \text{ m} - 50.00137 \text{ m} = -0.00377 \text{ m.}$

Hence in the difference equation $C_s = -0.00377$, $T = 15$, and $\Delta T = 0.042$ give

$$\Delta C_s = +0.00002, \text{ nearly;}$$

that is, an increase of 42 grams in the tension reduced the sag correction 0.00002 meter and correspondingly increased the effective length of the tape.

BASE LINE MEASUREMENT

CORRECTION FOR STRETCH

The correction for stretch is to the length of the tape as the increase of tension is to the area of the cross-section multiplied by the modulus of elasticity of the material;

or

$$C_s = L \left(\frac{\Delta T}{EA} \right).$$

The modulus of elasticity of steel is about 28 million pounds per square inch, and of invar about 15 metric tons per square millimeter, or somewhat greater than 21 million pounds per square inch. A metric ton is equivalent to 2204.62234 pounds, log 3.343,3343; a square millimeter is equivalent to 0.001,549,997 square inch, log 2.809,6684.

Illustrations.—1. For the invar tape of the preceding illustration $\Delta T = +0.042$ kilogram, $A = 3.2522$ millimeters, $E = 15000$ kilograms per square millimeter, L (nominal) = 50 meters; therefore $C_s = +0.00003$ meter.

2. The total correction due to a tension of 15.042 kilograms is—

Sag at standard tension.....	—0.00377 meter.
Change in sag due to increased tension.....	+0.00002 meter.
Stretch due to increased tension.....	+0.00003 meter.
Total.....	—0.00372 meter.

CORRECTION FOR TEMPERATURE

For ordinary atmospheric ranges of temperature the correction may be considered a linear function of the mean temperature of the tape, t_m , above that of standardization, t_s , and to be equal to this difference multiplied by the coefficient of expansion of the metal, a , and by the whole length, B , of metal used in the measurement; or

$$C_t = (t_m - t_s) a B$$

Field temperatures more than a few degrees outside the range for which the certificate gives an average value should be avoided.

REDUCTION TO SEA LEVEL

The length of a base line reduced to sea level is to the measured length (with all other corrections applied) as the radius of curvature of the earth is to the radius increased by the mean elevation of the base. The latter may be rounded off to whole meters.

The radius of curvature of a section having the azimuth Z , computed from the formula

$$R_z = \frac{RN}{R \sin^2 Z + N \cos^2 Z}$$

will apply to the vertical section of the base line, its azimuth being Z , but not to that of any other side of the triangulation for which the adopted value of the base line furnishes the unit of length. In most hydrographic surveys the elevation of the base is small, so that the choice of a particular radius is of little consequence. Usually the mean of the equatorial and polar radii may be used. It is

$$\frac{1}{2} (a+b) = 6,367,395.1 \text{ meters.}$$

The following table is taken from United States Coast and Geodetic Survey reports. The Geodetic Survey of Canada, Publication No. 7, publishes a more complete table for latitudes 42° to 56° N.

TABLE 8.—Radii of earth's curvature

R, in meridian; N, in prime vertical

Latitude	log R	log N	Latitude	log R	log N
0°	6. 8017489	6. 8046985	45°	6. 8039574	6. 8054347
1	7502	6990	46	6. 8040346	4604
2	7543	7003	47	1117	4861
3	7610	7025	48	1887	5118
4	7704	7057	49	2653	5373
5	7824	7097	50	3416	5628
6	7971	7146	51	4175	5880
7	8144	7203	52	4928	6131
8	8343	7270	53	5674	6380
9	8568	7345	54	6413	6627
10	6. 8018819	6. 8047428	55	6. 8047144	6. 8056870
11	9094	7520	56	7866	7111
12	9395	7620	57	8578	7348
13	9720	7729	58	9279	7582
14	6. 8020070	7845	59	9968	7811
15	0443	7970	60	6. 8050644	8037
16	0839	8102	61	1307	8258
17	1258	8242	62	1956	8474
18	1701	8389	63	2590	8685
19	2165	8544	64	3208	8891
20	6. 8022649	6. 8048705	65	6. 8053809	6. 8059092
21	3155	8874	66	4393	9287
22	3680	9049	67	4959	9475
23	4225	9231	68	5506	9658
24	4788	9418	69	6034	9834
25	5370	9612	70	6542	6. 8060003
26	5968	9812	71	7029	0165
27	6584	6. 8050017	72	7495	0321
28	7215	0227	73	7938	0468
29	7862	0443	74	8361	0608
30	8522	0663	75	8759	0742
31	9197	0888	76	9135	0867
32	9883	1117	77	9487	0984
33	6. 8030582	1350	78	9814	1093
34	1292	1586	79	6. 8060118	1195
35	2012	1826	80	0394	1287
36	2741	2069	81	0646	1371
37	3479	2315	82	0873	1446
38	4224	2564	83	1074	1513
39	4976	2814	84	1248	1571
40	6. 8035734	6. 8053067	85	6. 8061395	6. 8061620
41	6496	3321	86	1517	1661
42	7262	2576	87	1611	1692
43	8031	3822	88	1715	1679
44	8802	4089	89	1719	1728
45	6. 8039574	6. 8054347	90	6. 8061733	6. 8061733

INVAR TAPES

For authoritative information on the behavior of invar consult Bureau of Standards Circular 58, edition of 1923, entitled "Invar and Related Nickel Steels", including a bibliography.

Invar, so named by Guillaume after his successful experiments in search of an alloy invariable for temperature, is a nickel steel containing about 36 percent of nickel, 0.5 percent or less of manganese, about 0.1 percent of carbon, and negligible amounts of other elements. Below 200° C. and at a temperature dependent on its history and exact composition it undergoes a reversible transformation of such a nature that for any sample the transformation may be incomplete. This condition of thermo-chemical instability gives rise to both slowly changing and

quickly changing values of its physical properties—changes particularly manifested in the expansion.

The mean coefficient of linear expansion between 0° and 40° C. is of the order of one or two millionths, or as usually expressed, 1 to 2 microns per meter, a micron (μ) being the millionth part of a meter. This is a convenient basis of comparison and a convenient unit for estimating other variations that may be considered negligible in geodetic work only if the proper precautions are taken in view of the molecular instability of invar.

The coefficient of expansion of invar tapes is about one-eleventh that of steel tapes, using for comparison average coefficients (per degree Centigrade) of 0.000011 for invar and 0.000117 for steel.

The following table, due to the experiments of Benoit and Guillaume, shows the **elongation of invar with lapse of time**, following a treatment of cooling for 3 months from 100° to 25° C.

TABLE 9.—*Elongation of invar with lapse of time*

Days after completion of treatment	Elongation in microns per meter	Days after completion of treatment	Elongation in microns per meter
0	0.0	500	4.4
100	1.5	600	4.9
200	2.5	700	5.4
300	3.2	800	5.8
400	3.9	900	6.2
		1,000	6.6

Thus, if a 50-meter tape, standardized after being aged for 300 days by the manufacturer, is used 200 days later to measure a base, the change in length to be expected is

$$50(4.4 - 3.2)\mu = 60\mu = 0.000060 \text{ meter.}$$

If the tape is carried over to another season without a new determination of its equation, say another 300 days, the total increase in its length, due solely to the time elapsed, is

$$50(5.8 - 3.2)\mu = 130\mu = 0.000130 \text{ meter.}$$

The experiments of the same metallurgists showed that **temporary length variations of invar due to rapid changes in temperature** amount to $-0.00325t^2$ microns per meter, 0° being the initial temperature and t the final temperature. Circular 58 interprets "seasonal" temperatures as having been obtained slowly, and daily temperatures, such as hold for several days during a base measurement, as **sudden**.

Example.—The mean temperatures for the two measurements of the Batabano base of 7,700 meters were 97.3° F and 108.2° F, mean 102.7° F. Adopting for the seasonal temperature that of the storeroom, say 86° F, how great was the total correction for rapid changes of temperature?

Reduction from 86° F (30° C) to 0° C, $+0.00325 \times 900\mu$ per meter.

Reduction from 0° C to 102.7° F (39.3° C) $-.00325 \times 1544\mu$ per meter.

Resultant correction, 2.093μ per meter.

Correction for 7,700 meters, $-16,116.1\mu$, or -1.61 centimeter. (This correction, of which no account was taken, is nearly as great as the difference between the two measurements, 1.95 centimeters.)

It is well to avoid this correction by employing the tape with due regard for the peculiarities of invar. If applied, it is to be considered as additional to the corrections required by the equation of the tape.

Invar is sensitive to changes in strong magnetic fields. It resists corrosion to a remarkable extent as long as the surface polish remains intact. Spots should be removed as soon as detected, by rubbing with a greasy rag. Tapes not in use should be protected with vaseline. Invar in bars, before polishing, is always cracked on the surface. These cracks may be found sometimes on a tape that has deteriorated. Such a tape will break easily at any one of the numerous check marks.

The tensile strength of invar varies from 50,000 to 100,000 pounds per square inch, and its elastic limit from 30,000 to 70,000 pounds per square inch. Its density is about 8 grams per cubic millimeter.

Invar is very ductile. Care must be exercised in reeling and unreeling it and in carrying it, to avoid bends, which strain the outer fibers beyond the elastic limit, so that though the tape may be straightened easily its original length cannot be restored.

The type of reel sometimes supplied by manufacturers, having spokes with deep narrow channels and a short wobbly axis, is a source of danger to the tape. The following qualities are desirable for safety:

- (a) A diameter of 20 inches or more.
- (b) Continuous rims bordering a channel somewhat wider than the tape.
- (c) A long, true-fitting axis, preferably ball-bearing.
- (d) A handle near the circumference, not near the center.

A safety bicycle wheel answers these requirements fairly well, though the channel is somewhat too wide and too shallow. It may be mounted conveniently on light wooden A-frames.

Two persons should be employed in reeling up, one to turn the reel and the other to fair-lead the tape with a light tension.

The following is intended to suggest what may be required, when inexperienced persons are to carry an invar tape, to guard the tape against bends, turns, and shocks.

For each end of the tape make a handle about 18 inches long of 1½-inch heavy elastic, with a wide rope handle sewed to one end in a way to prevent turning. Sew a button or hook into the elastic near the rope handle end to engage the ring of the tape, and between the button and the free end attach two close-fitting loops. Placing the tape with the marked side up under the loops and engaging the button, paint the upper side of the rope handle, of the elastic, and of a small portion of the stray line of the tape. Also paint the top of the tape for an inch or more on either side of the intermediate marks. **Carry the tape by the ends only, with all painted parts up.**

If intermediate tape carriers are to be used, they should be previously trained in the technique of carrying a tape. The rear carrier is in the best position to command the march, to guard against breaking step, and to see that the tape is supported only on the open palm of the hand.

RECORDS

The most convenient book for base line notes is the level book, containing 18 lines to a page. Each of the parties detailed for reconnaissance, for levels, and for taping should be provided with one, and it is important that all the books should be alike, bearing the common title "Base Line", the name of the place, the date, and the name of the ship; and the particular subtitles "reconnaissance", "levels", "measurements", "computations", and so on, sufficient to describe the contents.

The reconnaissance party will naturally take side shots to all land features, so that the base line may be properly described and related to the land. There is opportunity, too, for gathering information not easily obtained from distant triangulation stations, such as the average width of marshes, palm belts, deciduous tree belts, and grassy lands; railway, highway, and communication terminals; and other general information useful to the survey.

The smooth copy of the base-line book should contain all the numerical facts bearing on the length of the base line, including a copy of the level notes and an abstract from the reconnaissance book of descriptions of the site. In addition it should contain the full field computation of the length, including the history and constants of the tape and a description of the apparatus and methods used.

Probable error.—

In

$$p. e. = \pm 0.6745 \sqrt{\frac{\sum v^2}{n(n-1)}}$$

the *v*'s are the residuals, or the differences from the mean, and *n* is the number of measurements. The "probable error" in this sense is merely an index of the uniformity of performance in applying the tape. The real probable error is best given as an estimate of constant errors in a descriptive summary of the measurement.

1. $\frac{1}{2} \log 2$
 2. $\frac{1}{2} \log 2$
 3. $\frac{1}{2} \log 2$
 4. $\frac{1}{2} \log 2$
 5. $\frac{1}{2} \log 2$
 6. $\frac{1}{2} \log 2$
 7. $\frac{1}{2} \log 2$
 8. $\frac{1}{2} \log 2$
 9. $\frac{1}{2} \log 2$
 10. $\frac{1}{2} \log 2$

[illegible]

CHAPTER VII

TRIANGULATION

CONVENTIONAL DEFINITIONS OF ORDERS OF PRECISION

In 1925 the Federal Board of Surveys and Maps adopted the following terminology and definitions relating to **horizontal control** by triangulation and by traverse. In the table, the factor R , is an index of the inherent strength of the figure, as explained in the chapter on **Reconnaissance**; and the factor K denotes the number of kilometers in a section of a base line.

For triangulation of order	First	Second	Third
<i>Strength of figures:</i>			
Desirable limit, ΣR , between bases	80	100	125.
Maximum limit, ΣR , between bases	110	130	175.
Desirable limit, R_1 , single figure	15	25	25.
Maximum limit, R_1 , single figure	25	40	50.
<i>Length discrepancy between computed and measured length of base or adjusted length of check line, not to exceed</i>			
	1 in 25,000	1 in 10,000	1 in 5,000.
<i>Triangle closure:</i>			
Average, not to exceed	1 second	3 seconds	6 seconds.
Maximum, not to exceed	3 seconds	8 seconds	12 seconds.
<i>Usual number of observations:</i>			
Positions with 1-second direction theodolite	16	4	2.
Positions with 2-second direction theodolite	20 to 24	4 to 8	2 to 4.
Sets with 10-second repeating theodolite	5 to 6	2 to 3	1 to 2.
<i>Base measurement</i>			
Actual error of base not to exceed	1 in 300,000	1 in 150,000	1 in 75,000.
Probable error of base not to exceed	1 in 1,000,000	1 in 500,000	1 in 250,000.
Discrepancy between 2 measures of a section, not to exceed	10 mm \sqrt{K}	20 mm \sqrt{K}	25 mm \sqrt{K} .
<i>Astronomic azimuth, probable error of result</i>	0.5 second	2 seconds	5 seconds.
For traverse of order	First	Second	Third
Closing error in position, not to exceed	1 in 25,000	1 in 10,000	1 in 5,000.
Probable error of main scheme angles	1.5 seconds	3 seconds	6 seconds.
Number of stations between astronomic azimuths	10 to 15	15 to 25	20 to 35.
Astronomic azimuth, discrepancy per main angle station, not to exceed	1 second	2 seconds	5 seconds.
Astronomic azimuth, probable error of result	0.5 second	2 seconds	5 seconds.

The foregoing conventional definitions of 1925 are not to be construed as specifications except as they may agree with specifications issued by the Hydrographic Office for individual surveys. In a general way it may be stated that the second-order definitions have proved to be the most suitable for coastal surveys, with the following modifications:

It is reasonable to continue to expect that base discrepancies will seldom exceed 1 part in 20,000.

In difficult cases, where lateral refraction is suspected, triangle closures exceeding 5 seconds should be reduced by additional observations in more favorable weather or at night.

In hydrographic surveys, the scarcity of tower sites often forces an abandonment of the requirement that the value of R_1 shall not exceed the maximum of the table.

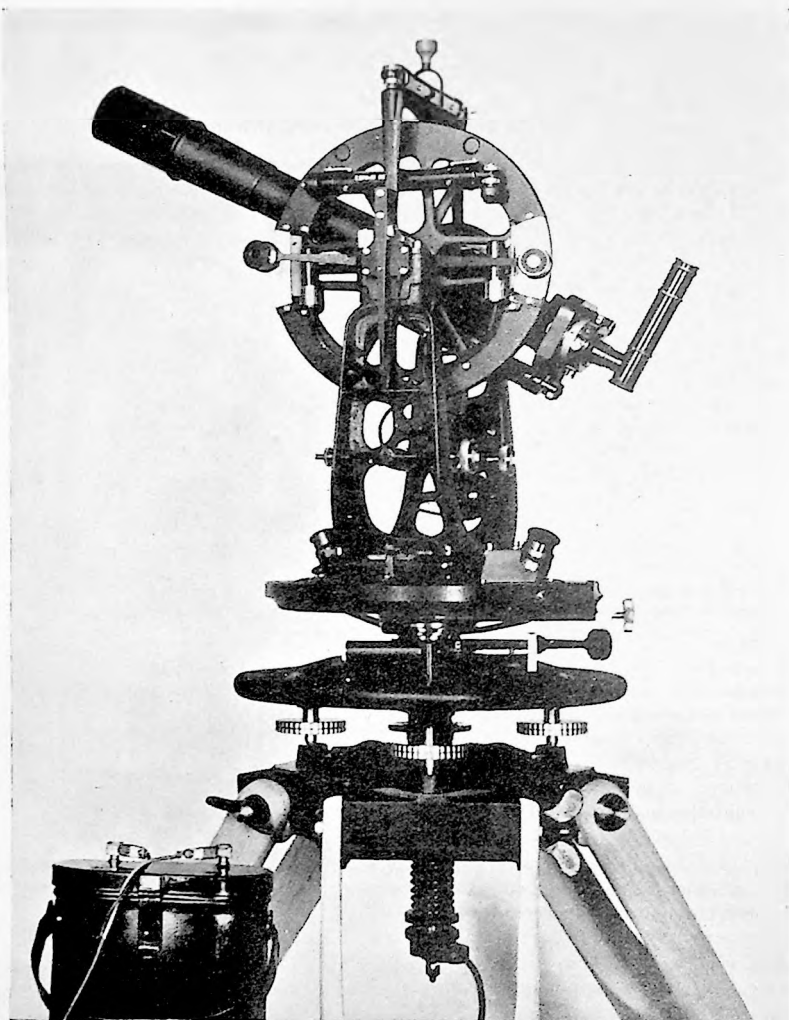


FIGURE 91.—Eight-inch transit-theodolite.

Third-order triangulation is suitable for spurs from the main triangulation, which by the nature of the case cannot carry far nor return in all cases to a side of the main system. Whether triangulation of the third order of precision is suitable for detached harbor surveys is questionable, and depends on circumstances. Due regard must be given to the probability of future extensions of such surveys, and to the possible connection of two or more of them by a coastal triangulation. .

INSPECTION AND CARE OF INSTRUMENTS

In planning triangulation for hydrographic surveys, one of the most important problems relates to the prevention of the deterioration of instruments due to the dampness of storerooms.

Soon after the arrival of the ship at her home port, all instruments should be removed from the storeroom to a dry place on shore, to be cleaned, dried, and minutely inspected for imperfections and possible damages during transit, in order that a list of necessary repairs may be compiled. In anticipation of the next voyage, the instruments should be well oiled and protected from dust.

When the triangulation instruments are again taken aboard for transit to the field, the careful triangulation man will see that they are securely and immovably packed in their cases and stored immovably in the storeroom, in a high place, knowing that there may be several feet of water in the storeroom during a rough passage.

On the way to the field the ship may go into port for coal. This is the time to adjust all theodolites and transits, for there may be scant opportunity in the field to make such adjustments as those for collimation and inclination of the horizontal axis. The excess oil may be removed from instruments expected to be used soon and the instruments put in the best possible condition for observing. Necessary accessories for observing, such as tower heads, hoisting gear, and sunshades, should be prepared and, if possible, tried out.

INSTRUMENT SUPPORTS

In harbor surveys, the distances are usually short and there is considerable protection from wind and sea. Under these circumstances, for lines not more than 6 miles long, the most practical support for the theodolite is its own tripod, provided that the sights are water-borne and the atmospheric conditions good. But if some of the lines pass over heated land or alternately over land and water, it is better to use towers, in order to avoid the heated strata of air near the surface.

Heavy steel towers, four-posted (to reduce twist) and in other respects of good design, are suitable for triangulation of second-order precision, if they are built up from the ground (not hoisted) and if they are fastened to good foundations, such as rock, heavy concrete blocks imbedded in the soil, or piling.

Double towers, one supporting the instrument and the other the observer, are necessary for first-order triangulation.

MAJOR AND MINOR PRECAUTIONS IN OBSERVING ON TOWERS

With reference to second-order triangulation, using a 10-second theodolite, the major precautions (in a numerical sense) to be taken may be defined as those which in their totality reduce the uncertainty in any angle of the main triangulation from 1 minute to 10 seconds of arc in bad weather, or to 5 seconds in good weather. Among these may be mentioned the choice of a suitable height of eye, of the proper instrument support, of level and stable founda-



FIGURE 92.—Triangulation observer camping on the job.

tions, and of proper targets at the towers sighted; also the proper erection of the tower. Finally, the choice of the best weather in each observing day is a major consideration. If any one of these precautions is neglected, the initial uncertainty assumed by the observer, when he mounts the tower, will not have been reduced to the point where it is worth while to observe with a theodolite rather than with a small transit.

It is unnecessary to mention the precautions with reference to methods of observing that are taken as a matter of course by every experienced and skillful observer. He will need to be on guard, however, against certain conditions peculiar to observing on single towers in hydrographic work.

Platforms, dressings, sunshade, windshield—all these tower fittings, if rigidly attached to the tower so that the whole vibrates as a unit, cause a short symmetrical vibration not harmful to observations unless the wind is very strong or gusty. In fact, a certain amount of vibration is favorable to good pointing, especially when the targets are small enough to be covered by the vertical wire. If the tower does not react in this way, in a moderate breeze, it is not a good triangulation tower.

It may not be necessary to remove the skirts from a properly dressed and well-constructed tower, but with standard skirts, fitted to the tower and substantially lashed down each leg with 6-thread manila rove through grommets, it requires but little time either to tighten or to remove the skirts, as may be deemed best. Torn and flapping dressings are intolerable during observations.

A good tower needs no guys. The use of wire guys is unfavorable to pointing in calm weather on account of their change in length, due to temperature, if they are tight; and in windy weather on account of their swaying, if they are slack.

It is necessary to consider the vibration of the telescope as well as the vibration of the tower, because the former often prevents getting a long sight. For this purpose, a small wind shield may be used. A large part of the wind checked by the tower dressings comes up over the edge of the platform. The telescope, therefore, must be shielded from below as well as from the direction of the wind.

The Navy uses heavy transit-theodolites on towers, with the feet of the trivets resting in conical holes on the tower headplates, which are thick and heavy, and of a diameter greater than that of the horizontal plate of the instrument. This keeps the hands off the instrument except in the act of observing, adjusting, etc. It is bad practice to fasten the instrument base to the support while observing. In windy weather it may be prudent to pass a preventer line over the arms of the base. This should be loose enough to permit passing a pencil between each arm and the line.

Though the use of a sunshade increases somewhat the vibration of the tower, this is not a harmful vibration except when the wind is too heavy to permit the sunshade to remain in place. The effect of vibration is one that can be observed on every pointing, and, if necessary, largely avoided during gusts. If the instrument is shaded, it will remain in adjustment for a long time. Unless the instrument is shaded, it cannot be adjusted in the first place; and, in the second place, any approximate adjustment that may be made will change continually, due to unequal expansion of the parts of the plate, the ends of the bubbles, the standards, and the reticule ring, so that the resulting movements will result in variable and unknown errors impossible to correct. Such errors are undoubtedly large on hot, still days, and small in moderately windy weather. For days when the shade cannot be used, it is well to provide a heavy white canvas cover for the plates, with lead-weighted flaps over the bubbles.

Every shift of the point of application of the observer's weight, while observing on the platform of a tower, produces a flexure and a twist in the tower. This will not necessarily be revealed by the failure of the horizon to close, for the same program that is used to compensate instrumental errors will tend to conceal the errors due to systematic shifting of the observer's weight.

This source of error may be minimized, but not entirely eliminated, if the observer keeps his weight on one foot while observing each set, or if he uses a stool with a small base in the same way. Some observers prefer to find, for each angle about to be observed, a comfortable mean

position of the feet such that pointings can be taken in both directions, keeping the weight of the body equally on both feet. For obvious reasons it is advisable to plan the triangulation so that no angle to be observed shall much exceed 70° ; and when larger angles are unavoidable, to use, or provide if necessary, a distant intermediate object for observing them in two parts.

The present towers are fitted with special braces to carry weights on the platform down to a lower section of the tower, and give but little trouble from flexure.

In observing from a single tower the presence of a recorder or other person on the platform or on any other part of the tower, is intolerable.

OBSERVATIONS AT A TOWER

The usual observations at a tower may be classed as follows:

- (a) Main triangulation angles.
- (b) Secondary triangulation angles.
- (c) Single cuts to signals, beacons, buoys, and landmarks.
- (d) Tangents to islands, shoals, and reefs.
- (e) Horizontal and vertical angles to peaks.
- (f) References for the station.

The best part of the day is to be reserved for (a), and, after that, the best visibility for (e). Upon arrival at the tower, the triangulator will ordinarily undertake observations, even in very windy weather, if the tower is safe. Having in mind the needs of the drafting office, he will take single angles (cuts) to everything from (a) to (e) that can be seen—a continuous round of angles, in observing which the whole horizon is searched and new objects sighted, such as beacons, buoys, chimneys, conspicuous trees, etc., are described; frequently checking the zeros, and changing zeros at intervals to avoid large angles. It is well to check the zeros every time a sounding signal comes into the field of view. The list of trial or **finder angles** for sounding signals should be consulted from time to time, and the name of each one crossed off the list as found. Full notes on the state of the weather as affecting the reliability of angles should be kept. After reading and recording each angle, it should be checked by looking at the plate and vernier again. The observations should include estimates of distances, reading vertical angles, if necessary, to assist the judgment. A short description of each tangent is required—as “sharp mangrove point”, “low rocky shore”, etc. When mirage indicates that the true tangent is probably outside of the apparent one, this will be indicated by the estimated distance and by the term “indefinite”; or the triangulator may decline to observe, merely noting that distant land appears between certain limiting directions, recorded approximately.

When the round of cuts has been completed, more intensive search may be made for sounding signals not yet crossed off the list. Floating signals and signals that bear any resemblance to sails should be pointed upon again to find whether they have moved. Finally, when all possible cuts have been recorded, **set the vernier on the recorded angles and verify the pointings.**

If fair observing weather arrives during the single angle observations, they should be discontinued in favor of (a). After going through the list of (a), with the weather constantly improving, the angles at the top of the list, observed in barely tolerable weather, should be observed again, to the end that the final observed triangulation angles shall be the best obtainable on that day. If fair weather does not arrive during the course of the day, the triangulator should decline to observe repeated angles; or, at most, he should observe only three repetitions of angles, to serve for immediate needs.

PROGRAM FOR OBSERVING

It is considered good practice to observe four half-sets of six repetitions, of the angle and of its explement. The program is arranged to eliminate as far as possible collimation error, inclination of the horizontal axis, eccentricity, unequal divisioning of the plate and verniers, creeping of the clamps, and twist of the instrument support.

The degrees and minutes of the one-time angle are carefully read and recorded, to prevent gross errors in dividing by 6; and when this has been done, they are written down with confidence as the first part of the quotient, leaving only the seconds to be found.

The seconds of the quotient or reduced angle are found in two steps. First, since $\frac{1}{4}$ of 1 minute = 10 seconds, the first figure of the seconds of the quotient is the same as the remainder obtained by dividing the minutes of the dividend by 6; and second, the remaining figures of the seconds of the quotient, usually carried to tenths, are obtained by dividing the mean seconds of the sextuple angle by 6.

These well-known rules, though applicable only for a zero setting of the *A*-vernier, may be used, with obvious modifications, whenever the setting is an integral number of degrees.

Another need for the one-time angle frequently arises when distant objects fade or disappear during a half-set. It then becomes necessary to compute the approximate reading of the circle to obtain the sight. When the sight is likely to be very difficult, but not otherwise, it is well to read the one-time angle as closely as possible, and to set the circle so that for the multiple angle the wire shall be surely on the left of the signal, or else surely on the right, and as close to it as may be. Under these circumstances, fatigue of the eye sometimes causes retained images of the wire to resemble the distant object, if it is slender. To distinguish them, two tests may be applied. If the image is seen on the same setting of the circle when the tangent screw is turned and brought back by the same amount (without looking at the vernier); or if the image is seen to move in the direction in which it ought to move when the tangent screw is turned, it is probably the true image of the distant object.

It often chances that one of the signals practically disappears during a set, making it impossible to measure both the angle and its explement in the same direction. It is more expedient, of course, to measure from the distinct object to the faint one, but if this amounts to measuring the same angle in opposite directions, it is an improper procedure, excusable only under absolute necessity, or if the action of the instrument in measuring angles in opposite directions is known by previous tests on angles of about the same size.

Usually this dilemma may be avoided by proper management and by a knowledge of the usual weather conditions. To mention some of the ways of overcoming such difficulties:

The triangulator should plot all information bearing on signals "up ahead"—his own cuts, those of the ship, and those of the tower erection parties—on an accurate small-scale sheet for his own particular use; and upon arriving at a tower should know, within a few minutes of arc, the directions of all stations to be sighted. He should then arrange his program to try the most difficult sights during the periods of best visibility. If, for instance, experience has taught him that long shots are difficult or impossible between 10 a. m. and 4 p. m., upon arrival, say at 8:30 a. m., with everything visible, it would be well to set range stakes or poles for all the distant sights, at least. Even if there were too much wind for observing triangulation angles at the time, the ranges might become a necessity with abatement of the wind and rising of heat haze. The range stakes will save time and needless manipulation of the instrument, and may be the means of avoiding incomplete sets.

When there is no place to set a range stake, any quickly recognized distant object, not necessarily on the range but preferably close to it, may be used as a "backsight" or reference direction.

On calm days the horizon may disappear or may become confused with shadows, when viewed through a telescope of high power; and at night, of course, there is no horizon to assist in picking up distant objects. It is well, therefore, to have on hand a list of approximate elevations for the longest sights, obtained, it may be, under unfavorable observing conditions, or with an instrument of low power, or if necessary by computation.

Spaced initial settings, designed to compensate for inequalities in the divisioning of the verniers, are ignored in the following illustrations. If desired, they may be obtained from the formula

$$\text{Interval} = \frac{360}{vs} \text{ degrees} + \frac{d}{s} \text{ minutes,}$$

in which *v* is the number of verniers, *s* the number of sets, and *d* the number of minutes in one division of the limb.

Of greater consequence is the avoidance of zero settings in the case of any instrument showing wear or damage in the vicinity of 0° or 90°. In such a case, to derive the sextuple angle it

is necessary to subtract the initial reading from the final reading, or, in the second and fourth half sets of form 2, to subtract the final reading from the initial one.

FORMS FOR OBSERVING MULTIPLE ANGLES

The following forms are convenient for use in the ordinary Mining Transit book. Even with an added column for the positions D and R of the telescope, there is room for writing brief observing notes alongside the angles, reserving the right-hand page, which is cross-sectioned, for more extended notes and for sketches.

The algebraic signs (inserted in the forms only for explanation) indicate that negative angles are to be subtracted from positive ones. In the first form, and in the first and third half-sets of the second form, initial angles, as is natural, are subtractive from terminal angles. But in the second and fourth half-sets of the second form there is a gain in reversing this rule in order to derive the second of the desired left-to-right angle instead of those of the explement, though the latter is the angle actually measured. All angles should be measured in the same direction.

(a) Four initial settings

May 27 1924	Lump, 1921	XYZ B. 8-inch	
Pip	— 0 00 00 05 1 39 14 40 +6)235 28 25 35	27.5	
Po	39 14 44.6		44.6
Po	— 0 00 00 05 1 320 45 20 +6)124 31 30 30	27.5	
Pip	320 45 14.6		45.4
Pip	— 90 00 00 05 +6)325 28 20 30	22.5	
Po	39 14 43.8		43.8
Po	— 90 00 00 05 +6)214 31 35 40	35.0	
Pip	320 45 15.8		44.2
			4)178.0
Mean, Pip to Po,		39 14	44.5

(b) Two initial settings

May 27 1924	Lump, 1921	XYZ B. 8-inch		
Pip	— 0 00 00 05 1 39 14 40 +6)235 28 25 35	35 30	27.5	
*Po	39 14 44.6			44.6
*Po	+ 235 28 25 35 -6)359 59 55 60	30 35	32.5	
	39 14 45.4			45.4
Pip	— 90 00 00 05 +6)325 28 20 30	20 25	22.5	
*Po	39 14 43.8			43.8
*Po	+ 325 28 20 30 -6)89 59 55 65	25 25	25.0	
Pip	39 14 44.2			44.2
				4)178.0
Mean, Pip to Po,		39 14	44.5	

Obviously the second form may be shortened by omitting the lines marked by an asterisk, if the seconds in the last column are written in the line above. This is often done, due to press of time. But it cannot be recommended on the score of clarity.

The two forms correspond to different methods of observing. The first gives an opportunity of adjusting the plate levels after each half-set, as is often necessary on towers, or in sunlight, or when there are long waits between sights. The second denies this opportunity to a careful observer, after the first and third half-sets; but it is probably the better method at night, if the sights can be obtained quickly, for it saves a quarter of the difficult settings and readings of verniers.

It is customary to write the seconds of the B-vernier immediately after those of the A-vernier on all initial and final pointings. The second form shows the travel of each vernier, but this is ascertained mentally in the first form, and the mean travel is set down in the next column.

Whether it is advisable to reverse the telescope in the middle of each half-set, or between the half-sets only, depends on the stability of the levels. The first method involves more

manipulation of the instrument, but affords an opportunity to relevel, if necessary, between the half-sets. In the event of difficult sights or capricious weather, if the seventh or any subsequent pointing is unduly protracted or lost, by the first method the result will be one measurement of the angle, compensated for collimation and inclination of the horizontal axis; while by the second method, the result will be no measurement because it will still contain these errors.

In pointing, the observer may correct his personal error or tendency to underrun or overrun the target by bringing the two targets from the apparent right in the first repetition, from the apparent left in the second repetition, and so on. This applies particularly to small, distant, and indistinct targets. Another advantage is thus gained in keeping the tangent screw lugs in the middle of their runs. If the tangent screw springs are too weak to give positive action when the tension is relaxed the smallest amount perceptible to the hand, they should be replaced by stronger ones.

GEODETIC SYSTEMS

The following figures of the earth, deduced from early triangulations, are still in use in many countries:

Figure	Semimajor axis, meters	Flattening 1 part in
Bessel's spheroid.....	6, 377, 397	299. 15
Clarke's spheroid, 1866.....	6, 378, 206. 4	294. 98
Clarke's spheroid, 1880.....	6, 378, 249. 2	293. 46
Hayford's ellipsoid, 1909.....	6, 378, 388	297. 0

Hayford's ellipsoid, based on the most complete data, was adopted by the International Geodetic and Geophysical Union in 1924, and has since been called the "International ellipsoid."

The international ellipsoid assigns to the earth larger dimensions than those of the Clarke spheroid of 1866, used by Canada, the United States, and Mexico, by about 1 part in 35,000 for the major axis and 1 part in 20,000 for the minor axis. Comparing these ratios with the limits of tolerance in base line measurement, it is evident that for an extended coast survey, the selection of the figure of the earth that will best fit the region may, theoretically, become more important than striving for the greatest precision in base-line measurement.

Reduction tables for the international ellipsoid, in the sexagesimal system, may be found in Bulletin Geodesique, No. 12, 1926. They require 8-place logarithms. Reduction tables for the Clarke spheroid of 1866 may be found in C. & G. S. Special Publication No. 8, requiring 7-place logarithmic tables; also in Publication No. 7, 1922, of the Geodetic Survey of Canada, employing a different method of reduction and requiring 7-place tables for the direct solution and 8-place tables for the inverse solution.

Except for occasional large figures, 7-place logarithms are adequate for hydrographic surveys. In computing triangle sides and geodetic positions, numbers are usually carried to the following limits:

Quantities to be expressed	To nearest—
Lengths (except of base lines).....	0.01 meter.
Mean observed triangulation angles.....	0.1 second.
Adjusted triangulation angles.....	0.01 second (if preferred 0.001 second).
Azimuths and spherical excesses.....	0.01 second (if preferred 0.001 second).
Geodetic positions.....	0.0001 second.

These limits consort well with the use of a 10-second theodolite and the method of repetitions, as 0.001 second, roughly 0.1 foot on a great circle of the earth, is appreciable with such an instrument for sights of moderate length.

SPHERICAL EXCESS

Table 12, page 159, gives values for every tenth minute of latitude, of the logarithm of the spherical excess coefficient m applying to the formula

$$\epsilon = a_1 b_1 \sin c_1 \cdot m,$$

in which ϵ is the spherical excess in seconds; a_1 , b_1 , and c_1 are two sides and the included angle of an imaginary plane triangle whose sides, expressed in meters, are equal in length to those of the triangle of which the spherical excess is sought; and m is a factor depending on the mean latitude of the vertices of the triangle and the dimensions of the Clarke spheroid of 1866.

Since the coefficient of m is twice the area of the plane triangle, in any given latitude the spherical excess of a triangle depends only on the area. It follows that the spherical excess of a figure composed of triangles is equal to the sum of the spherical excesses of the component triangles. Within a single triangle, however, spherical excess is an indivisible quantity, and cannot be distributed to the individual angles.

Thus, in the figure, an absurdity would be encountered in attempting to assign to A a spherical excess of 4 seconds, which is one-third that of the triangle ABC , and at the same time a spherical excess of 2 seconds, which is one-third that of the triangle ADC .

The only reason for computing the spherical excess of a triangle lies in the need of finding the true sum of the angles in order that the corrections for closure error may be distributed to the observed angles.

The spherical excess of a figure may be found approximately by scaling its dimensions and applying the following:

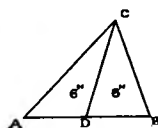


FIGURE 93.

Spherical excess per unit area

Unit area	Latitude						
	0°	10°	20°	30°	40°	50°	60°
	Seconds of arc						
100 square nautical miles....	1. 7533	1. 7525	1. 7505	1. 7474	1. 7435	1. 7394	1. 7355
100 square kilometers.....	0. 5105	0. 5103	0. 5097	0. 5088	0. 5076	0. 5064	0. 5053

SOLUTIONS OF TRIANGLES

The sea level distances from two fixed known points A and B on the surface of the geoid to an unknown point C are computed on the assumption that they are equal to the sides of a spherical triangle whose dihedral angles are the adjusted measured angles at the three points. This involves the assumption that the radii of curvature of the arcs that join the points in pairs are constant and equal, whereas, in fact, they vary with the latitude and with the azimuth in the same latitude. The assumption is justified, however, by the relatively small dimensions of the figure.

The direct solution of the triangle involves an inconveniently large radius. Two indirect solutions are available.

(1) *By passing to a plane triangle.*—If the lengths, in linear units, of the arcs forming the sides of the spherical triangle were known, they could be laid out on a plane surface to form a plane triangle. The angles of such a triangle, according to Legendre's theorem, are, to a high degree of approximation, separately equal to the corresponding angles of the spherical triangle each diminished by one-third of the spherical excess. Conversely, if one side and all the angles of an assumed plane triangle are known, the remaining two sides may be found, and when found may be associated with the known side and the known spherical angles of the spherical triangle to complete its solution.

The law of sines for plane triangles gives:

$$a=c \cdot \csc C \cdot \sin A \text{ and } b=c \cdot \csc C \cdot \sin B. \quad (1)$$

(2) *By passing from arcs to sines.*—The law of sines for spherical triangles differs from that for plane triangles in involving the sines of the angles subtended by the sides instead of the linear values of the sides. If x, y, z , are the central angles subtended by a, b, c , and if the radius is R ,

$$\frac{R \sin x}{\sin A} = \frac{R \sin y}{\sin B} = \frac{R \sin z}{\sin C}. \quad (2)$$

If x is expressed in radians, the arc a subtending it is equal to Rx . To pass from Rx to $R \sin x$ requires the introduction of the factor $\sin x \div \text{arc } x$, or $\sin x \div x$.

$$R \sin x = R \text{ arc } x \cdot \frac{\sin x}{\text{arc } x} = a \frac{\sin x}{\text{arc } x} = a \left[1 - \frac{x^2}{3} + \frac{x^4}{5} - \dots \right]$$

If the angle is expressed in seconds,

$$R \sin x'' = a'' \cdot \left[1 - \frac{1}{6}(x'' \div 206264.8)^2 - \dots \right] \quad (3)$$

or

$$R \sin x'' = a'' \cdot [1 - x''^2 \cdot (0.000,000,000,003,917) - \dots].$$

The factor in brackets is that required to reduce any number of seconds from the arc form to the sine form $R \sin x''$, or vice versa. By giving x successive values of $100''$, $200''$, $300''$, etc., in (3), a table of values of $\log x'' - \log R \sin x''$, or of $\log \text{arc} - \log \text{sine}$, may be constructed. By adding to each $\log x''$ a constant logarithm representing the number of meters in a second for a mean value of R , the logarithms of bases expressed in meters corresponding to the same reductions are found, and tabulated to four decimal places as arguments ($\arg. \log K$).

With the aid of this table, the given side of the triangle is solved by the law of sines, and the resulting sides, when found, are reduced from the sine form to the arc form by the same table. (See table 10, p. 142.)

For latitudes and azimuths farthest from the mean, the error introduced by using a mean value of R amounts to 1 in the seventh place of logarithms only in the last part of the table. If desired, account may be taken of the particular latitude.

SPECIMEN COMPUTATION OF A SPHERICAL TRIANGLE

Let X denote the angle at the unknown station, R the station on the right, and L the station on the left. (See pp. 144, 145.)

Given $\log RL = 4.781,9717$ (meters); X, R , and L as adjusted below; and latitude $8^\circ 38'$; to find the logarithms of LX and RX in meters.

The sine functions come down in the same order as the angles (the cosecant is a sine function).

To find RX , cover $\sin R$ and add the three remaining logarithms.

To find LX , cover $\sin L$ and add the three remaining logarithms.

To check, write the sum of the first two logarithms on the edge of a card, and add the sum separately to each of the remaining logarithms, in the order in which they stand.

In practice the headings are omitted. The side headings of the left-hand form are used for either form.

When the sides are long and differ considerably in length, the solution by spherical angles is more precise than that by plane triangles, which is correct only for equilateral triangles. But in practice, either solution may be used, except in adjusting a triangulation net. In this case the adjustment is carried through the sine forms, to avoid small discrepancies.

The conversion factors, arc to sine and sine to arc (66, 109, and 19, p. 141) are used again in the computation of the geodetic positions.

Solution by plane angles

Solution by spherical angles

		Plane angles				Spherical angles
		°	'	''	°	''
X	Rey	46	35	38.08	1.59	39.67
R	Pacheca	110	11	57.77	1.59	59.36
L	Otoque	23	12	24.15	1.58	25.73
		180	00	00.00	4.76	4.76
are form						
RL		4.781,9717	Pacheca—Otoque			
csc X		0.138,7634				
sin R		9.972,4327				
sin L		9.595,5508				
LX		4.893,1678	Otoque—Rey			
RX		4.516,2859	Pacheca—Rey			

		Spherical angles			
		°	'	''	
X	Rey	46	35	39.67	
R	Pacheca	110	11	59.36	
L	Otoque	23	12	25.73	
		180	00	04.76	
sine form					
R. sin RL		4.781,9651	66	9717, RL	
csc X		0.138,7602		Pacheca—	
sin R		9.972,4315		Otoque	
sin L		9.595,5585			
arc form					
R. sin LX		4.893,1568	109	1677, LX	
R. sin RX		4.516,2838	19	2857, RX	

The spherical excess is usually known from preliminary computations, as with the approximate plane angles 46-35-38, 110-11-58, and 23-12-24. But it is well to check it at this point by adding log sin X and log m to the last two logarithms of either solution, thus:

LX	4.893,1678	R. sin LX	4.893,1568
RX	4.516,2859	R. sin RX	4.516,2838
sin X	9.861,2366 (complement of csc X)	sin X	9.861,2398
m	1.406,82 - 10 (from table)	m	1.406,82 - 10
	0.677,51 = 4.759''		0.677,50 = 4.759''

In the solution above, LX is derived from R sin LX as follows:

R. sin LX	4.8932	R. sin LX	4.893,1568
Correction for latitude 8°38'	+ .0006		+ 109
R. sin X, corrected argument	4.8938	LX	4.893,1677
K from table	4.8875 gives log ratio 106.3		
63 × .0505		3.18	
		109	

In Table 10, page 142, the tabular values of argument log K correspond to a mean value of the radius of the earth, for azimuth 45° and in latitude 30°. For other latitudes, when K is very large, add the following to log K before entering the table:

Lat. 0°	Lat. 10°	Lat. 20°	Lat. 30°	Lat. 40°	Lat. 50°
+ .0007	+ .0006	+ .0004	.0000	-.0005	-.0010

TRIANGULATION

TABLE 10.—*Logarithms (log R) for reducing arc to sine or sine to arc*

Direct solution.—With Arg. log K use $-\log R$; with Arg. $\Delta\lambda$ use $+\log R$ Inverse solution.—With Arg. $\frac{1}{2}\Delta\lambda$ and Arg. $\frac{1}{2}\Delta\phi$ use $-\log R$; with Arg. $\sin \frac{1}{2}K$ use $+\log R$				
(1) Argument: difference of latitude or difference of longitude. seconds	(2) log seconds	(3) Arg. K (4-place log K)	(4) Log R To reduce arc to sine, $-\log R$ sine to arc, $+$	(5) Interpolation factor f , Units, increase of (4) $= f$ times increase of (3) $= f$ times increase of (2)
100	2.0000	3.4895	0.000,0000.2	.0002
200	2.3010	3.7906	0.7	.0005
300	2.4771	3.9667	1.5	.0010
400	2.6021	4.0916	2.7	.0015
500	2.6990	4.1885	0.000,0004.2	.0024
600	2.7782	4.2677	6.1	.0033
700	2.8451	4.3346	8.3	.0045
800	2.9031	4.3926	10.9	.0057
900	2.9542	4.4438	13.8	.0070
1,000	3.0000	4.4895	0.000,0017.0	.0082
1,100	3.0414	4.5309	20.4	.0103
1,200	3.0792	4.5687	24.3	.0126
1,300	3.1139	4.6035	28.7	.0143
1,400	3.1461	4.6357	33.3	.0163
1,500	3.1761	4.6656	0.000,0038.2	.0189
1,600	3.2041	4.6937	43.5	.0213
1,700	3.2304	4.7200	49.1	.0242
1,800	3.2553	4.7448	55.1	.0268
1,900	3.2788	4.7683	61.4	.0296
2,000	3.3010	4.7906	0.000,0068.0	.0330
2,100	3.3222	4.8118	75.0	.0361
2,200	3.3424	4.8320	82.3	.0394
2,300	3.3617	4.8513	89.9	.0433
2,400	3.3802	4.8697	97.9	.0474
2,500	3.3979	4.8875	0.000,0106.3	.0505
2,600	3.4150	4.9045	114.9	.0549
2,700	3.4314	4.9209	123.9	.0595
2,800	3.4472	4.9367	133.3	.0636
2,900	3.4624	4.9519	143.0	.0680
3,000	3.4771	4.9667	0.000,0153.0	.0730
3,100	3.4914	4.9809	163.4	.0776
3,200	3.5051	4.9947	174.1	.0823
3,300	3.5185	5.0080	185.1	.0879
3,400	3.5315	5.0210	196.5	.0937
3,500	3.5441	5.0336	0.000,0208.3	.0999
3,600	3.5563	5.0458	220.3	.1050
3,700	3.5682	5.0577	232.8	.1097
3,800	3.5798	5.0693	245.5	.1161
3,900	3.5911	5.0808	258.6	.1219
4,000	3.6021	5.0916	0.000,0272.0	-----

GEODETTIC EVALUATION, DIRECT AND INVERSE

The direct solution may be performed conveniently in the following steps (pp. 144, 145):

1. Write the names of the stations at the vertices of the triangle.
2. Write the solution of the triangle in the frames AA. Write the log sides and the spherical excess in the figure.
3. Fill the first three lines of the frames ZZ, the forward azimuths.
4. In frames BB write the log sines and cosines of the forward azimuths, Z , at the same time supplying the algebraic signs of their naturals.
5. In frames BB write $\log K = \log LX$ (left) and $\log K = \log RX$ (right).
6. In frames BB add the logs of $\sin Z$ and K , and write the sums with the proper algebraic signs opposite $K \sin Z$, both sides. Double $\log K \sin Z$, and write the results in frames CC opposite $(K \sin Z)^2$,—twice, using five decimal places to associate with the constant C and four to associate with the constant E .
7. With arguments ϕ , left and right, take from the tables the logs of the positive constants B , C , D , E , and F , writing them where asterisks occur in the forms.
8. Write log b , with appropriate signs (like those of $\cos Z$) in frames BB, and write the same below in CC with the opposite signs and reduced to four decimal places. Write the naturals of log b in the frames NN.
9. Complete frames CC, write the naturals in frames NN, and find $-\Delta\phi$, both sides, in seconds. Convert into minutes and seconds, and write with changed signs as $+\Delta\phi$ in frames FF. Add to ϕ and so find two values of ϕ' . They should agree, within 1 to 3 units in the fourth decimal place. If they do not agree, check the preceding steps from 3 to 9. If they still fail to agree, recompute the triangle.
10. With the new latitude ϕ' as an argument, find log A' and write it in frames BB, the same on both sides. Also write $\sec \phi'$, the same on both sides. Add the sum to log $K \sin Z$, and so find the approximate values of $\Delta\lambda$, namely $\Delta\lambda_1$.
11. Apply the corrective factors, arc to sine for K and sine to arc for $\Delta\lambda_1$, the first logarithmic correction being negative and the second positive; and so find the logarithms of $\Delta\lambda$ in seconds of arc. Find the corresponding naturals, reduce them to minutes and seconds, write the results in frames FF, and add them to the initial longitudes λ . The two values of the new longitude, λ' , should agree within a few units in the fourth decimal place.
12. The remaining frames are concerned with ΔZ , which with a small correction $\sec \frac{1}{2}\Delta\phi$ is equal to the difference of longitude multiplied by the sine of the middle latitude. The corrective term involving F , which is a maximum for latitude $35^\circ 15' 50''$, does not enter the third decimal place when $\Delta\lambda$ is less than $1100''$.
13. Sum the terms of $-\Delta Z$, write the two results in the frames ZZ, and add, together with 180° , to the initial azimuths Z , to give the reverse azimuths Z' .
14. In frames ZZ check the reverse azimuths by noting whether their difference is or is not equal to the angle X . It should agree within $0.01''$. If the discordance is greater than $0.01''$, examine the spherical excess again.

The inverse solution is performed in the following steps (p. 146):

1. In frames AA set down the coordinates of the two stations, and subtract in such a way as to derive $-\Delta\phi = \phi - \phi'$ and $+\Delta\lambda = \lambda' - \lambda$, these being the signs as they come out in the direct solution. In the example, subtract the second latitude from the first, but the first longitude from the second. Find the half-differences and reduce them to seconds. Find the mean latitude.
2. In frames AA find the log factors required to change the half-angles from the arc forms to their corresponding sine forms, using the same table as in the direct solution. Applying the log factors as negative corrections, write the resulting logarithmic sine forms in frame B in the first and seventh lines.

GEODETIC POSITION EVALUATION

DIRECT SOLUTION

1. From left.

Z	LR	Otoque—Pacheca.....	Z ₀	263	50	06. 36	<div style="text-align: center;"> Otoque 60,530±14 Pacheca </div>
	L	Otoque.....	+	23	12	25. 73	
	LX	Otoque—Rey.....	Z	287	02	32. 09	
	ΔZ	Rey—Otoque.....	⊕	6	6	01. 29	
	XL	Rey—Otoque.....	Z'	107	08	33. 38	

A	Angles			plane	spherical	½c	RL	4.781	9717
	X	R	L	°	'	''	csc X	0.138	7634
				46	35	38. 08	prod.	4. 920	7351
				110	11	57. 77	sin R	9. 972	4327
				23	12	24. 15	sin L	9. 595	5508

B	Latitude			0. 00	4. 76	4. 76	Longitude		
	sin Z	⊖	9. 980	4983	K sin Z	⊖	4. 873	6661	Arg.
	K	⊕ <td>4. 893<td>1678</td><td>*A'</td><td>⊖<td>8. 509<td>6952</td><td>K</td></td></td></td>	4. 893 <td>1678</td> <td>*A'</td> <td>⊖<td>8. 509<td>6952</td><td>K</td></td></td>	1678	*A'	⊖ <td>8. 509<td>6952</td><td>K</td></td>	8. 509 <td>6952</td> <td>K</td>	6952	K
	cos Z	⊕ <td>9. 466<td>9814</td><td>sec φ'</td><td>⊖<td>0. 004<td>6822</td><td>Δλ₁</td></td></td></td>	9. 466 <td>9814</td> <td>sec φ'</td> <td>⊖<td>0. 004<td>6822</td><td>Δλ₁</td></td></td>	9814	sec φ'	⊖ <td>0. 004<td>6822</td><td>Δλ₁</td></td>	0. 004 <td>6822</td> <td>Δλ₁</td>	6822	Δλ ₁
	*B	⊕ <td>8. 512<td>5774</td><td>Δλ₁</td><td>⊖<td>3. 388</td><td>0435</td><td>Corr.</td></td></td>	8. 512 <td>5774</td> <td>Δλ₁</td> <td>⊖<td>3. 388</td><td>0435</td><td>Corr.</td></td>	5774	Δλ ₁	⊖ <td>3. 388</td> <td>0435</td> <td>Corr.</td>	3. 388	0435	Corr.
	log b	⊕ <td>2. 872</td> <td>7266</td> <td>Δλ</td> <td>⊖<td>3. 388</td><td>0428</td><td>- 109</td></td>	2. 872	7266	Δλ	⊖ <td>3. 388</td> <td>0428</td> <td>- 109</td>	3. 388	0428	- 109
					Δλ =	⊖ <td>2443. 6714''</td> <td></td> <td>+ 102</td>	2443. 6714''		+ 102

N	b	=	⊕	745. 9789
	c	=	⊕	2. 1582
	$b+c$	=	⊕	748. 1371
	d	=	⊕	0. 0041
	e	=	⊖	0. 0183
	$-\Delta\phi$	=	⊕	748. 1229
C	$(K \sin Z)^2$			9. 74733
	$\ast C$		⊕	0. 58678
	$\log c$		⊕	0. 33411
	$\log (b+c)^2$			5. 7480
	$\ast D$		⊕	1. 8624
	$\log d$		⊕	7. 6104
	$\log (-b)$		⊖	2. 8727
	$(K \sin Z)^2$			9. 7473
	$\ast E$			5. 6412
	$\log e$		⊖	8. 2612

Azimuth				
$(\Delta\lambda)^2$		⊖		0. 164+10
$\ast F$		⊕		7. 458-20
$-\Delta Z_2$		⊖		7. 622-10
$\Delta\lambda$				3. 388
$\sin \phi_m$		⊕		9. 169
$\sec \frac{1}{2}\Delta\phi$				0. 000
$-\Delta Z_1$		⊖		2. 557
$-\Delta Z_1$	=	⊖		361. 290
$-\Delta Z_2$	=	⊖		0. 004
$-\Delta Z$	=	⊖		361. 294
ϕ	=	8	36	21. 8700
$\frac{1}{2}\Delta\phi$	=	⊖	6	14. 0615
ϕ_m	=	8	30	07. 8085

F	ϕ	8	36	21. 8700	----- Otoque -----	λ	79	36	09. 7270
	$\Delta\phi$	\square	12	28. 1229		$\Delta\lambda$	\square	40	43. 6714
	ϕ'	8	23	53. 7471		λ'	78	55	26. 0556
					----- Rev -----				

Formulae

$$-\Delta\phi = b + c + d + e, \text{ in which } b = K \cos Z \cdot B, c = (K \sin Z)^2 \cdot C, d = (b + c)^2 \cdot D, e = -b (K \sin Z)^2 \cdot E$$

$$\log \Delta\lambda = \log (K \sin Z \cdot A' \cdot \sec \phi') - \log \text{Arg } K + \log \text{Arg } \Delta\lambda$$

$$-\Delta Z = \Delta\lambda \sin \phi_m \sec \frac{1}{2}\Delta\phi + (\Delta\lambda)^2 \cdot F; Z' = Z + 180^\circ + \Delta Z \quad \phi' = \phi + \Delta\phi, \phi_m = \phi + \frac{1}{2}\Delta\phi$$

GEODETIC POSITION EVALUATION

2. From right.

DIRECT SOLUTION

Computed by G. M., 1929.

Otoque 4.781.9717	Pacheca	RL	Pacheca—Otoque.....	Z_0	83	55	01. 92	Z
		R	Pacheca.....		110	11	59. 36	
		RX	Pacheca—Rey.....	Z	333	43	02. 56	
		ΔZ		\oplus		1	10. 50	
		XR	Rey—Pacheca.....	Z'	153	44	13. 06	
			Z' from station on left.....		107	08	33. 38	
			Check, $X = 46^\circ 35' 39.67'' =$		46	35	39. 68	

$LX = RL \csc X \cdot \sin R;$	LX	4. 893 1678	$\sin X$	1. 406 82	A
$RX = RL \csc X \cdot \sin L;$	RX	4. 516 2859	$LX \cdot RX$	9. 861 24	
$c = m \sin X \cdot LX \cdot RX$	$LX \cdot RX$	9. 409 45	$\log c$	0. 677 51	
			$c =$	4. 7759	

Latitude			
$\sin Z$	\square	9. 646	2069
K		4. 516	2859
$\cos Z$	\oplus	9. 952	6088
$*B$		8. 512	5761
$\log b$	\oplus	2. 981	4708
"			
b	\oplus	958. 2320	
c	\oplus	0. 0822	
$b+c$	\oplus	958. 3142	
d	\oplus	0. 0067	
e	\oplus	0. 0009	
"			
$-\Delta\phi$	\oplus	958. 3200	
"			
$(K \sin Z)^2$	\oplus	8. 32499	
$*C$		0. 58976	
$\log c$	\oplus	8. 91475	
$\log (b+c)^2$		5. 9630	
$*D$	\oplus	1. 8652	
$\log d$	\oplus	7. 8282	
"			
$\log (-b)$	\square	2. 9815	
$(K \sin Z)^2$		8. 3250	
$*E$		5. 6416	
$\log e$	\square	6. 9481	

Longitude			
$K \sin Z$	\square	4. 162	4928
$*A'$		8. 509	6952
$\sec \phi'$		0. 004	6822
$\Delta\lambda_1$	\square	2. 676	8702
$\Delta\lambda$	\square	2. 676	8687
$\Delta\lambda =$	\square	475. 1915	
"			
\arg		K	-19
		$\Delta\lambda_1$	+ 4
		Corr.	-15

Azimuth			
$(\Delta\lambda)^2$	\square	8. 031	
$*F$	\oplus	7. 461-20	
$-\Delta Z_2$	\square	5. 492-10	
"			
$\Delta\lambda$	\square	2. 676	8687
$\sin \phi_m$	\oplus	9. 171	2897
$\sec \frac{1}{2}\Delta\phi$		0. 000	12
$-\Delta Z_1$	\square	1. 848	1596
"			
$-\Delta Z_1$	$=$	\square	70. 4952
$-\Delta Z_2$	$=$	\square	0
$-\Delta Z$	$=$	\square	70. 4952
"			
ϕ	$=$	8	39
$\frac{1}{2}\Delta\phi$	$=$	\square	7
ϕ_m	$=$	8	31

ϕ	8	39	52. 0670	Pacheca.....	λ	79	03	21. 2470
$\Delta\phi$	\square	15	58. 3200		$\Delta\lambda$	\square	7	55. 1915
ϕ'	8	23	53. 7470	Rey	λ'	78	55	26. 0555

 \square , algebraic sign to be supplied.In south latitudes ϕ and $\sin \phi$ are negative.In south latitudes C , D , and F are negative.

Eastern longitudes are considered negative.

N.H.O. 733

* From Tables for the Computation of Geodetic Positions, C. & G. S. Sp. Pub. 8.

GEODETIC POSITION EVALUATION

INVERSE SOLUTION

Computed by C. D., 1931

8	36	21.8700	From Ologue	*A	79	36	09.7270	<input type="checkbox"/> supply sign
8	23	53.7471	To Rey	A'	78	55	26.0556	<input type="checkbox"/> negative number
-Δa = (a - a')	12	28.1229	Δa = (A' - A)	40	43	57.14		
-Δa =	6	14.0615	Δa =	20	21	8.587		
*Δa = +Δa	8	30	07.8085	log Δa seconds				
log (-Δa) seconds				log Δa seconds				
are (-Δa) to sin (-Δa)				are Δa to sin Δa				

log	3.087	0.103	log	3.388	0.428	(Δa)²	0.164	10
Δa, sine form	0.995	2008	* sin Δa	0.169	8120	P	7	458 - 20
cos Δa	1.490	3055	sec Δa	0.000	7	-ΔZ	7	622 - 10
1 + Δa	4.572	5166	-ΔZ	2.557	8555	-ΔZ	361	290° - 20
sin ½K sin Z _m								
① + ①, tan Z _m	0.512	1612	Tabular angle	72	54	27.25	Quadrant	4th
sin ½K cos Z _m	4.060	3554	1 sum = -ΔZ				Z _m = Z + ΔZ	
-Δa, sine form	2.572	0427						
cos Δa	0.999	9924						
1 + Δa	1.487	4203						
Z = ① + ①	287	02	32	10				
Z = ① - ① + 180°	107	08	33	40				

* South latitudes and east longitudes are considered negative.

N.H.O. 734

3. In frame B complete the computation of $\tan Z_m$ as indicated, taking out A_m and B_m for the middle latitude ϕ_m . Give the tabular angle Z_m the sign of its tangent, and determine the quadrant as that of a line passing from the first station to the second, or if preferred, from the algebraic signs of terms ① and ②, $\sin \frac{1}{2}K$ being positive.

4. In frame C derive two values of $\sin \frac{1}{2}K$ by dividing ① by $\sin Z_m$ and ② by $\cos Z_m$. They should check. Change to the arc form, the log correction this time being positive, and multiply by 2 to derive arc K . Write K in frame D.

5. The azimuth difference, frame E, is found exactly as in the direct solution. Half of this difference, being added to Z_m , gives the forward azimuth Z ; and being subtracted from Z_m , gives the reverse azimuth Z' .

ANALYTICAL SOLUTION OF A THREE-POINT FIX

The matter of scale sometimes intervenes in a way to make it desirable to compute a three-point fix, rather than to plot it graphically on a small-scale sheet made for the express purpose, afterwards transferring the position to the large-scale sheet on which the position is required.

The problem in its general form may be stated thus:

Given the geodetic positions of three stations, A on the left, B in the center, and C on the right, and the fix angles at an unknown point X , namely, L from A to B , and R from B to C ; to find the geodetic position of X .

The first step is to find the distances and mutual azimuths between A and B and between B and C . These are found by the inverse geodetic solution process, if they are not already known from the triangulation system. In other cases, the geodetic positions are unknown, and in that event the needed distances or bases, AB and BC , and their azimuths are scaled. In the following analysis, it will be assumed that they are known or have been determined by computation or otherwise.

Connecting X with the three points, also drawing the left and right bases, thus forming two triangles, let A and C be the interior angles at A and C , and let B be the clockwise angle from the direction BA to the direction BC . Let L be the left fix angle and R the right fix angle, with l and r denoting the opposite bases. Further, let U be an auxiliary angle such that—

$$\tan U = \frac{l \sin R}{r \sin L}$$

Then

$$\frac{1}{2}(A+C) = \frac{1}{2}(B-(L+R))$$

and

$$\tan \frac{1}{2}(A-C) = \cot(U+45^\circ) \tan \frac{1}{2}(A+C)$$

Having replaced the second members by their values and found $\frac{1}{2}(A-C)$, add the half-angles to find A , and subtract the second from the first to find B .

Thus will become known the base and two angles in each triangle, which will permit the solution of each as a plane triangle. If their solution as spherical triangles is desired, it will be necessary at this point to compute the spherical excess and to solve again. The conditions of observation seldom warrant this. If the observed angles, however, are considered perfect, then all of the spherical excess will have to be assigned to the angles at C .

The solutions of the two triangles will give the distances and forward azimuths from the three points to the unknown point X . The distance BX should be the same in both triangles. It seldom will. But a mean may be taken, and using this the geodetic position of X may be computed by the direct solution.

Examples

1. (See H. O. chart 2615.) Loma Banao to Potrerillo, log 4.593,3798 (meters), azimuth 92-51-10.27, Loma Banao to Obispo, log 4.023,5983, azimuth 255-41-54.99, fix at Paz Bank, $L=34-51$, $R=7-01$. Show that—

Left triangle, Banao to Paz, log 4.814,5401, azimuth 19-40-09;

Right triangle, Banao to Paz, log 4.814,5439, azimuth 19-40-07.

The azimuths are geodetic, clockwise from south as zero.

Three-point fixes are also used to check stadia traverses, as in the next example.

2. With FORT as origin and north as zero a stadia traverse was run through the woods to station 51, the coordinates of which came out S2276.7, W7288.3. At 51 three triangulation stations were sighted, giving the angles L and R . Going back to FORT the angle from a north meridian mark to 51 was found to be 252-36. Given, in addition, l =FORT to PICO, log=3.882,5968 (feet), azimuth=294-14-27.2, and r =FORT to LOMA, log=3.937,1744, azimuth=245-11-10.0, $L=69-09$, $R=130-18$: solve the triangle 51-FORT-PICO and find the plane coordinates of 51 by triangulation.

3. Solve the three-point fix at 51, and find the coordinates of 51 by three-point fix.

4. Find the different values of the azimuth FORT-51.

ADJUSTMENT OF TRIANGLES

When only two angles of a triangle have been measured, it must be assumed that the measured angles are of equal weight and that the third angle is that which will bring the sum to 180° plus the spherical excess.

When all of the angles have been observed, the observer should weight them in accordance with the known and recorded conditions of observation. The corrections are proportional to the reciprocals of the weights. He may find it convenient to devise a scale of errors, based on experience, to assist the judgment. For example, suppose that the corrections, greatly magnified, are assumed to be proportional to 20 seconds for lack of rigidity of towers; 20 seconds for the force of the wind; 20 seconds for heat waves, refraction, etc.; 10 seconds for lack of distinctness of targets, bad background, etc.; 18 seconds for the sizes of angles, large angles being affected by the shift of weight of the observer; and 10 seconds for the number of sets observed.

The distribution may take the form:

Station	Weak-ness of tower	Force of wind	Heat effects	Light, back-ground	Size of angles	Number of sets	Sum of squares	Approx-imately $1/w$	Use $1/w$
Total.....	Seconds 20	Seconds 20	Seconds 20	Seconds 10	Seconds 18	Seconds 10			
Sal.....	10	2	15	2	10	2	437	20.9	21
Rey.....	6	4	4	2	4	4	104	10.2	10
Lex.....	4	14	1	6	4	4	281	16.8	17
									48

Station	Observed	Sets	$1/w$	Correc-tion	Adjusted
Sal.....	93 10 06.5	4	21	0.92	93 10 07.42
Rey.....	42 46 31.3	2	10	0.44	42 46 31.74
Lex.....	44 03 21.0	2	17	0.74	44 03 21.74
	179 59 58.8	$\epsilon=0.90$			180 00 00.90
	180 00 00.90	2.10, total to be added			

The resultant error at each station is assumed to be equal to the square root of one-sixth of the sum of the squares of the six component errors; and the reciprocals of the weights are made proportional to these resultant errors. Or, dropping the common factor $\sqrt{1/6}$, they are made proportional to the square roots of 437, 104, and 281, respectively. Using rounded-off values of the reciprocals, the corrections assigned to Sal, Rex, and Ley are, respectively, 21, 10, and 17 forty-eighths of the total correction for closure error.

Here the heaviest corrections are assumed to be due to heat waves at Sal, refraction between Rey and Sal, and wind at Lex.

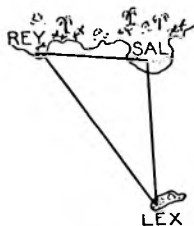


FIGURE 94.

REDUCTION TO CENTER

When the eccentric distance is small in comparison with the triangle sides and when the lengths of the sides are known approximately, the corrections required to convert the directions observed at the eccentric station into corresponding directions at the central station may be found by either of the two methods illustrated.

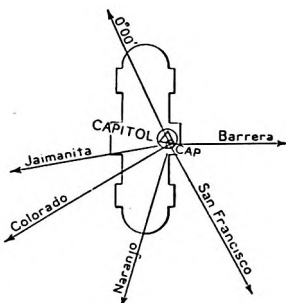


FIGURE 95.—Reduction to center, by computation.

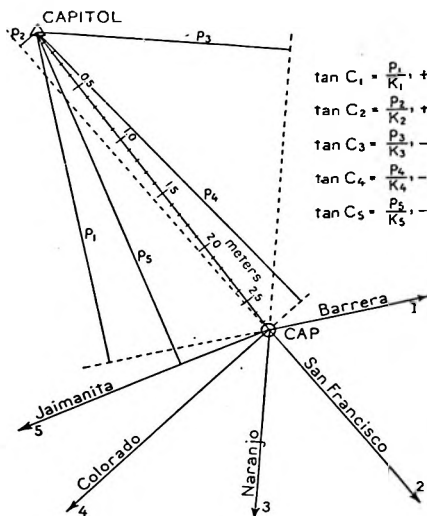


FIGURE 96.—Reduction to center, by scaling perpendiculars.

The rule of signs for the graphical solution is as follows:

If the eccentric station, seen from the central station, appears on the right of the distant station, the correction is positive; if it is seen on the left of the distant station, the correction is negative.

The formulae may be written in the form—

$$C'' = \frac{p \cot 1''}{K}$$

p being any perpendicular as shown in figure 96, and K the approximate distance corresponding to p .

If the perpendiculars are computed, each is equal to $d \sin a$, a being a direction to the distant station based on the zero direction of eccentric station to central station, and d the offset distance of the eccentric station. The formula is—

$$C'' = d \cot 1'' \frac{\sin a}{K}, \log \cot 1'' = 5.314,4251.$$

Occupying CAP, eccentric station, zero on Capitol, $d=9.065$ feet

Station	a			K	$\sin a$	$\frac{\sin a}{K}$	$\log C''$	C'''
	$^{\circ}$	$'$	$''$					
Capitol, center	0	00	00	<i>Log meters</i>				<i>Seconds</i>
Barrera	114	11	42.21	4.224, 2552	9.960, 0688	5.735, 8136	1.49162	+31.02
San Francisco	175	46	53.51	3.980, 8415	8.866, 6398	4.885, 7983	0.64161	+4.38
Naranjo	220	03	16.25	4.025, 2868	9.808, 5595	5.783, 2727	1.53988	-34.60
Colorado	263	15	42.08	4.398, 3328	9.996, 9896	5.598, 6568	1.35446	-22.62
Jaimanita	284	04	00.00	4.158, 0293	9.986, 7778	5.828, 7483	1.58456	-38.42

Caution.—In this case d must be reduced to meters.

LOCAL ADJUSTMENTS

When the sum of n observed angles about a point is $360^{\circ} + x$, the correction to each angle is $-x$ divided by n , if the angles are equally well measured; otherwise, the correction to each angle is found by dividing $-x$ by the sum of the reciprocals of the weights, and multiplying the resulting unit correction by the reciprocal of the weight for that angle.

When the sum of the observed angles about a point, for the same instrument and the same observer, persists in coming out either less or greater than 360° , or when observed sum angles repeatedly underrun or overrun the sum of the observed component angles, the presence of a constant error, affecting all angles equally, is indicated.

The example presents a different problem, in that there is an observed value of a sum angle, s , to be adjusted along with its independently observed partials, a , b , and c .

			Accidental errors					Constant error		
			Equal weights		Unequal weights			Equal weights		
Angle	Observed			Correc- tion	Adjusted	1/w	Correc- tion	Adjusted	Correc- tion	Adjusted
	°	'	''	''	''		''	''	''	''
s.....	140	14	54.9	-0.6	54.3	3	-0.9	54.0	+1.2	56.1
a.....	32	17	15.3	+0.6	15.9	1	+0.3	15.6	+1.2	16.5
b.....	65	26	24.5	+0.6	25.1	2	+0.6	25.1	+1.2	25.7
c.....	42	31	12.7	+0.6	13.3	2	+0.6	13.3	+1.2	13.9
Sums.....	52.5				54.3	8		54.0		56.1
				Divisor 4		Divisor 8			Divisor 2	
-x	+2.4			Unit, 0.6''		Unit, 0.3''			Unit, 1.2''	

For accidental errors, the divisor is $n+1$; for a constant error, $n-1$, n being the number of constituent angles.

If all the angles have been equally well measured, and if, as is usual, the errors are regarded as accidental, as likely by the performance of instrument and observer to be positive as negative, the smallest and most probable corrections are simply $\frac{1}{n}$ of the total adjustment, or $-\frac{x}{n}$, to each angle. Here the observed sum, s , is adjusted toward the sum, $a+b+c$, of the observed partials.

With accidental errors and unequal weights the same rule of signs applies, but the corrections are made numerically proportional to the reciprocals of the weights.

When a constant error is suspected (due, perhaps, to faulty clamp action), it must be assumed that all the corrections have the same sign. This results in an adjustment of the observed sum away from the sum of the observed partials, thus leading to large adjustments and a rapid accumulation of error.

Strictly speaking, constant errors cannot be adjusted out, though they may be estimated from similar observations. They call for a change in methods or instruments, and for additional observations, especially for frequent azimuth observations to check the accumulation of error.

ADJUSTMENT OF QUADRILATERALS

The quadrilateral of the illustration is adjusted in three steps:

(a) The eight separate angles are weighted according to the conditions of observing, and adjusted to the sum $360^\circ + E$, E being the spherical excess of the whole quadrilateral.

(b) The angles now having equal weights, the triangles are balanced in the most probable manner by an equal distribution of errors.

(c) The side adjustment, by which the two values of the terminal side are brought to the same value, is made in such a manner, without destroying the balance of the triangles, that the sum of the squares of the corrections applied is the least possible. They are, mathematically, the most probable corrections.

The first step is the most important, especially when single towers are employed for observing. In this case, the small angles of a figure should be superior to the large ones, other conditions being the same. The least square adjustment implies an assumption of equally good angles at the start, and bears heavily on the small angles through their large tabular differences. When there is good reason for believing that the small angles are superior, the weights assigned to them should be sufficient to prevent the subsequent mechanical adjustment from taking undue charge.

With weighted angles adjusted to $360^\circ + E$, the third residual, l_3 (see Johnson's Surveying) disappears, and the angle equations reduce to the simple forms indicated.

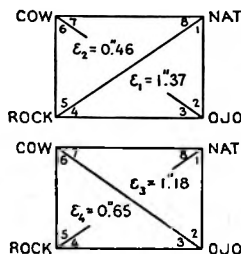
Quadrilateral, angle adjustment

No.	Observed	1/w	Weighted	First adjustment		Second adjustment
	° ' "		" "	" "	" "	" "
1	58 31 06.0	2	06.36	06.26	06.34	
2	38 35 35.7	1	35.88	35.78	35.86	
3	20 54 28.3	1	28.48	28.38	28.30	
4	61 58 50.7	2	51.06	50.95	50.87	
	0.7		$S_1 = 1.78$		$e_1 = 1.37$	
			$e_1 = 1.37$			
			$-4) + 0.41$			
			-104			
5	42 19 19.4	1	19.58	19.68	19.60	
6	54 47 21.5	2	21.86	21.96	21.88	
7	59 43 37.8	3	38.33	38.44	38.52	
8	23 09 40.1	1	40.28	40.38	40.46	
	58.8	13	$S_2 = 0.05$	0.86	0.97	
			$e_2 = 0.46$		$e_2 = 0.46$	
	59.5		$e_2 = 0.46$	$e_3 = 1.18$	$e_4 = 0.65$	
	$E = 1.83$		$-4) - 0.41$	$4) 0.32$	-0.32	
	13) 2.33 (+.18)		$+104$	$+0.08$	-0.08	

COW
ROCK

COW
ROCK

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QJO



Angle equations, applicable to the weighted angles:

$$\begin{aligned}
 l_1 &= (S_1 - e_1) - (S_1 - e_1) = 0.41 - 0.33 = 0.08 & u_1 &= u_2 = -u_3 = -u_4 = -\frac{1}{4}l_1 = -0.02 \\
 l_2 &= (S_1 - e_2) - (S_1 - e_2) = 0.33 + 0.41 = 0.74 & u_2 &= u_1 = -u_3 = -u_4 = -\frac{1}{4}l_2 = -0.18\frac{1}{4}
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Total corrections}$$

Quadrilateral, side adjustment

No.	Second adjusted angles	Log sines		Difference per second		Sums $t+l$	Squares $(t+l)^2$	Correction v	Final angles
		+	-	+	-				
1.....	06.34	9.930,8514		12.9				-0.38	58 31 05.96
2.....	35.86		9.795,0371		26.4	39.3	1544	+0.45	38 35 36.31
3.....	28.30	9.552,5054			55.2			-0.74	20 54 27.56
4.....	50.87		9.945,8575	11.2		66.4	4409	+0.67	61 58 51.54
5.....	19.60	9.828,2073		23.1				-0.37	Check: $e_1 = 1.37$
6.....	21.88		9.912,2425		14.8	37.9	1436	+0.44	42 19 19.23
7.....	38.52	9.936,3310			12.3			-0.69	54 47 22.32
8.....	40.46		9.594,7461	49.2		61.5	3782	+0.62	59 43 37.83
Sums.....		9.247,8951 -8832	9.247,8832	96.4	108.7				23 09 41.08
		$l_1 = +119$		$C_0 = -12.3$					Check: $e_2 = 0.46$
				$\frac{1}{2}C_0 = -3.1$		$\frac{1}{2}C_0 = 38$			Checks:
						$\Sigma C^2 = 11209$		$e_1 = 1 + 2 + 7 + 8 - 180^\circ = 1.18$	
								$e_2 = 3 + 4 + 5 + 6 - 180^\circ = 0.65$	

$$\frac{x}{-3.1} = \frac{x'}{39.3} = \frac{x''}{66.4} = \frac{x'''}{37.9} = \frac{x'''}{61.5} = \frac{-l_1}{\frac{1}{2}C_0 + \Sigma C^2} = \frac{-119}{11209} = -0.010625. \quad x = 0.0329.$$

$$\begin{array}{llll} +x = +0.0329 & -x = -0.0329 & +x = +0.0329 & -x = -0.0329 \\ x' = -0.4172 & x'' = -0.7049 & x''' = -0.4024 & x'''' = -0.6529 \end{array}$$

$$\text{Check: } \Sigma v = 0$$

$$\begin{array}{ll} v_1 = -0.38 & v_2 = -0.74 \\ v_3 = -0.45 & v_4 = -0.67 \\ v_5 = -0.37 & v_6 = -0.44 \\ v_7 = -0.69 & v_8 = -0.62 \end{array}$$

$$\Sigma(v_1 - v_2) = -5 - 12 - 41 - 8 - 9 - 6 - 8 - 30 = -119 = -l_1, \text{ check}$$

Formulae:

$$l_1 = \log \frac{\sin 1 \cdot \sin 3 \cdot \sin 5 \cdot \sin 7}{\sin 2 \cdot \sin 4 \cdot \sin 6 \cdot \sin 8}; \quad C_0 = l_1 - l_2 - l_3 + l_4 + l_5 - l_6 - l_7 + l_8; \quad C_1 = l_1 - l_2; \text{ etc.}$$

$$\frac{x}{\frac{1}{2}C_0} = \frac{x'}{C_1} = \frac{x''}{C_2} = \frac{x'''}{C_3} = \frac{x''''}{C_4} = \frac{-l_1}{\frac{1}{2}C_0 + \Sigma C^2}; \quad v_1 = x + x', \quad v_2 = x - x', \quad v_3 = -x + x'', \quad v_4 = -x - x'', \text{ etc.}$$

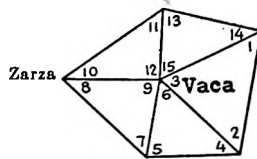
The signs + and - in the second line of the form are merely signs of operation, not of quality. When a tabular difference is essentially negative, as in the case of an obtuse angle, it must be entered in the appropriate column with the negative sign. In such a case the sums in the fifth, sixth, and seventh columns become algebraic sums.

ADJUSTMENT OF CENTRAL-POINT POLYGONS

Central-point polygon, side adjustment (when all of the periphery angles are acute)

Manati

Charcas



Lobo

Rabihorcado

Conditional equations

$$\sin 1 \cdot \sin 4 \cdot \sin 7 \cdot \sin 10 \cdot \sin 13 = 1$$

$$\sin 2 \cdot \sin 5 \cdot \sin 8 \cdot \sin 11 \cdot \sin 14$$

$$3 + 6 + 9 + 12 + 15 - 360^\circ = 0; \text{ sines do not enter}$$

$$1 + 2 + 3 - (180^\circ + \epsilon_1) = 0 \quad 10 + 11 + 12 - (180^\circ + \epsilon_4) = 0$$

$$4 + 5 + 6 - (180^\circ + \epsilon_2) = 0 \quad 13 + 14 + 15 - (180^\circ + \epsilon_3) = 0$$

$$7 + 8 + 9 - (180^\circ + \epsilon_5) = 0$$

No.	First adjusted angles	Log sines of first adjusted angles		Difference per 1"	t_1^2 t_2^2 l_1, l_2	$-2t_1 - t_2$ $+t_1 + 2t_2$ $+t_1 - t_2$	Corrections v	Final adjusted values	
		+	-					Angles	Log sines
1.....	54 41 52.69	9.911 7525		+14.9	222.01	-44.9	+0.322	53.01	9.911 6530
2.....	54 27 27.38		9.910 4566	+15.1	228.01	+45.1	+0.212	27.59	9.910 4570
3.....	70 50 41.98				+224.99	-0.2	-0.534	41.45	9.975 2634
	$\epsilon_1 = 2.05$					0.0	0.000	2.05	
4.....	47 11 09.97	9.865 4386		+19.5	399.75	-43.3	+0.320	10.29	9.865 4393
5.....	78 32 18.68		9.991 2520	+4.3	18.49	+28.1	+0.233	18.91	9.991 2521
6.....	54 16 32.67				+83.85	+15.2	-0.553	32.12	9.909 4677
	$\epsilon_2 = 1.32$					0.0	0.000	1.32	
7.....	51 41 03.44	9.894 6518		+16.7	278.89	-53.1	+0.332	03.77	9.894 6524
8.....	46 50 39.23		9.863 0234	+19.7	388.09	+56.1	+0.199	39.43	9.863 0238
9.....	81 28 18.63				+328.99	-3.0	-0.531	18.10	9.995 1711
	$\epsilon_3 = 1.30$					0.0	0.000	1.30	
10.....	36 34 27.35	9.775 1473		+28.4	806.56	-74.6	+0.358	27.71	9.775 1483
11.....	49 44 42.55		9.882 6257	+17.8	316.84	+64.0	+0.189	42.74	9.882 6261
12.....	93 40 51.20				+505.52	+10.6	-0.547	50.65	9.999 1032
	$\epsilon_4 = 1.10$					0.0	0.000	1.10	
13.....	81 40 46.70	9.995 4046		+3.1	9.61	-32.6	+0.307	47.01	9.995 4047
14.....	38 35 36.29		9.795 0382	+26.4	696.96	+55.9	+0.199	36.49	9.795 0387
15.....	59 43 38.19				+81.84	-23.3	-0.506	37.68	9.936 3299
	$\epsilon_5 = 1.18$					0.0	0.000	1.18	
Sums.....		9.442 3948	9.442 3959		+4590.40	-0.7		Check: 1 Sum ..	9.442 3977
			3948		$\Sigma(t_1^2 + t_2^2)$	$\Sigma(l_1 - t_2)$		2 Sum ..	9.442 3977
	$-l'' = +2.67'' = 3 + 6$				+ $l_1 l_2$			Check:	
	$+9 + 12 + 15 - 360^\circ$			$-l'' = -11$				$3 + 6 + 9 + 15 = 360^\circ$	

Normal equations:

$$2A\Sigma(t_1^2 + t_2^2 + l_1 l_2) + B\Sigma(t_1 - t_2) = -3l''$$

$$-A\Sigma(t_1 - t_2) - B.2n = -3l''$$

$$9180.8A - .7B = -33 \quad A = -0.00366$$

$$.7A - 10B = +8.01 \quad B = -0.80126$$

Correlate equations:

$$3v_1 = A(-2t_1 - t_2) - B$$

$$3v_2 = A(+t_1 + 2t_2) - B$$

$$3v_3 = A(+t_1 - t_2) + 2B$$

Form of corrections for each triangle

$$0 = 0, \text{ check}$$

NOTATION

t_1 and t_2 are the tabular differences of the log sines of the first and second angles of each triangle. Strict account must be taken of their algebraic signs, plus for acute angles and minus for obtuse angles, as in the form on the next page.

$-l''$ is the closure error of the log of the quotient of the sines.

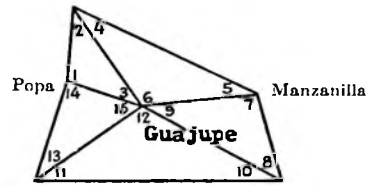
$-l''$ is the closure error of the sum of the central angles.

The sigma sign indicates the summation of terms of the same type.

$-A$ and B are correlates, or multipliers, of the differential forms of the first two conditional equations.

Central-point polygon, side adjustment (when some of the periphery angles are obtuse)

Santo Domingo



Conditional equations

$$\sin 1 \cdot \sin 4 \cdot \sin 7 \cdot \sin 10 \cdot \sin 13 = 1.$$

$$\sin 2 \cdot \sin 5 \cdot \sin 8 \cdot \sin 11 \cdot \sin 14$$

$$3+6+9+12+15-360^\circ = 0; \text{ sines do not enter.}$$

$$1+2+3-(180^\circ + \epsilon_1) = 0 \quad 10+11+12-(180^\circ + \epsilon_4) = 0$$

$$4+5+6-(180^\circ + \epsilon_2) = 0 \quad 13+14+15-(180^\circ + \epsilon_5) = 0$$

$$7+8+9-(180^\circ + \epsilon_3) = 0$$

No.	First adjusted angles	Log sines of first adjusted angles		Difference per 1'' l	t_1^2 t_2^2 t, t_2	$-2t_1 - t_2$ $+t_1 + 2t_2$ $+t_1 - t_2$	Corrections v	Final adjusted values	
		+	-					Angles	Log sines
1.....	107 26 35.68	9.979 5550		-6.6	44.56	-7.7	+0.093	35.773	¹ 9.979 5549
2.....	45 11 00.98		9.850 8722	+20.9	436.81	+35.2	+0.075	01.055	² 9.850 8724
3.....	27 22 23.38				-137.94	-27.5	-0.168	23.212	9.662 5530
$\epsilon_1 = 0.04$									
4.....	25 13 56.73	9.629 7065		+44.7	1998.09	0.0	0.000	0.040	
5.....	28 51 48.23		9.683 6980	+38.2	1459.24	+121.1	+0.039	48.269	² 9.683 6981
6.....	125 54 15.12				+1707.54	+6.5	-0.182	14.938	9.908 4846
$\epsilon_2 = 0.08$									
7.....	96 29 50.88	9.997 2015		-2.4	5.76	0.0	0.000	0.080	
8.....	49 22 10.68		9.880 1996	+18.0	324.00	+33.6	+0.095	50.975	¹ 9.997 2015
9.....	34 07 58.50				-43.20	-20.4	-0.171	58.329	² 9.880 1997
$\epsilon_3 = 0.06$									
10.....	27 28 59.00	9.664 1588		+40.4	1632.16	-114.2	+0.137	59.137	¹ 9.664 1593
11.....	32 09 39.20		9.726 1552	+33.4	1115.56	+107.2	+0.045	39.245	² 9.726 1554
12.....	120 21 21.91				+1349.36	+7.0	-0.182	21.728	9.935 9614
$\epsilon_4 = 0.11$									
13.....	47 03 41.49	9.864 5619		+19.5	380.25	-42.5	+0.107	41.597	¹ 9.864 5621
14.....	80 42 16.60		9.994 2594	+3.5	12.25	+26.5	+0.079	16.679	² 9.994 2594
15.....	52 14 01.98				+68.25	+16.0	-0.186	01.794	9.897 9111
$\epsilon_5 = 0.07$									
Sums.....		9.135 1837	9.135 1844		10352.69	0.0	0.000	0.070	
$-l'' = +0.89'' = 3+6$			1837		$\Sigma(t_1^2 + t_2^2 + t_1 t_2)$	-18.4	Check:	¹ Sum...	9.135 1850
$+9+12+15-360^\circ$				$-l' = -7$		$\Sigma(t_1 - t_2)$	Check:	² Sum...	9.135 1850
								$3+6+9+12+15=360^\circ$	

Normal equations:

$$2A\Sigma(t_1^2 + t_2^2 + t_1 t_2) + B\Sigma(t_1 - t_2) = -3l'$$

$$-A\Sigma(t_1 - t_2) - B \cdot 2n = -3l''$$

$$20705.38A - 18.4B = -21 \quad A = -0.001254$$

$$18.4A - 10B = +2.67 \quad B = -0.269307$$

Correlate equations:

$$3v_1 = A(-2t_1 - t_2) - B$$

$$3v_2 = A(+t_1 + 2t_2) - B$$

$$3v_3 = A(+t_1 - t_2) + 2B$$

Form of corrections for each triangle

$$0=0, \text{ check}$$

BASE AND AZIMUTH ADJUSTMENTS

Any side of a triangulation net, on becoming known, becomes a base. The lengths of the initial and terminal bases, between which an adjustment is proposed, may have been fixed either by measurement or by previous triangulation.

Let D_b be the logarithm, in units of the seventh decimal place, that must be added to the logarithm of MN derived by geodetic transport from AB to reduce it to the logarithm of the fixed value of MN .

In any triangle, let the angle opposite the known side be denoted by K , and that opposite the required side by Q , the corresponding tabular differences being k and q , all with subscripts corresponding to the number of the triangle.

Since all distances are involved in the form—

$$\frac{\text{required side}}{\text{known side}} = \frac{\sin Q}{\sin K}, \quad (1)$$

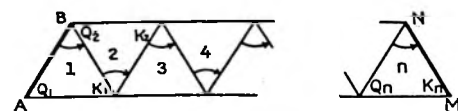


FIGURE 97.—Distance and azimuth angles.

the requirements of the adjustment are that K and Q , the distance angles in each triangle, are to be given corrections that are numerically equal but opposite in sign, so as to leave the third angles, called the **azimuth angles**, undisturbed. The probability condition is that the corrections applied must be proportional to the several values of $(k+q)$. Let x and y be the general symbols for the numerical values of the corrections to be added to K and Q . Then $x = -y$; and x has a certain sign in all the triangles and y has the opposite sign.

When D_b is positive, it is necessary to **adjust long**, causing the successive bases from AB to MN to receive a cumulative positive adjustment. In this case, the corrections are applied in the sense $-x, +y$; $-x$ serving to decrease the denominator in (1) and $+y$ to increase the numerator. When D_b is negative, it is necessary to **adjust short**, and accordingly the corrections are applied in the sense $+x, -y$.

In the following illustration, a base adjustment is made between the Peninsula of Zapata (Mont-Gor) and the Isle of Pines (Pip-Lump):

Pip-Lump = 4,354,6647, fixed by triangulation of 1923-25.
 Pip-Lump = 4,354,6264, derived by triangulation of 1926-27.
 $D_b = 383$, to be adjusted long.

Base adjustment

Mont-Gor to Pip-Lump					
Triangle No.	Tabular differences		Sums $k+q$	Squares $(k+q)^2$	Angle corrections $-x(=+y)$
	k	q			
1.....	11.5	25.3	36.8	1,354	-0.43
2.....	8.5	39.5	48.0	2,304	-0.55
3.....	35.4	18.3	54.7	2,992	-0.63
4.....	-11.3	32.1	20.8	433	-0.24
5.....	40.1	15.5	55.6	3,091	-0.64
6.....	4.1	32.3	36.4	1,216	-0.42
7.....	32.0	4.0	36.0	1,296	-0.42
8.....	14.2	1.3	15.5	240	-0.18
9.....	21.6	18.6	40.2	1,616	-0.46
10.....	13.0	30.6	43.6	1,888	-0.50
11.....	25.6	-0.1	25.5	650	-0.29
12.....	23.2	15.0	38.2	1,459	-0.44
13.....	0.6	12.5	13.1	172	-0.15
14.....	6.7	12.6	19.3	372	-0.22
15.....	18.0	1.1	19.1	365	-0.22
16.....	23.2	45.7	68.9	4,747	-0.79
17.....	17.9	-4.1	13.8	190	-0.16
18.....	22.6	25.5	48.1	2,314	-0.56
19.....	31.2	18.6	49.8	2,480	-0.58
20.....	4.5	12.7	17.2	296	-0.20
21.....	15.1	-6.2	8.9	79	-0.10
22.....	6.6	31.1	37.7	1,421	-0.44
23.....	8.8	19.0	27.8	773	-0.32
24.....	33.6	3.8	37.4	1,399	-0.43
			$\Sigma(k+q)^2$	33,147	

Formulae:

$$\frac{1}{2}C = \frac{-D_b}{\Sigma(k+q)^2} = \frac{-333}{33147} = -0.01155.$$

$$x_n = -y_n = \frac{1}{2}C(k_n + q_n), \text{ correction to any angle.}$$

CAUTION.—The tabular differences are negative for obtuse angles.

AZIMUTH ADJUSTMENT; SIMULTANEOUS BASE AND AZIMUTH ADJUSTMENT

As is evident from Figure 97, in which the azimuth angles are marked by arcs, the azimuth angles rule the orientation of the whole net.

Let D_z be the angle that must be added to the azimuth of MN derived by geodetic transport from AB to reduce it to the azimuth of MN fixed by astronomical observations or by previous triangulation.

The base adjustment having been made, leaving the azimuth angles unchanged, the whole accumulation of azimuth correction, D_z , is divided by the number of triangles, n , to find the portion to be assigned to each triangle. This is applied to the azimuth angles in the sense of increasing azimuths when D_z is positive, and in the opposite sense when D_z is negative. When D_z is positive, it will be found that the common correction, D_z/n will require to be applied positively to L -angles and negatively to R -angles, L and R being the left-hand and right-hand stations as seen from the unknown station in each triangle. When D_z is negative, the opposite rule will hold.

Having applied a correction, D_z/n , to the azimuth angle of any triangle, the same correction with opposite sign must be applied to the sum of the distance angles, to keep the triangle in balance. Further, to avoid disturbing the base adjustment, the latter correction must be distributed to the two angles in inverse proportion to the tabular differences of their sines. The corrections to the three angles of a triangle, then, are:

$$\text{Azimuth angle, } +\frac{D_z}{n}; \text{ K-angle, } -\frac{D_z}{n} \frac{q}{p+q}; \text{ Q-angle, } -\frac{D_z}{n} \frac{k}{p+q};$$

or the same with opposite signs for all.

The base and azimuth adjustments, and the final solution of each triangle, may be performed simultaneously, as illustrated below.

Position	Tabular differences	Station	Angles, 1926	Adjustment		Final angles		Sides (arc)	Sides
				Base	Azimuth				
X	K	0.6	Rose-----	88 31 53.42	-.15	+ .58	53.85		
R	Q	12.5	Vis-----	59 11 40.52	+.15	+.03	40.70		
L	Z	(33.3)	Lone-----	32 16 26.76	-----	-.61	26.15		
				0.70	.00	.00	0.70		
			1926 values			Final			
RL			4 387, 8725	-----	-----	8949	11	8960	Vis-Lone
csc X			0.000, 1427	-----	-----	1426			
sin R			9.933, 9484	-----	-----	9487			
sin L			9.727, 5170	-----	-----	5150			
LX			4 321, 9636	-----	-----	9662	8	9870	Lone-Rose
RX			4 115, 5322	-----	-----	5525	3	5528	Vis-Rose

Explanation.—This is triangle 13, page 154. The base is Vis-Lone, which has already received an increment of 8949-8725, or +224 units, by adjustment of triangles 1 to 12. The base to go forward is Rose-Lone. Its adjustment, 9862-9636, or +226 units, is composed of the initial 224 units plus 2 units derived from the present adjustment. Its contribution to the total adjustment of 383 units, an average of 16 for each triangle, is small, due to the fact that its $(k+q)^2$, 172, is such a small part of the total $\Sigma(k+q)^2$ for all the triangles, which is 33147.

The base adjustments of X and R cancel each other and leave the angles of the triangle in balance. Their length effect, however, is cumulative, the functions concerned being $\csc X$ and $\sin R$.

The sum of the azimuth adjustments of X and R is to be $0.61''$. Being likewise involved in $\csc X$ and $\sin R$, the tabular increments arising from them will have opposite signs. In order that they may cancel each other and leave the base adjustment apportioned to the triangle unchanged, since the tabular difference of X is about $\frac{1}{10}$ of that of R it will be necessary to distribute $\frac{3}{10}$ of $0.61''$ to X and $\frac{7}{10}$ of $0.61''$ to R , or $0.58''$ to X and $0.03''$ to R . The final angles may now be obtained, and the triangle solved anew. Only the last four figures of the logarithms need be written.

It will be noted that the adjustment of a net is carried through the sine forms of the bases. For instance, if ρ is the radius of curvature of the original measured base and is held unchanged for all sides of the net, and if x and r are the central angles subtended by X and R , we have in the form:

$$\begin{array}{ll} 4.387,8725 = \log \rho \sin x, \text{ unadjusted,} & 4.321,9636 = \log \rho \sin r, \text{ unadjusted,} \\ 4.387,8949 = \log \rho \sin x, \text{ adjusted,} & 4.321,9862 = \log \rho \sin r, \text{ adjusted,} \\ 4.387,8960 = \log \text{ arc } RL, \text{ adjusted,} & 4.321,9870 = \log \text{ arc } LX, \text{ adjusted.} \end{array}$$

The last two logarithms have no part in the adjustment, but they are carried along because their naturals are the actual lengths of the sides.

The tabular difference for the azimuth angle (33.3) takes no part in the adjustment, but is useful as a check in looking out $\sin L$.

ADJUSTMENT OF INCOMPLETE FIGURES

Incomplete figures attached to the sides of a triangulation net may be conveniently called **blisters**.

In the first figure, bases I and II and all the angles about Viz having been fixed by the main triangulation, there remained only a base adjustment from I to II through four triangles.

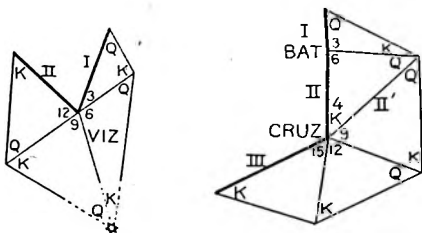


FIGURE 98.—Incomplete figures.

In the second figure, after balancing and solving the triangles, the following discrepancies remained:

At Bat, $v_3 + v_6 = 0.57$ second; at Cruz, $v_4 + v_6 + v_{12} + v_{15} = -0.18$ second; D , I to II, 38 units too long; D , I to III, 52 units too long.

Angle 3 was increased +0.30 second, and -0.30 second was distributed to K and Q in inverse proportion to their tabular differences. A similar adjustment of +0.27 second and -0.27 second took place in the next triangle. Next the discrepancy of 38 units between I and II was eliminated by a base adjustment.

This adjustment increased the sum of the v 's at Cruz to -0.36 second, which was divided into three equal parts for the angles 9, 12, and 15, while +0.12 second was distributed to the distance angles in each triangle in inverse proportion to the tabular differences. The remaining base discrepancy between II' and III, amounting to 23 units, was eliminated by a base adjustment.

A complete central-point polygon may be adjusted by the same methods, which amount to a least square adjustment, if the initial base is a radial. In all such figures, it is best to make the azimuth adjustment first, as this often amounts to a mere local adjustment, which may be made with or without weights.

ADJUSTMENT OF OVERLAPPING TRIANGLES

The cut represents triangulation carried forward with only one line of occupied stations, M, N, P , etc., being either water towers or inaccessible inland features. It is assumed that the nature of the shore forbids the measurement of sides, except the short base AB .

To derive any base from the preceding, it is necessary to resort to overlapping triangles producing figures resembling folded or reversed polygons. The first figure in the chain is like $MABCN$, in which the terminal side BC is doubly derived from the initial side through two pairs of triangles, one pair having the vertex M , the other the vertex N . All other figures are like $NBCDP$, in which one of the triangles of the first pair is frozen by the adjustment of the figure.

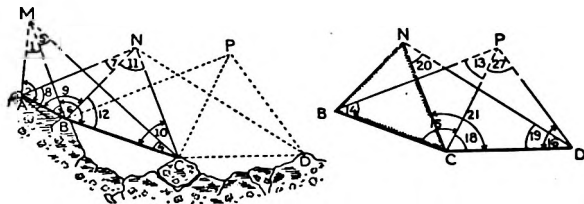


FIGURE 98.—Overlapping triangles.

The net can be adjusted, with two determinations of every base, if it is assumed that the total angle at each shore station is fixed. The shore angles, therefore, must be measured with more than ordinary care, and frequent azimuth observations must be made, to justify such an assumption.

If occupations are made on the ground, the angles of the separate triangles should be measured as numbered, in order to provide independent measurements. The proper lines of sight to cause the figures to recur are, first, the shore bases, and second, those lines, and no others, which would be drawn from the unoccupied stations to indicate three-point fixes on the ends of two adjacent shore bases.

In any triangle, the angles are numbered in the order K, Q, Z . Thus, the azimuth angles, numbered as multiples of 3 and subject to local adjustment only, will be visible at a glance.

The order of adjustment is as follows:

(a) Locally adjust the azimuth angles at B to the fixed sum. Take 2, 8, 4, and 10 as measured. Conclude the triangles to derive the angles at M and N .

(b) Derive BC from AB through the triangles having the common vertex M ; then through the triangles having the common vertex N . Bring the two values of BC together by a base adjustment.

(c) Locally adjust the azimuth angles at C to the fixed sum. Take 21 as the fixed sum less the adjusted value of 10. Take 14, 16, and 19, as measured. Conclude the triangles to derive the angles at N and P .

(d) Subject the two values of CD derived from NC and BC to a base adjustment.

The operations (c) and (d) may be continued indefinitely.

If M, N, P , etc., are fixed signals in positions selected with due regard to the strength of figures, the resulting net will carry forward distance and azimuth fairly well. If they are not fixed objects, nevertheless the system can be carried forward by observing **simultaneous cuts** from the shore stations to each one; but the survey based on the positions of the shore stations thus determined will necessarily be of inferior precision.

TRIANGULATION

TABLE 11.—*Logarithmic secants of small angles, unit 0.000 0001*

Angle	00	10	20	30	40	50
1	0	0	0	0	1	1
2	1	1	1	1	1	1
3	2	2	2	2	2	3
4	3	3	3	4	4	4
5	5	5	5	6	6	6
6	7	7	7	8	8	9
7	9	9	10	10	11	11
8	12	12	13	13	14	14
9	15	16	16	17	17	18
10	18	19	20	20	21	22
11	22	23	24	24	25	26
12	26	27	28	29	29	30
13	31	32	33	33	34	35
14	36	37	38	39	40	40
15	41	42	43	44	45	46
16	47	48	49	50	51	52
17	53	54	55	56	57	58
18	60	61	62	63	64	65
19	66	67	69	70	71	72
20	73	75	76	77	78	80
21	81	82	84	85	86	88
22	89	90	92	93	94	96
23	97	99	100	101	103	104
24	106	107	109	110	112	113
25	115	116	118	119	121	123
26	124	126	127	129	131	132
27	134	136	137	139	141	142
28	144	146	148	149	151	153
29	155	156	158	160	162	164
30	165	167	169	171	173	175
31	177	178	180	182	184	186
32	188	190	192	194	196	198
33	200	202	204	206	208	210
34	212	214	217	219	221	223
35	225	227	229	232	234	236
36	238	240	243	245	247	249
37	252	254	256	258	261	263
38	265	268	270	272	275	277
39	279	282	284	287	289	292
40	294	296	299	301	304	306
41	309	311	314	316	319	322
42	324	327	329	332	334	337
43	340	342	345	348	350	353
44	356	358	361	364	367	369
45	372	375	378	380	383	386
46	389	392	394	397	400	403
47	406	409	412	415	417	420
48	423	426	429	432	435	438
49	441	444	447	450	453	456
50	459	462	466	469	472	475

TRIANGULATION

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TABLE 12.—*Logarithms of the spherical excess coefficient m*

Lat.	00'	10'	20'	30'	40'	50'
°	—10	—10	—10	—10	—10	—10
0	+1. 40695	+1. 40695	+1. 40695	+1. 40695	+1. 40695	+1. 40695
1	95	95	94	94	94	94
2	94	94	94	94	93	93
3	93	93	93	92	92	92
4	92	92	91	91	91	91
5	+1. 40690	+1. 40690	+1. 40690	+1. 40689	+1. 40689	+1. 40689
6	88	88	88	87	87	86
7	86	86	85	85	84	84
8	83	83	83	82	81	81
9	80	80	79	79	78	78
10	+1. 40677	+1. 40677	+1. 40676	+1. 40675	+1. 40675	+1. 40674
11	73	73	72	71	71	70
12	69	69	68	67	66	66
13	65	64	63	63	62	61
14	60	60	59	58	57	56
15	+1. 40655	+1. 40655	+1. 40654	+1. 40653	+1. 40652	+1. 40651
16	50	49	48	47	46	45
17	44	44	43	42	41	40
18	39	38	37	36	34	34
19	32	31	30	29	28	27
20	+1. 40626	+1. 40625	+1. 40624	+1. 40623	+1. 40622	+1. 40620
21	19	18	17	16	14	13
22	12	11	10	09	07	06
23	05	04	02	01	00	+1. 40599
24	+1. 40597	+1. 40596	+1. 40595	+1. 40594	+1. 40592	91
25	90	88	87	86	84	83
26	82	80	79	78	76	75
27	73	72	71	69	68	66
28	65	64	62	61	60	58
29	56	55	54	52	51	49
30	+1. 40548	+1. 40546	+1. 40545	+1. 40543	+1. 40542	+1. 40540
31	39	37	36	34	33	31
32	30	28	26	25	23	22
33	20	19	17	15	14	12
34	11	09	07	06	04	03
35	01	00	+1. 40498	+1. 40496	+1. 40495	+1. 40493
36	+1. 40491	+1. 40490	88	86	85	83
37	82	80	79	77	75	75
38	72	70	68	67	65	63
39	62	60	58	57	55	53
40	+1. 40452	+1. 40450	+1. 40448	+1. 40446	+1. 40445	+1. 40443
41	41	40	38	36	35	33
42	31	29	27	26	24	22
43	21	19	17	16	14	12
44	11	09	07	05	04	02
45	+1. 40400	+1. 40399	+1. 40397	+1. 40395	+1. 40393	+1. 40392
46	90	88	86	85	83	81
47	80	78	76	75	73	71
48	69	68	66	64	63	61
49	59	58	56	54	52	51
50	+1. 40349	+1. 40347	+1. 40346	+1. 40344	+1. 40342	+1. 40341
51	39	37	36	34	32	31
52	29	27	26	24	22	21
53	19	17	16	14	12	11
54	09	07	06	04	03	01
55	+1. 40299	+1. 40298	+1. 40296	+1. 40295	+1. 40293	+1. 40291

CHAPTER VIII

TRIGONOMETRIC LEVELING

TOPOGRAPHICAL MATERIAL DESIRED FOR CHARTS

In respect to landmarks, hills, and mountain peaks, the requirements are:

- (a) Accurate location of summits.
- (b) Approximate elevations of all recognizable features.
- (c) Delineation of forms of summits and striking features, both by sketch contouring and photography, sufficient to afford ready identification.
- (d) Emphasis of important features and generalization of subordinate features, not, however, to the point of suppressing them when they are needed to give a background and a basis of comparison for the main features. For example, if there are two conical peaks of much the same appearance, one near the west end of an east-and-west range of hills, and the other at the southeast end of a descending ridge, these details cannot be suppressed without danger of failing to provide a proper identification of the peaks.

From the mariner's standpoint, the important features of a coast are those visible 20 to 50 miles from shore. The distance at which they can be best studied, separated, and cut upon with a theodolite is, perhaps, 15 to 20 miles. For photography, to seize the same general natural or usual appearance, the distance must be shortened to 10 miles or less.

DISTANT CUTS, MOUNTAIN SHEETS

For distant cuts, the principle of the long plotting base should be kept in mind. If cuts are taken from subordinate stations at water level, as is often advisable for identification of peaks, the zero of the instrument should be placed on a well-defined distant peak, either already well located or certain to be well located later. In general, locations are accepted only from cuts at triangulation stations, where an accurate azimuth is available, and the cuts are laid down by azimuths, rather than by angles, on an accurate small-scale projection. Even an accurate polyconic projection does not suffice for distant cuts intersecting near the corners, and recourse must be had to computing principal peaks at intervals, and using the directions to these as controlling directions.

The rounded tops of peaks are sharpened by distance, so that there is often a remarkable agreement in the computed lengths of sides common to adjacent triangles having the same peak as a vertex. The point so defined is the real summit. Shorter cuts often give a sizable triangle of error, partly because nearer points are mistaken for the summit and partly because of irregularities in refraction over land lines.

ACCURACY OF ELEVATIONS

Elevations obtained by trigonometrical transport are always uncertain because of great variations in the index of refraction. An extreme case occurred in the survey of India, in which the elevation angle of an object near the horizon changed more than 7 minutes of arc in about 6 hours. More stable conditions are usually encountered in hydrographic surveys than in land surveys, because of water-borne sights and a better circulation of air by breezes, but anomalous conditions may be caused even by a small patch of heated sand near the observer.

The best elevations probably come from water-borne sights at moderate distances of 10 to 20 miles. For greater distances, the combined correction for curvature and refraction, which varies as the square of the distance, enters with preponderant effect. Short cuts, though numerous and concordant, are to be regarded with suspicion for reasons stated above.

PHOTOGRAPHY AS AN AID

Photographing a range of mountains in sections from points near shore, enlarging the prints, and designating the main features by name or number, will serve to identify the same features when cut upon from greater distances, and to prevent confusion due to overlapping profiles.

Additional photographs taken at triangulation stations will save sketching time. Without the aid of photography, sketches must often be carried in the field books for several years. Photographs also aid in sketch contouring and in writing sailing directions.

SKETCHING

Sketches assist the memory in identifying land forms expected to be seen from subsequent stations. For this purpose, a notched pencil held at arm's length, or a short comb with coarse teeth, will be found convenient. Vertical dimensions must be exaggerated to give a natural appearance. It is well to keep the notes and the corresponding angles, numbered to correspond, on opposite pages, alternately sketching in sections and observing angles. The sketches should be fairly large. It is useless to measure altitudes with the sketching tool. Instead, the highest elevation may be taken as a certain number of squares of cross-section paper, and all other elevations estimated as parts of it as they appear to the eye. Enlarged or thumbnail sketches of summit forms, notches, etc., may be placed on the same lines as the angles relating thereto. They give forms as seen through the telescope, with identifying marks, such as leaning trees, boulders, etc.

The following kinds of cuts may be noted:

- (a) *Peaks*.—Designation, horizontal angle, vertical angle, thumbnail sketch of summit.
 (b) *Notches*.—Designation, horizontal and vertical, thumbnail sketch showing the lap, thus:

17	15-07-30	+3-18½	+3-18
17/18	15-30-00	+2-09-30	
18	16-04-00	+3-18½	+3-18



- (c) *Sections*.—Horizontal angles at set elevations.
 (d) *Cliffs*.—Horizontal angle, vertical angles for top and bottom, notes on slope and appearance.
 (e) *Culture, direction of ridges, etc.*—Limits by angles.

The figure illustrates typical field sketches.

1. Sierra Trinidad, from outer harbor of Casilda.
2. Sierra de los Organos—sketch for sailing directions for entering Cortes Bay.
3. Profile of Colombo Hill, showing cliffs and terraces.
4. Typical cross section of Sierra las Casas.
5. Thumbnail sketches.

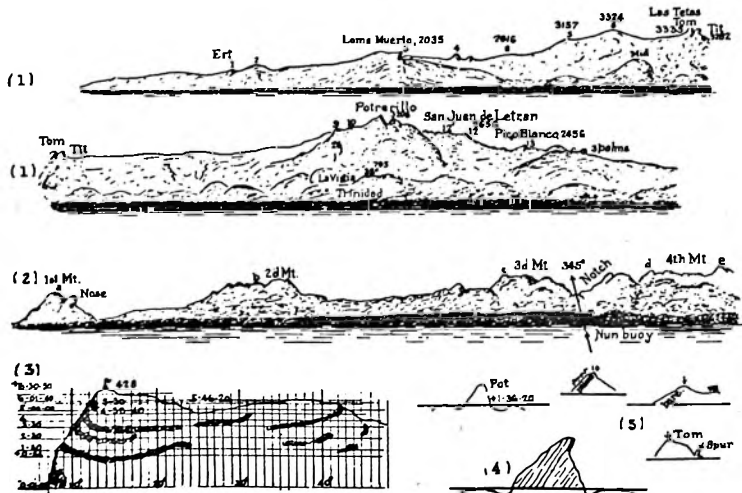


FIGURE 100.—Topographic sketches.

In the illustrations that follow, the distances in examples 1 to 7 and in 12 and 13 would be considered moderate in geodetic work. The distances in 8 to 11 are rather long, the elevation angle at the lower point to the higher point being negative in some cases. Those in 14 and 15 are shorter than the average.

COMPUTATION OF ELEVATIONS, DIRECT AND INVERSE

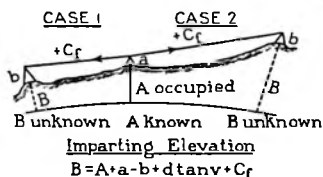
Illustrations. $B = A + a - b + d \tan v + C_f$; $A = B + b - a - d \tan v - C_f$; $\log (-v'') = \frac{1}{2} \log h + 2.027,5171$.

Δ Trinidad, 211.78 meters, 695 feet

Number and case	1, Case 2 Diaz-Trin	2, Case 2 Viv-Trin	3, Case 4 Trin-Pot	4, Case 4 Trin-Bald	5, Case 1 Bald-Trin
A to B	21, 190 c	13, 885 c	8, 458 c	35, 189 c	35, 189 c
d, log d	21, 190 c	13, 885 c	8, 458 c	35, 189 c	35, 189 c
v	+0° 29' 10"	+0° 47' 35"	+5° 08' 24"	+0° 32' 16"	-0° 49' 19"
w	3	2	1	1	1
A	+0.82	+3.28	A	A	+631.90
a	+1.53	0.00	-1.52	-1.52	+1.22
B	B	B	+977.09	+631.90	B
b	0	0	0	0	0
d tan v	+179.79	+192.90	-760.84	-330.29	-504.84
C _f	+30.35	+12.99	-4.85	-83.61	+83.61
h	+212.49	+209.17	+209.88	+216.48	+211.89

Loma Cartujo, 354.67 meters, 1,164 feet, bare summit near Δ Lo.

Number and case	6, Case 3 Lo-Ag	7, Case 2 Ag-Lo
A to B	25, 492 c	25, 492 c
d, log d	25, 492 c	25, 492 c
v	-0° 50'	+0° 37' 15"
w	1	2
A	A	0.00
a	-0.61	+30.18
B	0.00	B
b	+29.88	0.00
d tan v	+370.82	+279.84
C _f	-43.87	+43.87
h	+356.22	+353.89



A is the station occupied, and all "imparting" or "receiving" of elevation is done there. B is the station sighted. The equations are identical, the first solved for B, the second for A.

The beginner will do well to use the first equation, meant for imparting elevation, for all cases, until he finds by trial that the second equation is more convenient for the indirect cases of receiving elevation. Thus, example 3 may be solved as follows:

Pot ($B = +977.09$) = Trin (A , unknown) + a ($= +1.52$) + $d \tan v$ ($= +760.84$) + C_f ($= +4.85$) - b ($= 0$).

Hence Trin = $+977.09 - 1.52 - 760.84 - 4.85 + 0 = +209.88$,

which could have been obtained directly from the second equation.

Computed distances, or their logarithms, are marked c; scaled distances are marked s.

TRIGONOMETRIC LEVELING

Loma Lorenzo Sanchez, 398.84 meters, 1,309 feet, outline, probably tree tops

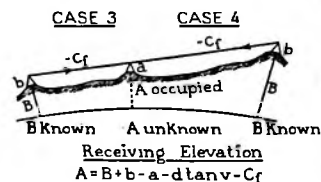
Number and case	8, Case 1 <i>Ar-Lore</i>	9, Case 2 <i>Juc-Lore</i>	10, Case 2 <i>Cow-Lore</i>	11, Case 1 <i>Rock-Lore</i>	
A to B	81, 130 s	54, 480 s	72, 405 s	77, 640 s	
d, log d	-0° 03' 10"	+0° 11' 30"	+0° 01' 20"	-0° 01' 30"	
v	1	1	1	1	
w	1	1	1	1	
A	0.00	0.00	0.00	0.00	
a	+25.30	+18.59	+19.20	+24.38	
B	B	B	B	B	
b	indef. 0	indef. 0	indef. 0	indef. 0	
d tan v	-74.73	+182.25	+28.08	-33.88	
C _f	+444.53	+200.45	+354.07	+407.12	
h	+395.10	+401.29	+401.35	+397.62	

Loma San Carlos, 183.19 meters, 601 feet, bare summit

Number and case	12, Case 2 <i>Lump-Carl</i>	13, Case 2 <i>Tri-Carl</i>	12 Horizon reads	13 Horizon reads	Corr. to v"
A to B	4.414, 2630	4.340, 3886			-12"
d, log d	+0° 01' 20"	+0° 19' 40"	-0° 20' 20"	-0° 09' 00"	-4"
v	1	3			
w	1	3			
A	+131.07	0.00			
a	+0.91	+24.99			
B	B	B			
b	0.00	0.00			
d tan v	+6.04	+125.70			
C _f	+45.44	+32.41			
h	+183.46	+183.10			

Loma Sigüanea, 90.09 meters, 296 feet, summit, ground

Number and case	14, Case 2 <i>Ware-Sig</i>	15, Case 3 <i>Sig-Ware</i>	
A to B	4.014, 9987	4.014, 9987	
d, log d	+0° 26' 00"	-0° 30' 30"	
v	1	1	
w	1	1	
A	0.00	A	
a	+4.39	-1.81	
B	B	0.00	
b	0.00	+7.47	
d tan v	+78.29	+91.84	
C _f	+7.25	-7.25	
h	+89.93	+90.25	



Examples 8 and 11 illustrate negative angles of elevation to a distant hill, observed from towers at sea level, and the commanding influence of the curvature and refraction term, which increases with the square of the distance.

In examples 12 and 13 the corrections required to reduce the observed dip of the horizon to their theoretical values, at two stations on different days, were applied to small observed angles of elevation, thus practically eliminating variations of refraction.

ELEMENTS OF A TRIGONOMETRICAL ELEVATION

If v is the vertical angle, positive or negative, of B sighted from A with an instrument having the zero of its vertical circle adjusted to level sights, and if d is the distance from A to B measured in the plane tangent to the surface of the earth at A , the first element of the difference in elevation between A and B , or the instrumental element, is $d \tan v$. For terrestrial sights, short in terms of R , the radius of the earth, the sea level arc AB may be used for d with negligible error.

The second element, that due to the curvature of the earth, which rapidly drops below the tangent plane, is a large positive correction equal to the extension of the radius of the earth at B necessary to attain the height of the tangent plane at A .

If C is the curvature of the earth between A and B , or the central angle subtended by AB , the curvature correction to v , expressed as an angle, is simply $+\frac{1}{2}C$. This correction is combined with the next and converted into linear measure before being applied.

The third element is due to refraction, which normally causes objects sighted to appear too high, whether v is positive or negative, and thus tends to decrease the effect of curvature. On the assumption that the density of the atmosphere increases toward the earth at a uniform rate, the path of a ray of light passing through this medium is usually considered to be the arc of a circle, convex in the same sense as the curve of the earth, but with a larger radius. The correction for refraction, therefore, may be treated as a certain fractional part, f , of the curvature. This fraction, or coefficient, is called the **coefficient of refraction**.

Expressed as an angle, the correction for curvature and refraction combined is $\frac{1}{2}C - fC$, or $\frac{1}{2}(1-2f)C$.

Let C_r = correction for curvature and refraction, in meters,

and R = mean radius of the earth in meters.

Take $d = RC$ = arc AB = chord AB , approximately,

and C_r = perpendicular from B to tangent plane through A .

In the right triangle having the base d , the perpendicular C_r , and the angle $-v = \frac{1}{2}(1-2f)C$ opposite the perpendicular, we have

$$C_r = d \tan \frac{1}{2}(1-2f)C = d \tan \frac{1}{2}(1-2f) \cdot \frac{d}{R};$$

or replacing the tangent of the small arc by the arc,

$$C_r = \frac{(1-2f)d^2}{2R} \quad (1)$$

The following illustrations give an idea of the variation of the coefficient of refraction (as determined from long high lines of sight, except in the last example).

Value	Country	Remarks
0.0753.....	Great Britain.....	Lines over water.
0.0813.....	Great Britain.....	Lines over land.
0.06 to 0.08.....	United States.....	Interior, New England.
0.0665 to 0.075.....	Egypt.....	Adopted mean, 0.065.
0.069 to 0.087.....	Gold Coast.....	By day, by night.
0.0568 to 0.0576.....	Nigeria.....	At 600 to 3,000 feet.
0.054.....	Colombia.....	Atlantic coast.

The following table is based on $f=0.070$ and the mean radius of curvature for Clarke's spheroid of 1866, $\log R=6.803,9665$, meters. Modifying (1) to express the distance as K kilometers instead of d meters, C_r remaining in meters,

$$C_r = 0.0675K^2. \quad (2)$$

TABLE 13.—Correction for curvature and refraction ($f=0.070$) $C_f=0.0675K^2$, C_f in meters, K in kilometers

K	C_f	K	C_f	K	C_f	K	C_f	K	C_f	K	C_f
1	0.06	21	29.8	41	113.5	61	251.3	81	443.1	101	688.9
2	0.27	22	32.7	42	119.1	62	259.6	82	454.1	102	702.6
3	0.61	23	35.7	43	124.9	63	268.0	83	465.2	103	716.4
4	1.08	24	38.9	44	130.7	64	276.6	84	476.5	104	730.4
5	1.69	25	42.2	45	136.8	65	285.3	85	487.9	105	744.5
6	2.43	26	45.6	46	142.9	66	294.2	86	499.5	106	758.8
7	3.31	27	49.2	47	149.2	67	303.1	87	511.1	107	773.2
8	4.32	28	52.9	48	155.6	68	312.3	88	523.0	108	787.7
9	5.47	29	56.8	49	162.1	69	321.5	89	534.9	109	802.3
10	6.75	30	60.8	50	168.8	70	330.9	90	547.0	110	817.1
11	8.17	31	64.9	51	175.6	71	340.4	91	559.2	111	832.0
12	9.72	32	69.2	52	182.6	72	350.1	92	571.6	112	847.0
13	11.41	33	73.5	53	189.7	73	359.9	93	584.1	113	862.3
14	13.23	34	78.1	54	196.9	74	369.8	94	596.7	114	877.6
15	15.19	35	82.7	55	204.3	75	379.9	95	609.5	115	893.1
16	17.28	36	87.5	56	211.8	76	390.1	96	622.4	116	908.7
17	19.51	37	92.4	57	219.4	77	400.4	97	635.4	117	924.4
18	21.87	38	97.5	58	227.2	78	410.9	98	648.6	118	940.3
19	24.37	39	102.7	59	235.1	79	421.5	99	661.9	119	956.3
20	27.01	40	108.0	60	243.1	80	432.2	100	675.3	120	972.4

For other values of f decrease the tabular values as follows:

f	0.071	0.072	0.073	0.074	0.075	0.076	0.077	0.078	0.079	0.080
Percentage of decrease—	0.23	0.47	0.70	0.93	1.16	1.40	1.63	1.86	2.09	2.33

CONVERSION TABLES

TABLE 14.—Heights of towers, feet to meters

Tens	Units									
	0 feet	1 foot	2 feet	3 feet	4 feet	5 feet	6 feet	7 feet	8 feet	9 feet
	<i>Meters</i>	<i>Meters</i>	<i>Meters</i>	<i>Meters</i>	<i>Meters</i>	<i>Meters</i>	<i>Meters</i>	<i>Meters</i>	<i>Meters</i>	<i>Meters</i>
5.....	15.24	15.54	15.85	16.15	16.46	16.76	17.07	17.37	17.68	17.98
6.....	18.29	18.59	18.89	19.20	19.50	19.81	20.11	20.42	20.72	21.03
7.....	21.34	21.64	21.94	22.25	22.55	22.86	23.16	23.47	23.77	24.08
8.....	24.38	24.69	24.99	25.30	25.60	25.91	26.21	26.52	26.82	27.13
9.....	27.43	27.74	28.04	28.35	28.65	28.95	29.26	29.57	29.87	30.18
10.....	30.48	30.79	31.09	31.39	31.70	32.00	32.31	32.61	32.92	33.22
11.....	33.53	33.83	34.14	34.44	34.75	35.05	35.36	35.66	35.97	36.27
12.....	36.58	36.88	37.19	37.49	37.80	38.10	38.41	38.71	39.01	39.32
13.....	39.62	39.93	40.23	40.54	40.84	41.15	41.45	41.76	42.06	42.37

Decimal equivalents, 1 to 9

Meters to feet						Feet to meters					
<i>Meters</i>	<i>Feet</i>	<i>Meters</i>	<i>Feet</i>	<i>Meters</i>	<i>Feet</i>	<i>Feet</i>	<i>Meters</i>	<i>Feet</i>	<i>Meters</i>	<i>Feet</i>	<i>Meters</i>
1.....	3.28083	4	13.12333	7	22.96583	1	0.30480	4	1.21920	7	2.13360
2.....	6.56167	5	16.40417	8	26.24667	2	.60960	5	1.52400	8	2.43840
3.....	9.84250	6	19.68500	9	29.52750	3	.91440	6	1.82880	9	2.74320

In hydrographic surveys, the value of f adopted is usually a matter of small consequence. An average value derived from numerous observations may be used. Or, if instruments of sufficient sensitiveness are at hand, the value momentarily holding at each station during observations may be measured, provided that the elevation of the station is well known, by vertical angles to points known both in elevation and position; or, if the elevation of the station is not known, by vertical angles to such points in pairs, one lower and one higher than the station.

Values of f derived from a pair of reciprocal sights are illusory, because of the differing local conditions at the two stations, and the differing general conditions at the two times of observation.

Mean sea level is commonly taken as the zero of trigonometric elevations.

Let A be the point of reference at A , the station occupied; a the height of the instrument support above A ; B the elevation of the reference point at B , the station sighted; and b the height above B of the target, if any.

Then
$$-A+B=a-b+d \tan v+C, \quad (3)$$

CORRECTION TO OBSERVED ANGLES OF DEPRESSION

The dip of the horizon may be obtained from the approximate formula

$$\log (-v'')=\frac{1}{2} \log h+2.027,5171, \text{ or } -v''=106.54 \sqrt{h} \text{ seconds} \quad (4)$$

in which v'' is the dip in seconds and h is the elevation in meters above sea level.

With the same notation, the approximate formula given in C. & G. S. Sp. Pub. 145, page 81, may be written—

$$h=\frac{1}{2} R \sin ^2 1'' \frac{(-v'')^2}{1-2f} \quad (5)$$

or taking $\log R=6.803,9612$, and again the same and $f=0.07$,

$$\log h=2 \log (-v'')-\log (1-2f)+5.874,0810-10, \quad (6)$$

and
$$\log h=2 \log (-v'')+5.939,5825-10. \quad (7)$$

In illustration number 12 in the tabular computation of elevations, page 164:

By (6), if $h=138.98$ meters and $-v''=1220''$, $f=0.078$.

By (7), if $h=138.98$ meters, $-v''=1232''$; using (4), $-v''=1224''$.

By (7), if $-v''=1220''$ and $f=0.07$, $h=129.51$ meters; by (4), 131.12 meters.

The correction for mean refraction, for refraction differing from the mean, for curvature, and for index error are all angular corrections reckoned from the plane defined by the instrument set for level sights, and so may be combined. Therefore, when the height of eye is accurately known, the difference between the computed dip and the observed dip may be used, as in example 12, to correct the observed dip, and all other small angles observed at the same time and place, for combined index error and refraction differing from the mean tabular value. This is the principal use of formula (7). In using this formula, which refers to sea level at the time of observation, elevations referred to mean sea level must first be reduced for stage of tide.

Formula (4), in the form

$$D=58.82'' \sqrt{x},$$

with D in seconds and x in feet, is most useful in reconnaissance. It is based on a higher value of f , namely $f=0.0784$.

DISTANCES BY ANGLES OF DEPRESSION

Objects at sea level nearer than the horizon may be located by horizontal angles and distances determined by depression angles observed at an elevated point, but unless the height of eye is comparable to the distance, the determination of the latter will be feeble. With a vertical circle reading to 20 seconds, for example, distances computed from vertical angles numerically less than 15 minutes should be considered merely provisory, as in the 10-foot pole or masthead angle method, the inverse of this. Such small angles, however, may be used in reconnaissance. See Reconnaissance table 3.

The depression angle method is useful for obtaining details of shore, reef lines, rocks, and breakers near towers or elevated points, and for estimating distances to distant dangers.

Let h be the height of eye and d the distance to the object, both in meters; and let x and y be the positive values in seconds of the depression angles to the object at water level and to the horizon, respectively. Regarding h as an elevation received from each of these two zero elevations through the depression angles, an equation between the two values of h may be formed involving d , x , and y .

From x ,

$$h = d \tan x - C_f = d \tan x - \frac{1-2f}{2R} d^2,$$

and from y ,

$$h = \frac{R}{2(1-2f)} y^2 \sin^2 1'';$$

whence, writing $x \tan 1''$ for $\tan x$ and $y^2 \tan^2 1''$ for $y^2 \sin^2 1''$, equating the expressions for h , and solving for d ,

$$d = \frac{R \tan 1''}{1-2f} (x - \sqrt{x^2 - y^2}), \quad (8)$$

which is determinate for d provided that both x and y are observed. Note that the formula is independent of the height of eye, and may be used at a station of unknown elevation. The height of eye is that corresponding to the depression of the horizon at the time of observation, and is not necessarily the best value thereof. If the height of eye is accurately known, the depression of the horizon for mean refraction may be computed to derive a correction, which should then be applied to both x and y . In practice, this is seldom done, but x and y are merely freed of index error.

Taking
(8) becomes

$$\log R = 6.803.9612 \text{ and } f = 0.07, \\ d = 35.895 (x - \sqrt{x^2 - y^2}), \quad (9)$$

the numerical coefficient coming from the logarithm 1.555,0376.

Example 1.—From a hill of unknown elevation, the depression angles to a breaker and to the horizon are $0^\circ 42'$ and $0^\circ 18'$, respectively. Find the distance to the breaker.

By (9), $d = \log^{-1} 1.555,0376 (2520 - \sqrt{3600 \times 1440}) = 8729$, meters.

Example 2.—Find the distance to an object nearer than the horizon and $0^\circ 20'$ lower than the horizon as observed from an elevation of 100 feet.

(a) By tables 3 and 5, pages 27 and 29, in which $f = 0.0784$,

$$d = 11.44 - 9.52 \text{ nautical miles} \\ = 3558 \text{ meters. } 1 \text{ nautical mile} = 1853.25 \text{ meters.}$$

(b) By table 15, page 169, arguments $y = 591.9''$ and $x = 1791.9''$, $d = 3611$ meters, corresponding to $f = 0.07$.

(c) By the approximate cotangent formula, table 15, second part, $d = 100 \cot (1791.9'') = 3508$ meters. Reject.

Tables 3 and 5 correspond to sextant measurements, and table 15 to transit or theodolite measurements, of angles of depression.

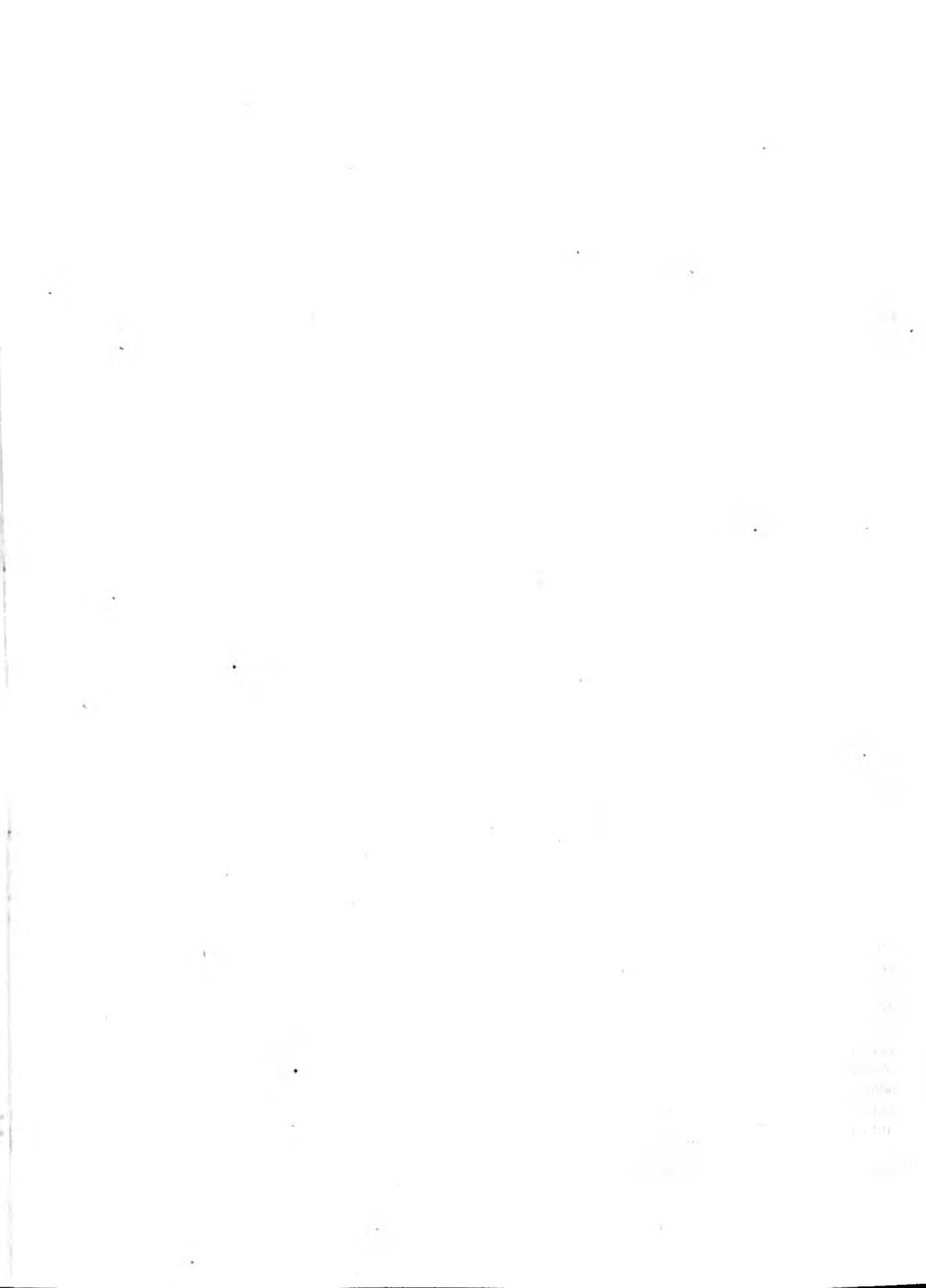
TABLE 15.—Distances in meters from towers to water lines of shores, reefs, breakers, etc.

By two depression angles, x'' to water line of object, y'' to horizon.								By single angle, — v to object.		
$d = 35.895(x - \sqrt{x^2 - y^2})$								$d = h \cot(-v)$		
Height, feet, ——— or meters, $h =$		60 18.29	70 21.34	80 24.38	90 27.43	100 30.48	110 33.53		100 30.48	
To horizon, seconds, $y =$		458.4	495.2	529.4	561.5	591.9	620.7		591.9	
$-v$	x	d						$-v$	d	Error
0 15	900	4,505	5,330	6,180	7,058	7,969	8,712	0 15	6,985	984
16	960	4,173	4,940	5,713	6,509	7,329	8,172	20	5,227	377
17	1,020	3,906	4,604	5,318	6,047	6,795	7,560	25	4,191	178
18	1,080	3,657	4,315	4,977	5,651	6,341	7,042	30	3,493	100
19	1,140	3,454	4,062	4,680	5,308	5,948	6,597	35	2,994	62
0 20	1,200	3,266	3,839	4,419	5,006	5,604	6,210	40	2,619	42
21	1,260	3,099	3,639	4,186	4,739	5,301	5,868	45	2,328	30
22	1,320	2,949	3,461	3,977	4,500	5,031	5,565	50	2,095	22
23	1,380	2,813	3,299	3,790	4,286	4,788	5,294	55	1,905	16
24	1,440	2,689	3,152	3,620	4,091	4,568	5,048	1 00	1,746	13
0 25	1,500	2,582	3,019	3,465	3,915	4,369	4,826	10	1,497	
26	1,560	2,472	2,896	3,323	3,753	4,188	4,623	20	1,310	
27	1,620	2,377	2,783	3,193	3,605	4,020	4,438	30	1,164	
28	1,680	2,288	2,679	3,072	3,468	3,867	4,268	40	1,048	
29	1,740	2,206	2,583	2,961	3,341	3,725	4,109	50	952	
0 30	1,800	2,130	2,493	2,858	3,224	3,593	3,963	2 00	873	
31	1,860	2,059	2,410	2,761	3,115	3,471	3,827	10	806	
32	1,920	1,993	2,332	2,672	3,013	3,357	3,701	20	748	
33	1,980	1,931	2,259	2,588	2,918	3,250	3,583	30	698	
34	2,040	1,873	2,190	2,509	2,830	3,150	3,472	40	654	
0 35	2,100	1,818	2,126	2,435	2,745	3,056	3,368	50	616	
36	2,160	1,766	2,065	2,365	2,666	2,968	3,270	3 00	582	
37	2,220	1,717	2,008	2,299	2,591	2,885	3,179	10	551	
38	2,280	1,671	1,954	2,237	2,520	2,806	3,091	20	523	
39	2,340	1,627	1,902	2,178	2,454	2,732	3,009	30	498	
0 40	2,400	1,586	1,854	2,122	2,391	2,661	2,931	40	476	
41	2,460	1,547	1,808	2,069	2,331	2,594	2,857	50	455	
42	2,520	1,509	1,764	2,019	2,274	2,531	2,787	4 00	436	
43	2,580	1,473	1,722	1,971	2,220	2,470	2,720	10	418	
44	2,640	1,439	1,682	1,925	2,168	2,413	2,656	20	402	
0 45	2,700	1,407	1,644	1,881	2,119	2,358	2,596	30	387	
46	2,760	1,376	1,608	1,840	2,072	2,305	2,538	40	373	
47	2,820	1,346	1,573	1,800	2,027	2,255	2,483	50	360	
48	2,880	1,318	1,540	1,761	1,984	2,207	2,430	5 00	348	
49	2,940	1,291	1,508	1,725	1,943	2,161	2,379	6	290	
0 50	3,000	1,265	1,477	1,690	1,903	2,117	2,330	7	249	
51	3,060	1,239	1,448	1,656	1,865	2,074	2,283	8	217	
52	3,120	1,215	1,420	1,624	1,829	2,033	2,238	9	192	
53	3,180	1,192	1,393	1,593	1,793	1,995	2,195	10	173	
54	3,240	1,170	1,367	1,563	1,720	1,958	2,154	11	156	
0 55	3,300	1,149	1,341	1,534	1,727	1,921	2,114	12	143	
56	3,360	1,128	1,317	1,507	1,696	1,886	2,076	13	132	
57	3,420	1,108	1,294	1,480	1,666	1,853	2,039	14	122	
58	3,480	1,088	1,271	1,454	1,637	1,820	2,003	15	114	
59	3,540	1,070	1,249	1,429	1,609	1,789	1,968	16	106	
1 00	3,600	1,052	1,228	1,405	1,582	1,759	1,935	17	100	

When $-v$ exceeds 1° , the distance for a height of eye less than 100 feet may be taken as a percentage of the tabular value for 100 feet.

When $-v$ exceeds 1° , the distance for a height of eye less than 100 feet may be taken as a percentage of the tabular value for 100 feet.

The heavy horizontal lines indicate that values above them are uncertain beyond practical use.



CHAPTER IX

AZIMUTH AND TIME

The angle, clockwise from the south, between the plane determined by the plumb line and the south celestial pole and that determined by the plumb line and a distant station, called the **mark**, is the astronomical azimuth, or simply the **azimuth**, of the mark. When longitude is observed at the same station, the azimuth is called a **La Place azimuth**. An azimuth derived by geodetic transport may be called a **geodetic azimuth**.

For orienting triangulation in the field an azimuth will always be observed over a principal long side of the triangulation net, if possible, rather than to an auxiliary mark, the use of which would remove the orientation one step and in effect waste one figure.

Check azimuths at suitable intervals are intended to furnish data for adjusting out an accumulation of azimuth error.

DIRECTION AND DISTANCE OF THE MARK

If the instrument is a repeating theodolite, the mark should not be near the meridian, but rather 30° to 60° from north or south, in order to utilize a large part of the horizontal circle in the multiple arc. For an astronomical transit, however, the mark should be placed near the meridian to minimize the travel of the micrometer. Three miles or more is the distance usually needed to afford a sidereal focus for both star and mark. For other reasons 8 miles is a good average distance for water-borne sights. The curvature of the earth, weather conditions, and the power of the lights must be considered in any case.

PREPARATIONS FOR OBSERVING

In geodetic work the only celestial bodies used to determine time and azimuth are stars. Therefore, a light is needed at the mark. To a limited extent, however, daylight observations without a light are possible. For example, in the northern hemisphere, Sirius for time and Polaris for azimuth may be used at twilight in moderate latitudes during March and April, and at dawn during September and October, Sirius at those times being less than 2 hours from culmination.

In preparation for night observations a light, run on battery, may be left at the distant tower late in the afternoon. For short distances use a light-box with a slit in the front subtending 2 seconds to 4 seconds. Take the theodolite ashore before sundown, shade it and allow it to cool before adjusting it and the stride level. Particular attention should be paid to the adjustment for inclination of the horizontal axis.

The instrument tripod is hardly a suitable support for precise work at night, in most cases. On rocky ground a pier may be built or improvised. In soft ground drive a pipe to receive the regular tower-head fitting for mounting the theodolite. When the pipe is a few inches higher than desired, drive three pipes at a steep angle for braces, touching the center pipe. Wire the pipes together, then drive down the braces. In quaking ground, add a platform for the observer, with the edges resting on short timbers. Comfort in observing being essential for accuracy, a set-up somewhat higher than the usual one will be required, for observing objects at an altitude. A substantial wind shield 8 to 10 feet high is often necessary. Ranges to help pick up the mark and star will contribute to the accuracy through reader pointings and more regular spacing of intervals between them. Flashlight bulbs may be mounted on poles at heights to bring them into the field of view when sighting the objects.

Stiff drawing paper with a matte surface makes a better reflector than the manufactured article. For illuminating the cross-wires, if necessary, run wires from a pocket battery down to

a small flashlight bulb at the end of a pencil. With the socket slightly loose the light can be turned on or off, or obscured to the proper dimness, with the forefinger. The same light, being small and in the clear, is invaluable for illuminating the verniers, as it may be held directly over the graduation under examination.

A built-in lighting system has recently been devised for triangulation instruments, combining reading lamps at all verniers and rheostat control of illumination to cross-wires. There are separate switches to further facilitate observations during darkness.

It is most convenient to use a chronometer with a loud tick. If a metronome is employed, lacking a chronometer, corrections must be made to reduce watch readings before and after use. A stop watch entails troublesome corrections.

With a watch alone, the assistant must tap out the seconds, announcing every fifth or tenth second. The result to be sought in tapping and counting is a light, staccato, unhurried cadence, with the seconds but slightly accented over the half-seconds, thus: "*Six-and, sev-en, eight-and,*" etc., "*twen-ty, one-and, two-and,*" etc., "*thir-ty, one-and, two-and,*" etc.

Some observers watch a time star come to the cross-wire and estimate tenths of a second between ticks of the clock. Others keep the star on the wire and release the tangent screw on an exact tick. The error when taps are employed is at least twice as great as when the observer hears the tick of the clock.

The chronometer may keep either mean time or sidereal time. In precise work a pendulum is sometimes used.

For azimuth observed with an astronomical transit see "Determination of time, longitude, latitude, and azimuth", Coast and Geodetic Survey Special Publication No. 14.

MEASURING THE AZIMUTH ANGLE

Measuring an angle from a mark to a star is similar to measuring an angle between two stations, but in addition it is necessary to mark time on the star, in order that its azimuth may be computed to furnish a known direction of reference.

If A is the azimuth of the star and Z that of the mark, a the reading of the horizontal circle on the star and z that on the mark, $a-z$ being the measured angle, then

$$Z = A - (a - z)$$

In the method of repetitions it is unnecessary, theoretically, to read the verniers except on the initial and final pointings. Suppose that there are n repetitions of the angle. In this method each z after the first is identical, on the circle, with the preceding a . Therefore,

$$Z = A_1 - (a_1 - z_1)$$

$$Z = A_2 - (a_2 - a_1)$$

$$Z = A_n - (a_n - a_{n-1}),$$

the mean of which is

$$Z = \frac{1}{n}(A_1 + A_2 + \dots + A_n) - \frac{1}{n}(a_n - z_1); \quad \text{that is,}$$

the mean observed azimuth of the mark is equal to the mean azimuth of the star, less the total travel of the verniers divided by the number of repetitions.

To avoid gross errors it is advisable to read carefully and to record the first angle of a set of repetitions. But reading other intermediates, involving a slowing of the work and unnecessary manipulation of the instrument, is unfavorable to precision of measurement and should be minimized to the point of taking approximate readings of the vernier that is under the eye. The following illustrates the general procedure for a 10-second 2-vernier transit in perfect adjustment:

Azimuth observations at Δ Petate									
Latitude: N. 20° 00' 00''							Observer: ARP		
Date: Jan. 21, 19--, evening.							Assistant: WHL		
Instruments: Theodolite No. -----							Mean time chronometer No. -----		
Star: 51 Cephei.							Mark: Δ Lobos.		
Clear, windy, shield used.									
No.	Object	Clock time	Verniers				Altitude	Predicted angles	Vernier
			A			B			
		h m s	° ' "	"	"	° ' "	° ' "	° ' "	
	Mark-----		0 00 00		55	0 00 20			A
1	Star-----	10 05 45	33 58 30		30	22 45 50			A
2	do-----	10 08 50	(67 55)				67 57		A
3	do-----	10 10 38	(101 49)				101 51		A
4	do-----	10 12 58	(315 43)				315 43		B
5	do-----	10 15 05	(349 34)				349 37		B
6	do-----	10 17 09	203 23 40		30	22 48 40	23 25		B
	Travel-----			40	35				
	Mean travel-----		203 23 37.5						
	Angle-----		33 53 56.2						

The notes indicate that the observer read both verniers on the first pointing on the mark, and as a precaution against losing the mark read its altitude as well. He did the same on the first pointing on the star, while the assistant noted the time. The latter, having recorded first the time, then the horizontal angle (both verniers), then the vertical angle, doubled the horizontal angle and set down the result as the predicted two-time angle. The observer, having set on the mark again, using the lower motion only, unclamped the upper motion, clamped it again on the predicted angle announced by the assistant, set off the same vertical angle as before, and found the star in the field of view, disclosing its apparent motion as upward and toward the left. As before, the star was bisected, using the upper motion tangent screw, on the even tick as heard, the assistant's time observation being limited to noting and recording the number of the second "marked." With a glance at the A-vernier, the observer announced the approximate angle as 67° 55' and the assistant, noting that it was 2 minutes less than predicted, added 33° 56' instead of 33° 58' to predict the next angle. As the star was moving westward, the increments added by the assistant to the preceding approximate angle heard, decreased. For the fourth pointing he added 180° as well as the increment, the telescope being reversed.

CORRECTIONS

Stride level readings are taken before and after an azimuth set, and during the set if cloudy weather occasions delays. They are considered applicable to the nearest group of pointings. It is convenient but not essential to have the stride level in perfect adjustment. The length of the bubble may have to be adjusted, if the movement is sluggish. Assist the bubble to come to equilibrium by rocking the level slightly without downward pressure, and by tapping the vial with a pencil. Do not allow it to be influenced by the heat of the hand or of the flashlight.

The correction for inclination of the horizontal axis is equal to the value of one division of the bubble in seconds of arc, multiplied by half the travel of the bubble between the direct and reversed positions, and by the tangent of the apparent altitude of the star. Numerically expressed, the two positions of the bubble are the means of the readings of the ends. The correction is applied in a sense away from the higher end of the axis. For horizontal circles numbered clockwise it is positive when the higher end of the axis is left, and negative when the higher end is right. If the stride level readings before and after a set show much change, it is better to reject the set and observe again.

The **inclination correction** I , applicable to clockwise circle readings of pointings on stars of apparent altitude h when the plate is truly horizontal and the horizontal axis is inclined, is given by the formula

$$I = \pm \frac{d \tan h}{b} (\Sigma w + \Sigma e),$$

in which the upper sign applies to north stars and the lower sign to south stars; d is the value of 1 division of the stride level; b is the total number of bubble end readings in $\frac{1}{2}b$ applications of the stride level direct and an equal number of companion readings in the reverse position of the level; and w and e are, separately, the algebraic sums of the west and east end readings, regarded as *positive* when the end of the bubble is *west of the zero* of the scale, and *negative* when the end of the bubble is *east of the zero*. The following examples show how the formula applies equally well to two common ways of graduating level vials. The observations are equivalent. When the zero is at the middle of the scale, either of the two positions in any observation may be regarded as the direct position. When the zero is at one end it is necessary to premise that the direct position shall be that in which the zero end is west. The position of the zero in each case is indicated by an asterisk.

	Zero at middle		
D	15	0	15
R	15	0	15

	+W	-E
D	+ 8.2*	- 10.2
R	+ 7.8*	- 9.6
D	+ 7.8*	- 11.8
R	+ 8.0*	- 10.6
$\Sigma w + \Sigma e$	+ 31.8	- 42.2

	Zero at one end		
	0	15	30
	30	15	0

	W	E
-D	* - 6.8	- 25.2
+R	+ 22.8	+ 5.4*
-D	* - 7.2	- 26.8
+R	+ 23.0	+ 4.4*
$\Sigma w + \Sigma e$	+ 31.8	- 42.2

The same formula applies to pointings on the mark, $\tan h$ being positive or negative according as the mark is above or below the horizon of the instrument.

The true azimuth angle between the mark and the star is the difference of the corrected pointings on them, expressed in the sense mark minus star. This angle, added algebraically to the computed azimuth of the star, will give the azimuth of the mark.

Instead of computing the azimuth of the star separately for each pair of pointings on star and mark, many computers prefer to compute a single azimuth at the mean epoch of the star pointings, and to reduce this, approximately, to the mean of the separate azimuths by applying a **curvature correction** derived from the formula

$$C'' = \frac{225}{2n} \sin 1'' \tan A_m \Sigma s^2 = F \cdot \tan A_m \Sigma s^2,$$

in which C'' denotes the corrections in seconds of arc, n the number of repetitions, A_m the azimuth at the mean epoch, and s the number of sidereal seconds of time from the mean epoch to any pointing on the star. When a sidereal chronometer is used, $\log F = 5.95858 - 10$. When a mean time chronometer is used $\log F = 5.96099 - 10$.

The formula should not be applied to other than circumpolar stars.

The sign of the curvature correction is that tending to swing the azimuth of the star toward the meridian.

Example of reduction of azimuth observations made with a repeating theodolite.

1926, April 18. At Δ MACCA, Almirante Bay, R.P., $9^{\circ}20'16''.93$ N., $82^{\circ}14'29''.70$ W. Observer GFK. Mark at Δ SIT, near horizon, distant 15 miles. Theodolite B45, chronometer 3140, mean time. Star, Polaris, $\alpha = 1^{\circ}39'19''.34$, $\delta = +88^{\circ}57'44''.98$, $\log \tan \delta = 1.742\ 0848$, $h = 8^{\circ}30'$. Stride level, $d = 12''$, zero at middle of

scale. $d \tan h = 0''.179$, $\log F = 5.96099 - 10$. $\tan x = \cos t \cot \delta$; $-\tan A = \tan t \sin x \sec(\phi + x)$; $C'' = F \tan A_m \Sigma s^2$;
 $I = \frac{1}{8} d \tan h (\Sigma w + \Sigma e)$

Set number		5D	6R	7D	8R
L.S.T.	hms	12 07 58.2	12 19 47.0	12 31 44.0	12 45 05.6
R.A.	hms	1 39 19.3	1 39 19.3	1 39 19.3	1 39 19.3
t in time	hms	10 28 38.9	10 40 27.7	10 52 24.7	11 05 46.3
t in arc	o' "	157 09 43.5	160 06 55.5	163 06 10.5	166 26 34.5
$\cos t$	log (p)	9.964 5455	9.973 3032	9.980 8340	9.987 7274
$\cot \delta$	log (n)	8.257 9152	8.257 9152	8.257 9152	8.257 9152
$\tan x$	log (n)	8.222 4607	8.231 2184	8.238 7492	8.245 6426
x , auxiliary angle	(-)	0 57 22.27	0 58 32.37	0 59 33.81	1 00 30.97
ϕ , latitude	(+)	9 20 16.93	9 20 16.93	9 20 16.93	9 20 16.93
$\phi + x$	(+)	8 22 54.66	8 21 44.56	8 20 43.12	8 19 45.96
$\tan t$	log (n)	9.624 4267	9.558 3373	9.482 5415	9.382 2561
$\sin x$	log (n)	8.222 4003	8.231 1555	8.238 6841	8.245 5753
$\sec(\phi + x)$	log (p)	0.004 6638	0.004 6421	0.004 6232	0.004 6055
$-\tan A_m$	log (p)	7.851 4908	7.794 1349	7.725 8488	7.632 4369
A_m , from north	(-)	0 24 25.24	0 21 23.97	0 18 17.16	0 14 44.83
A , clockwise from south		179 35 34.76	179 38 36.03	179 41 42.84	179 45 15.17

Nominal pointings on star		0 00 00.00	0 00 00.00	0 00 00.00	0 00 00.00
Curvature corrections	(+)	0.09	0.07	0.12	0.06
Inclination corrections	(-)	2.32	1.96	2.70	1.37
Corrected pointings on star		359 59 57.77	359 59 58.11	359 59 57.42	359 59 58.69
Angle, star to mark, nominal		157 25 07.08	157 22 06.67	157 19 00.41	157 15 28.75
Angle, star to mark, corrected		157 25 09.31	157 22 08.56	157 19 02.99	157 15 30.06
Azimuth of star from south, A		179 35 34.76	179 38 36.03	179 41 42.84	179 45 15.17
Azimuth of mark from south		337 00 44.07	337 00 44.59	337 00 45.83	337 00 45.23

Levels		+W	-E	+W	-E	+W	-E	+W	-E
Before set	D	8.2	10.2	8.5	10.0	8.9	9.6	8.2	10.1
	R	7.8	10.6	7.8	10.8	6.8	11.7	8.0	10.4
After set	D	7.8	10.8	8.0	10.4	7.0	10.5	9.4	9.1
	R	8.0	10.6	8.2	10.1	7.8	10.8	8.2	10.4
$\Sigma w + \Sigma e$		+31.8	-42.2	+32.5	-41.3	+30.5	-42.6	+33.8	-40.0
			-10.4		-8.8		-12.1		-6.2
$I = \frac{1}{8} d \tan h$, multiply by			.223		.223		.223		.223
$I = \frac{1}{8} d \tan h (\Sigma w + \Sigma e)$, unit 1''			-2.32		-1.96		-2.70		-1.37

Interval in seconds		s	s ²	s	s ²	s	s ²	s	s ²
Before mean epoch		273	74529	260	67600	330	108900	254	64516
		83	6889	95	9025	155	24025	124	15376
		14	196	20	400	25	625	44	1936
After mean epoch		57	3249	65	4225	60	3600	61	3721
		122	14884	120	14400	150	22500	141	19881
		192	36864	190	36100	300	90000	221	48841
Σs^2			136553		131750		249650		154271
Σs^2	log		5.13520		5.11975		5.39733		5.18828
F , coefficient	log		5.96099-10		5.96099-10		5.96099-10		5.96099-10
$\tan A_m$	log		7.85149-10		7.79413-10		7.72585-10		7.63244-10
Curvature correction	log (n)		8.94768-10		8.87487-10		9.08417-10		8.78171-10
Curvature correction	(-)		0.09		0.07		0.12		0.06

AZIMUTH STARS

The stars most used for azimuth in geodetic work are the northern circumpolars α and δ in Ursa Minor and 51 and 39 in Cepheus; and the southern circumpolars σ , χ , κ , and ζ in the constellation of Octans. All are within 5° of their respective poles, and are suitable for observation at any hour angle, when the latitude is known, with a determination of time far less precise than is necessary in longitude work. With stars 5° to 15° from the poles good azimuths may be obtained, provided that more attention is paid to hour angle position and to the method of finding time. Among the brightest of these, visible to the naked eye, are γ and κ Cephei, 5 and ζ Ursae Minoris, β Hydri, α Apodis, and β Chamaeleontis, which are available for use in equatorial regions.

The magnitude of an azimuth star is important in pointing. Polaris is rather too bright for the most precise pointings, especially when the state of the atmosphere causes the image to waver. λ Ursae Minoris, which is about 6 minutes nearer the pole than Polaris, is too faint for ordinary instruments. The best magnitudes range between 4 and 5.

Refraction effects make it desirable to observe for azimuth on more than one night, or else to observe on two or three azimuth stars during the same night. There is some advantage in observing on stars symmetrically placed in hour angle with reference to the meridian, as on 51 Cephei and δ Ursae Minoris at any time, or on Polaris at both morning and evening.

FINDER ANGLES

The use of the table 17, computed to serve for a term of years, will make it easy to find and identify 51 Cephei and δ Ursae Minoris. For example, the difference between the azimuths of Polaris and 51 Cephei will show how far right or left of Polaris the fainter star may be found; and the value of h at the head of the column, when added algebraically to the latitude, will give the approximate vertical angle. If desired, the small correction for refraction may be added. The table also shows the direction and rate of the apparent motion of the star. When the star is found, its angle from the mark (the zero of the instrument having been set originally on the mark), and its altitude, will take care of later pointings. The local sidereal time for the table may be found with sufficient accuracy by multiplying 3.94 minutes by the number of days since March 22, and adding the number of hours since noon, plus 10 seconds extra for each hour.

AZIMUTH STARS AND TIME STARS COMPARED

An azimuth star is one that has a slow apparent motion in azimuth. All circumpolars are azimuth stars. Also all stars of numerically greater declination than the latitude of the observer become azimuth stars temporarily twice a day, near the times of their elongations, when their azimuth is greatest and their apparent motion in azimuth least.

A time star is one that has a rapid apparent motion in the direction of measurement:

(a) Westward, if time is sought by noting the moment of transiting the meridian, thence deriving hour angles for other stars.

(b) In altitude, if hour angle is sought from altitude.

(c) In azimuth, if hour angle is sought by measuring a horizontal angle.

To illustrate the great difference between stars in this respect: In a time observation for azimuth use, in 1923, the azimuth star, Polaris, was moving in azimuth at the rate of $4''$ per minute, and the time star, Sirius, then about 1 hour 40 minutes from the meridian, was moving in azimuth $1015''$ per minute. In Figure 101 the combined rate of apparent motion of the two-time stars is about $2880''$ per minute.

The heavens present the aspect of the face of a clock, the vertical plane through each star and the zenith giving a direction like that of the hand of a clock. Time stars are on long clock hands, giving a plain indication of the time. Azimuth stars are on short clock hands, giving a difficult indication of the time.

FINDING TIME—VARIOUS METHODS

If star 1 is east of the observer's meridian, the local sidereal time (T) is earlier than that particular time (α) or angle, called the star's right ascension, when the star crosses the meridian, by an amount equal to the star's negative hour angle ($-t_1$); that is,

$$T = \alpha_1 - (-t_1) = \alpha_1 + t_1$$

If star 2 is west of the meridian, the local sidereal time is later than the time (α_2) when the star crossed the meridian, by the star's positive hour angle ($+t_2$); that is,

$$T = \alpha_2 + t_2$$

For any number of stars the local sidereal time at any instant is defined by the relation

$$T = \alpha_1 + t_1 = \alpha_2 + t_2 = \alpha_3 + t_3 = \dots$$

If one finds by observation the t of a time star, and adds the result to the α of that time star, obtained from the Ephemeris, the result will be the local sidereal time, T . If T and the α of an azimuth star (Ephemeris) are substituted in the fundamental relation

$$T = \alpha + t$$

corresponding to the azimuth star, the hour angle t of the azimuth star is obtained.

The meridian transit method consists in marking the clock time of the transit of a star across the meridian. At this moment $t=0$ and $T=\alpha$. Though by definition local sidereal time is the time of transit indicated by the right ascension of any star whatever, for greater precision only rapidly moving stars are so used, and many rather than one. This is the most accurate of all methods. For its use with a star transit instrument see the U. S. Naval Observatory's Manual of Field Astronomy. It may be employed in connection with azimuth observations with theodolites by using an extra theodolite for time, placing the latter in the meridian of the station a few feet away. With a prismatic eyepiece, or in lieu of it a sextant mirror, stars may be observed near the zenith, the most favorable position. Lacking such an eyepiece, the observer will find the selection of time stars somewhat limited. Time sets preceding and following the azimuth set may consist of two time stars north of the zenith, two time stars south of the zenith, and one azimuth star within 30° of the pole. The precision obtainable is about one-half second by the eye-and-ear method, but the method is difficult to use with theodolites.

ALTITUDE METHODS

In latitude ϕ the hour angle t of a star of declination δ may be obtained from the "time sight" formula

$$\sin^2 \frac{1}{2} t = \sin \frac{1}{2} (\zeta + \phi - \delta) \sin \frac{1}{2} (\zeta - \phi + \delta) \sec \phi \sec \delta$$

in which ζ is the zenith distance or 90° minus the altitude.

From an inspection of the differential formula

$$\frac{d\zeta}{dt} = \cos \phi \sin A = -\frac{dh}{dt}$$

in which A is the azimuth of the star, it is evident that the rate of change of zenith distance, or of altitude, is greatest, and therefore that the determination of time is most precise, when the star is in the prime vertical, $\sin A$ being then a maximum.

Observing the altitude of the same star, or of another star of nearly the same declination at about the same altitude on the opposite side of the meridian, tends to eliminate constant errors of assumed latitude, of the instrument, and of the observer.

The equal altitude method consists in observing the times of transit, across the horizontal wire of an instrument, of the same star at a fixed altitude on both sides of the meridian. The mean of the observed times is the clock time of meridian transit, provided that the rate of the clock is constant. This method is used with the sextant, with altazimuth instruments, and with specially designed instruments like the Jobin equal altitude instrument. It is a most inconven-

ient method for the ordinary transit-theodolite, for with stars of convenient altitude the choice is limited and the intervals between observations are unduly long; while with stars that transit near the zenith, chosen to shorten the intervals, it is difficult to decide on the instant of transit because of the obliquity of the star's motion across the thread.

The **method of Gauss**, and modifications of it, in which stars equally spaced in azimuth about the whole horizon and of nearly the same altitude are used, tends to eliminate uncertainties of refraction.

In the **method of Stechert**, facilitated by special tables, the clock times when two stars of nearly the same declination, one east the other west of the meridian, rise and descend, respectively, to the same altitude, which is fixed by means of a latitude level, are noted. In the fundamental formulae

$$\sin h = \sin \phi \sin \delta_e + \cos \phi \cos \delta_e \cos (\mu_e + \Delta\mu_e - \alpha_e),$$

and

$$\sin h = \sin \phi \sin \delta_w + \cos \phi \cos \delta_w \cos (\mu_w + \Delta\mu_w - \alpha_w),$$

in which μ is the clock time and $\Delta\mu$ the clock correction, the two values of $\sin h$, being equated, give an equation in which ϕ and $\Delta\mu$ are the only unknowns. If either is assumed, the other becomes known.

For a transit-theodolite the choice of stars for the determination of time from measured altitude is limited.

(a) The only stars that can transit the prime vertical are those whose declinations lie between zero and a declination equal to the latitude of the observer.

(b) Near the equator, stars in this belt are too high.

(c) Except near the equator, say within 30° at most, the rate of change of altitude with time is too small to give a good determination of time, when the star is near the horizon. At that time, too, uncertainties of refraction are large.

(d) In high latitudes the determination of time from observed altitude is feeble, and fails completely at the poles.

The **vertical-of-Polaris method** (Hansen's or Döllén's) is much used in high latitudes. Essentially it consists in observing the transit of a time star across the vertical of a close circumpolar. The instrument is first pointed a little ahead of the circumpolar and clamped. The stride level is read. When the circumpolar is on the vertical wire the time is marked. The telescope is then transited and time is again marked when a time star transits the same vertical. The stride level is read again. The whole process is now repeated with the telescope in the reversed position. This, of course, is possible only with multiple-thread instruments, unless two time stars are used. For refined methods of reduction see Chauvenet's *Spherical and Practical Astronomy*, Vol. II. This method may be adapted for use with a vernier instrument by reading the plate angles in addition to marking time on both stars, as in the next method.

A modification of Döllén's method, suitable for the theodolite, will be found in the chapter on **Latitude**. If this method is used for time and azimuth, the latitude must be known.

TIME BY THE HORIZONTAL ANGLE METHOD

The method of finding local sidereal time by measuring horizontal angles with a transit-theodolite was developed in United States Naval surveys in 1923. A description of it and a discussion of its limitations will be found in Vol. 50, No. 252, of the United States Naval Institute Proceedings of February 1924. By permission of the publishers much of the paper is used here. It refers to the northern hemisphere, but its application to the southern hemisphere is obvious.

In briefest terms the steps of this method are as follows:

1. Point on a north star, mark the time, and read the plate.
2. Point on a south star, mark the time, and read the plate.
3. Assume a clock correction on local sidereal time, and compute the difference of azimuths of the stars. This is the **first computed angle**. If it is too small as compared with the measured angle, the clock, as tentatively corrected, is slow. If it is too great, the clock, as corrected, is fast.
4. Assume a second clock correction and obtain a **second computed angle**, which should nearly agree with the measured angle.

The following illustrates the determination of time from a single time star paired with a pole star. For brevity only one pair of pointings is reduced, the sixth pointings.

Example.—March 28, 1926, at Δ FAB, N 22-05-50.7, W 81-40-33.7, at 8:27:00 watch time Polaris read 201-34-55, and at 8:31:10.5 Sirius read 60-45-50; to find the watch error on local sidereal time at the instant of pointing on Sirius.

With the usual notation, x being an auxiliary angle,

$$\cot x = \sec t \tan \delta, \text{ and } -\tan A = \tan t \sin x \sec (\phi + x)$$

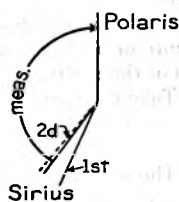
1.—Assume watch correct.—

	Polaris at 8:27:00	Sirius at 8:31:10.5	Polaris:
	$h^m s$	$h^m s$	$\alpha = 1:33:47.5. (h m s)$ $\delta = 88-54-31.60, N. (^{\circ} ' ")$
t	6:53:12.5 (h m s)	1:49:17.6 (h m s)	Sirius: $\alpha = 6:41:52.9. (h m s)$ $\delta = 16-37-06.25, S. (^{\circ} ' ")$
$\sec t$	103-18-07.5 ($^{\circ} ' "$)	27-19-24 ($^{\circ} ' "$)	
$\tan \delta$	0.638, 1115*	0.051, 3766	
	1.720, 1570	9.474, 8901*	
$\cot x$	2.358, 2785*	9.526, 2667*	
x	0-15-04.0, neg. ($^{\circ} ' "$)	71-25-50.0, neg. ($^{\circ} ' "$)	Polaris, 201-34-55
$\phi + x$	21-50-46.7 ($^{\circ} ' "$)	49-19-59.3, neg. ($^{\circ} ' "$)	Sirius, 60-45-40
$\tan t$	0.626, 3004*	9.713, 2001	
$\sin x$	7.641, 7273*	9.976, 7798*	Measured, 140-49-15
$\sec (\phi + x)$	0.032, 3653	0.185, 9791	
$-\tan A$	8.300, 3930	9.875, 9590*	Computed 141-55-45.6
A	358-51-21.9 ($^{\circ} ' "$)	216-55-36.3, difference ($^{\circ} ' "$)	
Motions, E. 0.07' per minute		W. 16' per minute	Difference, 1-06-30.6

Comparing the sixth pointings with the first (not given), it is found that the angle Sirius to Polaris is decreasing 16' per minute. Since the computed angle is 66.5' too large, the epoch to which it corresponds is about $(66.5' \div 16)$ minutes too early; that is, the watch is about 4½ minutes slow.

To derive a second clock correction that will place the measured angle between the computed values and near the second one, assume a clock correction a little greater than 4½ minutes.

2.—Assume watch 4 minutes 15 seconds slow at time of pointing on Sirius.—

	Polaris at 8:31:15	Sirius at 8:35:25.5	
	$^{\circ} ' "$	$^{\circ} ' "$	
t	104-21-52.5	28-23-09	
$\sec t$	0.605, 3888*	0.055, 6328	
$\tan \delta$	1.720, 1570	9.474, 8901*	
$\cot x$	2.325, 5458*	9.530, 5229*	
x	0-16-14.7, negative	71-15-37.3, negative	
$\phi + x$	21-49-36.0	49-09-46.6, negative	
$\tan t$	0.591, 5945*	9.732, 6981	
$\sin x$	7.674, 4994*	9.976, 3446*	
$\sec (\phi + x)$	0.032, 3056	0.184, 4820	First computed angle, 141-55-45.6
$-\tan A$	8.298, 3495	9.893, 5247*	Measured angle, 140-49-15
A	358-51-40.6	218-02-44.7	Second computed angle, 140-48-55.9

Interval, first and second, in time, 4^m15^s; in angle, 6'49.5", or 66.83'.

Interval, second from true, in time, c ; in angle, 0.32'.

$c:255^s = 0.32':66.83'$, whence $c = 1.2^s$, to be subtracted from 4^m15^s. Therefore the watch is 4^m13.8^s slow at watch time 8:31:10.5.

The example illustrates the mechanics of interpolation in a single determination of sidereal clock error. For dealing with mean time clocks, rating, and elimination of instrumental errors, the following procedure is recommended.

PROCEDURE FOR OBSERVATIONS

In north latitudes pair a pole star, or a star within 1 hour of the meridian as high as the instrument will permit, with a fast south star, preferably, but not necessarily, within 2 hours of the meridian. North and south here refer to the zenith. In south latitudes pair a pole star or a high south star within 1 hour of the meridian, with a fast north star.

Read the stride level before and after each set.

Reverse the telescope in the middle of each set.

First time set.—Observe a two-time angle between the stars of the pair.

First azimuth set.—Observe a six-time angle from the mark to the azimuth star.

Second azimuth set.—Observe a six-time angle from the azimuth star to the mark.

Second time set.—Observe a two-time angle between the stars of the same pair or any other similar pair.

Additional azimuth sets, if employed should be placed before the second time set.

This much may be done in the field, without an ephemeris or program sheet; without knowing the approximate sidereal time; and without performing any computations.

If the same stars have been used in both time sets, the change in the observed angle that has taken place in the known uncorrected interval between the sets will give the rate of change with sufficient precision for estimating how much the first assumed clock corrections must be changed to take up the discrepancy between the first computed angles and their measured values, in order to derive the second assumed corrections. But if the stars of the first set are too far west to be used again, it will be found convenient to observe directly the motion of each in azimuth for, say, 1 minute, to furnish this rate.

PROCEDURE FOR COMPUTATIONS

Apply stride level corrections to the times of transit.

If the clock keeps mean time, reduce all of the readings after the first to readings of a fictitious sidereal clock that started even with the mean time clock on the first reading.

Find the (sidereal) clock error at the mean epoch of each time set, when both stars are fast; or at the instants of pointing on the time stars, when they are associated with pole stars. Rate the clock between these epochs. Apply corrections for error and rate to obtain the local sidereal time of each pointing on the azimuth star. Find the corresponding hour angles of the latter.

In each azimuth set compute the azimuth of the star at each pointing, and find the mean azimuth of the six pointings. From this derive the azimuth of the mark by subtracting one-sixth of the multiple angle, mark to star.

Take the mean of the resulting azimuths of the mark.

AZIMUTH FORMULAE

The azimuth of a star clockwise from the north is given by

$$-\tan A = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}, \quad (1)$$

or for logarithmic computation, using an auxiliary angle x ,

$$\text{by} \quad \left[\begin{array}{l} \cot x = \tan \delta \sec t \\ -\tan A = \tan t \sin x \sec (\phi + x) \end{array} \right] \quad (2)$$

Example.—Find A when $\phi = +10^\circ$, $t = 11^h 50^m$, $\delta = +87^\circ 09' 35''$.

By (1):

$\cos \phi$	9. 993, 3515	
$\tan \delta$	1. 304, 4059	
	1. 297, 7574	+ 19. 84986
$-\sin \phi$	9. 239, 6702 ⁿ	
$\cos t$	9. 999, 5865 ⁿ	
	9. 239, 2567	+ 0. 17348
	1. 301, 5365	+ 20. 02334
$-\sin t$	8. 639, 6796 ⁿ	
$\tan A$	7. 338, 1431 ⁿ	359° 52' 30.7''

Hour angle at elongation—

$$\cos t = \tan \phi \cot \delta.$$

(3)

Azimuth at elongation—

$$\sin A = \sec \phi \cos \delta.$$

(4)

In either hemisphere only stars that transit between the zenith and the pole have elongations. Hence in (3) both factors of $\cos t$ have the same sign, and $\cos t$ is always positive. Therefore western elongation comes before $t = 6^h$ and eastern elongation after $t = 18^h$, the star in both cases being "above pole."

The rate of change of azimuth with latitude is a maximum at elongations, and is given by

$$\frac{dA}{d\phi} = \tan A \tan \phi, \quad (5)$$

that is, latitude errors affect azimuth most at elongations.

TABLE 16.—Maximum errors in azimuth per 1^s of time error

[C, at culminations, for time error = 1^s; E at elongations, for latitude error = 1']

Latitude N.	α Urs. Min. δ = 88° 56' 01''		51 H. Cephei δ = 87° 09' 35''		39 H. Cephei δ = 86° 55' 37''		δ Urs. Min. δ = 86° 36' 47''	
	C	E	C	E	C	E	C	E
0	"	"	"	"	"	"	"	"
5	0. 28	0. 10	0. 75	0. 26	0. 81	0. 28	0. 90	0. 31
15	0. 29	0. 31	0. 77	0. 83	0. 85	0. 89	0. 93	0. 98
25	0. 31	0. 57	0. 84	1. 53	0. 91	1. 66	1. 01	1. 83
35	0. 34	0. 95	0. 94	2. 55	1. 02	2. 76	1. 13	3. 04
45	0. 40	1. 58	1. 11	4. 22	1. 20	4. 56	1. 33	5. 03
55	0. 50	2. 78	1. 40	7. 43	1. 52	8. 06	1. 69	8. 87
65	0. 69	5. 67	1. 97	15. 19	2. 15	16. 49	2. 41	18. 17

Illustration.—For errors of 0^s.25 in time plus 15'' in latitude, to keep the azimuth error below 1.5'', the four stars must not be observed at elongations north of 59°, 44°, 42°, and 40°, respectively. North of latitude 40° Polaris at elongation, as an azimuth star, is inferior to any of the other three stars at culmination, when the uncertainty in latitude is as much as 1'.

FINDING CIRCUMPOLARS

Approximate local sidereal time for mental calculation.

In a leap year, the approximate sidereal time at Greenwich at 0^h civil time, midnight of the preceding day, is:

	<i>h</i>	<i>m</i>		<i>h</i>	<i>m</i>		<i>h</i>	<i>m</i>		<i>h</i>	<i>m</i>
Jan. 1.	6	37	Apr. 1.	12	36	July 1.	18	35	Oct. 1.	0	38
Feb. 1.	8	40	May 1.	14	35	Aug. 1.	20	37	Nov. 1.	2	40
Mar. 1.	10	34	June 1.	16	37	Sept. 1.	22	40	Dec. 1.	4	38

On January 1 and February 1, add 3 minutes for the first year after leap year, 2 minutes for the second year after, and 1 minute for the third year after. On the first of any other month, subtract 1 minute, 2 minutes, and 3 minutes, respectively, for the first, second, and third years after leap year.

To obtain local sidereal time on any date, with a precision sufficient for beginning azimuth computations, add to the Greenwich sidereal time at the beginning of the month:

- Four minutes for each whole day since the first of the month.
- The number of hours since the preceding local mean midnight.
- Ten seconds for each hour of that interval.
- Ten seconds for each hour of west longitude.

To find circumpolars with an altazimuth instrument.—The azimuth pair, 51 Cephei and δ Ursae Minoris, being in opposite quadrants and nearly equal in declination, are favorably placed to eliminate latitude error. The table of finder angles shows the relation of each of these faint stars to the bright star Polaris at any time.

Example.—An observer in approximate latitude 20° N., local sidereal time 3 hours 30 minutes, in January 1942, wishes to find 51 Cephei. His settings on Polaris are: Altitude $20^\circ 55'$, azimuth $32^\circ 10'$ right of the mark. Find the settings for 51 Cephei.

Table 17, page 185, fourth column, interpolating for 1942:

51 Cephei, altitude above pole,	+95. 8',	azimuth +153. 0'
Polaris	+54. 2	—29. 0
<hr/>		
Polaris to 51 Cephei,	+41. 6	+182. 0 = $3^\circ 02'$
Polaris reads	$20^\circ 55'$	32 10
<hr/>		
Settings for 51 Cephei,	$21^\circ 36. 6'$	$35^\circ 12'$

In latitudes -60° to $+60^\circ$ the azimuths and altitudes above pole of north or south circumpolars within 5° of either pole, at equal hour angles, are approximately proportional to their polar distances—a fact that may be utilized to obtain finder angles not covered by the table. The most convenient star for comparison is δ Ursae Minoris, nearly constant in declination.

δ Ursae Minoris, 1932, $\alpha = 17^h 54^m 08.87^s$, $p = 203.21'$.

Example.—Find the approximate azimuth and altitude above the south pole of χ Octantis, $\alpha = 18^h 16^m$, $\delta = -87^\circ 46'$, in latitude 40° S. at 2 o'clock local sidereal time.

The hour angle is 7 hours 44 minutes and the polar distance is $134'$, which is 66 percent of that of the standard star. When the hour angle of the latter is 7 hours 44 minutes, or at 1:38 o'clock, the azimuth and altitude above pole are $-233'$ and $-89'$, respectively. Multiplying by 66 percent, the corresponding quantities for the south circumpolar are $-154'$ and $59'$. Therefore it will be found in the field of view in approximate azimuth $182^\circ 34'$ and in altitude 40° less $59'$, or in $39^\circ 01'$, neglecting refraction.

Other stars in Octans: Ratio, 24 percent for δ , 136 percent for ζ and κ .

Altitudes derived from the approximate formula $h = \phi + p \cos t$, in which $p \cos t$ is the altitude above or below pole given by table 17, are subject to the following maximum corrections:

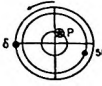
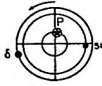

Latitude north,	10°	20°	30°	40°	50°	55°	60°
Refraction corrections, table 23,	+5'	+3'	+2'	+1'	+1'	+1'	+1'
Maximum corrections to $p \cos t$,	+1'	+2'	+3'	+5'	+7'	+8'	+10'

The maximum corrections apply to δ Ursae Minoris at hour angles 6 and 18 hours. They decrease toward zero at both culminations. The corresponding maximum corrections are .7 as great for 51 Cephei and .1 as great for Polaris.

FINDER ANGLES FOR NORTHERN CIRCUMPOLARS

TABLE 17.—*Simultaneous azimuths and angles of elevation above the pole*

(For 1932.0 and 1952.0, quantities in minutes, black face when negative)






Latitude	L. S. T. Year																
		000 o'clock	030 o'clock	100 o'clock	130 o'clock	200 o'clock	230 o'clock										
		1932	1952	1932	1952	1932	1952	1932	1952	1932	1952	1932	1952	1932	1952	1932	1952
10°	δ Urs. M. 51 Cephei Polaris	Altitude above pole															
		'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	
		5. 2	10. 9	31. 7	37. 4	57. 6	63. 2	82. 5	87. 9	106. 1	111. 1	127. 8	132. 4				
	50. 8	58. 2	29. 1	36. 5	6. 9	14. 1	15. 4	8. 5	37. 4	30. 9	58. 8	52. 8					
	57. 9	51. 1	60. 9	54. 2	62. 8	56. 3	63. 6	57. 4	63. 4	57. 6	62. 0	56. 8					
	Azimuth																
	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'		
	δ Urs. M.	206. 2	206. 2	203. 5	202. 7	197. 3	195. 8	187. 8	185. 5	175. 1	172. 2	159. 5	155. 9				
	51 Cephei	164. 8	164. 6	170. 3	171. 0	172. 9	174. 6	172. 6	175. 2	169. 2	172. 7	163. 0	167. 4				
	Polaris	26. 9	27. 1	19. 0	20. 1	10. 7	12. 7	2. 2	5. 1	6. 2	2. 5	14. 6	10. 2				
20°	δ Urs. M. 51 Cephei Polaris	216. 0	215. 9	212. 9	212. 0	206. 0	204. 4	195. 9	193. 5	182. 4	179. 3	166. 0	162. 2				
172. 3		171. 9	178. 2	178. 9	181. 1	182. 8	181. 0	183. 6	177. 7	181. 3	171. 4	175. 9					
28. 3		28. 5	19. 9	21. 1	11. 2	13. 4	2. 4	5. 4	6. 5	2. 7	15. 3	10. 7					
30°	δ Urs. M. 51 Cephei Polaris	234. 3	234. 1	230. 5	229. 4	222. 8	220. 9	211. 5	208. 8	196. 6	193. 2	178. 7	174. 6				
186. 3		185. 8	193. 0	193. 6	196. 4	198. 2	196. 5	199. 3	193. 3	197. 1	186. 6	191. 5					
30. 8		31. 0	21. 7	23. 0	12. 3	14. 6	2. 6	5. 9	7. 1	2. 9	16. 7	11. 6					
40°	δ Urs. M. 51 Cephei Polaris	264. 6	264. 3	259. 8	258. 5	250. 7	248. 5	237. 5	234. 4	220. 5	215. 6	200. 0	195. 4				
209. 8		209. 1	217. 7	218. 2	221. 9	223. 7	222. 4	225. 4	219. 0	223. 3	211. 9	217. 3					
35. 0		35. 2	24. 7	26. 1	13. 9	16. 6	2. 9	6. 7	8. 1	3. 3	19. 0	13. 2					
50°	δ Urs. M. 51 Cephei Polaris	314. 9	314. 4	308. 4	306. 7	296. 8	294. 0	280. 5	276. 7	259. 9	255. 2	235. 3	229. 7				
248. 6		247. 6	258. 5	259. 0	264. 1	266. 1	265. 3	268. 7	261. 9	266. 8	253. 9	260. 2					
41. 9		42. 2	29. 6	31. 3	16. 7	19. 9	3. 5	8. 0	9. 7	3. 9	22. 8	15. 8					
55°	δ Urs. M. 51 Cephei Polaris	352. 6	351. 8	344. 7	342. 6	331. 2	327. 9	312. 5	308. 2	289. 1	283. 8	261. 4	255. 2				
277. 5		276. 3	289. 0	289. 4	295. 7	297. 7	297. 5	301. 2	294. 1	299. 5	285. 6	292. 5					
47. 2		47. 5	33. 3	35. 2	18. 8	22. 3	4. 0	9. 9	10. 9	4. 4	25. 6	17. 8					
60°	δ Urs. M. 51 Cephei Polaris	403. 8	402. 7	393. 9	391. 4	377. 7	373. 8	355. 7	350. 6	328. 4	322. 3	296. 5	289. 3				
316. 8		315. 2	330. 4	330. 7	338. 7	340. 9	341. 4	345. 5	338. 3	344. 2	329. 1	336. 9					
54. 4		54. 7	38. 4	40. 6	21. 7	25. 8	4. 6	10. 4	12. 6	5. 1	29. 6	20. 5					

Apparent altitude of pole = latitude + refraction correction.

FINDER ANGLES FOR NORTHERN CIRCUMPOLARS—Continued

TABLE 17.—Simultaneous azimuths and angles of elevation above the pole—Continued

[For 1932.0 and 1952.0, quantities in minutes, black face when negative]




Latitude	L. S. T. Year												
		300 o'clock	330 o'clock	400 o'clock	430 o'clock	500 o'clock	530 o'clock	300 o'clock	330 o'clock	400 o'clock	430 o'clock	500 o'clock	530 o'clock
		1932	1952	1932	1952	1932	1952	1932	1952	1932	1952	1932	1952
Altitude above pole													
δ Urs. M.													
51 Cephei.													
Polaris													
Azimuth													
10°													
δ Urs. M.													
51 Cephei													
Polaris													
20°													
δ Urs. M.													
51 Cephei													
Polaris													
30°													
δ Urs. M.													
51 Cephei													
Polaris													
40°													
δ Urs. M.													
51 Cephei													
Polaris													
50°													
δ Urs. M.													
51 Cephei													
Polaris													
55°													
δ Urs. M.													
51 Cephei													
Polaris													
60°													
δ Urs. M.													
51 Cephei													
Polaris													

Apparent altitude of pole = latitude + refraction correction.

FINDER ANGLES FOR NORTHERN CIRCUMPOLARS—Continued

TABLE 17.—*Simultaneous azimuths and angles of elevation above the pole*—Continued

[For 1932.0 and 1952.0, quantiles in minutes, black face when negative]




Latitude	L. S. T. Year										
		600 o'clock	630 o'clock	700 o'clock	730 o'clock	800 o'clock	830 o'clock				
		1932	1952	1932	1952	1932	1952	1932	1952	1932	1952
Altitude above pole											
		203.1	203.2	200.7	200.0	194.9	193.4	185.7	183.5	173.3	170.5
δ Urs. M.		162.8	162.5	168.0	168.7	170.4	172.0	169.8	172.4	166.3	169.8
51 Cephei		26.4	26.7	18.6	19.7	10.5	12.5	2.2	5.0	6.1	2.5
Polaris											
		158.0	154.5	160.0	164.4	14.3	10.0				
Azimuth											
		5.2	11.0	31.8	37.6	58.0	63.6	83.1	88.5	106.8	111.9
δ Urs. M.		52.0	59.6	29.8	37.4	7.1	14.5	15.8	8.7	38.3	31.7
51 Cephei		58.9	52.0	61.9	55.1	63.8	57.2	64.6	58.3	64.3	58.5
Polaris											
		128.8	133.4	60.2	54.1	62.9	57.6				
20°		5.4	11.4	33.0	39.0	60.1	65.9	86.2	91.8	110.9	116.2
δ Urs. M.		55.0	63.0	31.5	39.5	7.5	15.3	16.7	9.2	40.5	33.5
51 Cephei		61.8	54.6	64.9	57.8	66.9	60.0	67.7	61.2	67.4	61.3
Polaris											
		133.8	138.7	63.7	57.2	65.9	60.4				
30°		5.8	11.2	35.4	41.8	64.5	70.7	92.5	98.5	119.1	124.8
δ Urs. M.		60.3	69.1	34.6	43.4	8.2	16.8	18.3	10.1	44.5	36.7
51 Cephei		67.2	59.3	70.5	62.8	72.6	65.1	73.5	66.4	73.1	66.5
Polaris											
		143.8	149.1	69.8	62.7	71.5	65.5				
40°		6.5	13.6	39.4	46.6	71.8	78.8	103.1	109.9	132.9	139.3
δ Urs. M.		69.1	79.1	39.6	49.7	9.4	19.2	21.0	11.5	50.9	42.1
51 Cephei		76.9	67.2	79.8	71.1	82.2	73.7	83.1	75.1	82.6	75.2
Polaris											
		160.7	166.6	79.9	71.9	80.7	74.0				
50°		7.5	15.9	46.1	54.4	84.0	92.2	120.7	128.6	155.7	163.2
δ Urs. M.		83.8	96.0	48.1	60.3	11.4	23.4	25.4	14.0	61.8	51.1
51 Cephei		91.0	80.3	95.3	84.9	98.0	88.0	99.1	89.5	98.4	89.5
Polaris											
		188.5	195.5	96.9	87.2	96.0	88.0				
55°		8.3	17.6	51.0	60.2	93.0	102.0	133.7	142.4	172.5	180.8
δ Urs. M.		95.0	108.8	54.6	68.4	13.0	26.5	28.9	15.9	70.1	58.0
51 Cephei		102.1	90.1	107.0	95.2	109.9	98.6	111.0	100.3	110.2	100.3
Polaris											
		209.0	216.8	107.5	98.6						
60°		9.4	19.9	57.6	68.0	105.0	115.2	151.0	160.9	195.1	204.5
δ Urs. M.		110.7	126.8	63.0	79.7	15.1	30.9	33.7	18.5	81.7	67.6
51 Cephei		117.4	103.6	122.9	109.4	126.2	113.3	127.4	115.2	126.3	115.1
Polaris											
		236.5	245.4	128.0	115.2	123.2	113.0				

Apparent altitude of pole = latitude + refraction correction.

FINDER ANGLES FOR NORTHERN CIRCUMPOLARS—Continued

TABLE 17.—Simultaneous azimuths and angles of elevation above the pole—Continued

[For 1932.0 and 1952.0, quantities in minutes, black face when negative]

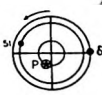

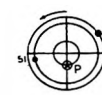
Latitude	L. S. T. Year													
		900 o'clock 1932 1952	930 o'clock 1932 1952	1000 o'clock 1932 1952	1030 o'clock 1932 1952	1100 o'clock 1932 1952	1130 o'clock 1932 1952							
10°		Altitude above pole												
	δ Urs. M.	140.0	135.9	119.5	115.0	97.1	92.1	72.9	67.6	47.6	42.0	21.4	15.7	
	51 Cephei	151.0	156.1	139.4	145.1	125.4	131.7	109.2	115.0	91.2	98.3	71.6	78.9	
	Polaris	22.3	17.3	29.9	24.3	37.0	31.0	43.4	37.0	49.1	42.5	54.0	47.2	
		Azimuth												
	δ Urs. M.	148.6	152.7	165.9	169.5	180.4	183.4	191.9	194.2	200.1	201.7	205.0	205.8	
	51 Cephei	81.1	75.6	100.5	95.7	118.2	114.2	133.8	130.7	147.0	144.9	157.7	156.6	
	Polaris	60.5	55.8	57.0	53.0	52.5	49.3	47.2	44.8	41.0	39.5	34.2	33.5	
	20°	δ Urs. M.	154.5	158.9	172.7	176.5	188.1	191.2	200.3	202.7	209.2	210.9	214.5	215.5
	51 Cephei	85.7	79.9	106.1	101.2	124.7	120.6	141.0	137.9	154.8	152.7	165.9	164.8	
Polaris	63.3	58.4	59.6	55.5	54.9	51.6	49.3	46.8	42.9	41.3	35.7	35.1		
30°	δ Urs. M.	166.2	171.0	186.0	190.2	202.8	206.3	216.3	219.0	226.2	228.2	232.4	233.5	
51 Cephei	93.9	87.6	116.2	110.8	136.4	131.9	154.1	150.7	169.0	166.7	180.8	179.7		
Polaris	68.6	63.3	64.6	60.1	59.5	55.9	53.4	50.7	46.4	44.7	38.6	37.9		
40°	δ Urs. M.	186.0	191.3	208.4	213.1	227.6	231.5	243.1	246.3	254.8	257.0	262.2	263.6	
51 Cephei	107.4	100.2	132.8	126.6	155.7	150.7	175.6	171.8	192.3	189.8	205.5	204.3		
Polaris	77.4	71.5	72.9	67.8	67.1	63.0	60.2	57.2	52.2	50.4	43.5	42.7		
50°	δ Urs. M.	218.5	224.9	245.3	250.9	268.4	273.2	287.4	291.3	301.9	304.8	311.6	313.4	
51 Cephei	130.1	121.4	160.6	153.3	187.9	182.0	211.6	207.3	231.3	228.5	246.7	245.4		
Polaris	92.1	85.1	86.6	80.6	79.6	74.9	71.4	67.9	62.0	59.8	51.5	50.7		
55°	δ Urs. M.	242.6	249.7	272.7	279.0	298.8	304.2	320.4	324.8	337.1	340.4	348.4	350.6	
51 Cephei	147.3	137.6	181.8	173.6	212.5	205.9	239.0	234.1	260.8	257.8	277.7	276.4		
Polaris	103.0	95.2	96.8	90.2	89.0	83.7	79.7	75.9	69.2	66.8	57.5	56.6		
60°	δ Urs. M.	274.9	283.1	309.5	316.8	339.7	346.0	264.9	370.1	384.7	388.7	398.5	401.2	
51 Cephei	171.4	160.2	211.2	201.8	246.5	239.1	276.8	271.4	301.5	298.2	320.4	319.2		
Polaris	117.9	109.0	110.7	103.3	101.8	95.8	91.1	86.8	79.0	76.3	65.7	64.7		

Apparent altitude of pole = latitude + refraction correction.

FINDER ANGLES FOR NORTHERN CIRCUMPOLARS—Continued

TABLE 17.—Simultaneous azimuths and angles of elevation above the pole—Continued

(For 1932.0 and 1952.0, quantities in minutes, black face when negative)




Latitude	L. S. T. Year												
		1200 o'clock	1230 o'clock	1300 o'clock	1330 o'clock	1400 o'clock	1430 o'clock						
		1932	1952	1932	1952	1932	1952	1932	1952	1932	1952		
		Altitude above pole											
		5.2	10.9	31.7	37.4	57.6	63.2	82.5	87.9	106.1	111.1	127.8	132.4
	δ Urs. M.	50.8	58.2	29.1	36.5	6.9	14.1	15.4	8.5	37.4	30.9	58.8	52.8
	51 Cephei	57.9	51.1	60.9	54.2	62.8	56.3	63.6	57.4	63.4	57.6	62.0	56.8
	Polaris												
		Azimuth											
		206.3	206.4	204.1	203.5	198.5	197.0	189.4	187.2	177.0	174.1	161.6	158.0
	δ Urs. M.	165.7	165.6	170.8	171.7	173.0	174.9	172.3	175.0	168.6	172.2	132.0	166.5
	51 Cephei	26.7	27.0	18.8	20.0	10.6	12.7	2.2	5.1	6.2	2.5	14.5	10.1
	Polaris												
10°	δ Urs. M.	216.3	216.4	214.3	213.7	208.6	207.2	199.4	197.1	186.6	183.6	170.5	166.8
	51 Cephei	174.1	174.0	179.3	180.3	181.4	183.4	180.4	183.3	176.3	180.1	169.3	173.9
	Polaris	27.9	28.2	19.7	20	11.1	13.2	2.2	5.3	6.5	2.6	15.1	10.6
20°	δ Urs. M.	234.7	234.9	232.9	232.3	227.2	225.7	217.4	215.0	203.8	200.4	186.5	182.5
	51 Cephei	189.5	189.5	194.9	196.0	196.9	199.1	195.5	198.8	190.8	195.1	183.0	188.1
	Polaris	30.2	30.5	21.3	22.6	12.0	14.3	2.5	5.8	7.0	2.8	16.4	11.4
30°	δ Urs. M.	265.3	265.7	263.9	263.3	257.8	256.2	247.3	244.7	232.2	228.7	212.9	208.4
	51 Cephei	215.8	215.1	220.8	222.1	222.6	225.2	220.7	224.5	215.1	210.9	205.9	211.7
	Polaris	34.0	34.4	23.9	25.4	13.5	16.1	2.8	6.5	7.8	3.2	18.4	12.8
40°	δ Urs. M.	316.1	316.7	315.2	314.7	308.9	307.2	297.0	294.1	279.7	275.6	257.0	251.8
	51 Cephei	257.5	257.8	263.7	265.6	265.4	268.7	262.5	267.2	255.2	261.2	243.8	250.9
	Polaris	40.3	40.7	28.4	30.2	16.0	19.1	3.4	7.7	9.3	3.8	21.8	15.2
50°	δ Urs. M.	354.1	355.0	353.8	353.3	347.3	345.5	334.6	331.4	315.6	311.1	290.6	284.8
	51 Cephei	289.5	290.0	296.0	298.2	297.4	301.2	293.7	299.1	285.2	291.9	272.0	280.0
	Polaris	44.9	45.5	31.6	33.7	17.8	21.3	3.8	8.6	10.6	4.3	24.4	17.0
55°	δ Urs. M.	405.9	407.1	406.5	406.2	400.1	398.2	386.3	382.9	365.3	360.3	337.1	330.5
	51 Cephei	333.3	334.1	340.2	343.0	341.1	345.7	336.2	342.5	325.8	333.7	310.3	319.6
	Polaris	51.3	52.0	36.1	38.5	20.4	24.3	4.3	9.8	11.8	4.8	27.8	19.4
60°	δ Urs. M.												
	51 Cephei												
	Polaris												

Apparent altitude of pole = latitude + refraction correction.

FINDER ANGLES FOR NORTHERN CIRCUMPOLARS—Continued

TABLE 17.—*Simultaneous azimuths and angles of elevation above the pole*—Continued

(For 1932.0 and 1952.0, quantities in minutes, black face when negative)




Latitude															
		L. S. T. Year		1500 o'clock 1932 1952		1530 o'clock 1932 1952		1600 o'clock 1932 1952		1630 o'clock 1932 1952		1700 o'clock 1932 1952		1730 o'clock 1932 1952	
		Altitude above pole													
		δ Urs. M.		147.3	151.4	164.3	167.8	178.5	181.4	189.7	191.9	197.6	199.1	202.1	202.8
		51 Cephei		79.2	73.8	98.2	93.5	115.6	111.7	130.9	127.9	144.1	141.9	154.7	153.6
		Polaris		59.6	55.0	56.2	52.3	51.8	48.7	46.6	44.2	40.5	39.0	33.7	33.1
		Azimuth													
10°		δ Urs. M.		143.3	139.2	122.5	117.9	99.6	94.5	74.9	69.4	48.8	43.1	21.9	16.1
		51 Cephei		152.7	157.9	140.8	146.7	126.6	133.0	110.2	117.0	91.9	99.1	72.2	79.5
		Polaris		22.6	17.5	30.3	24.6	37.4	31.4	44.0	37.5	49.8	43.1	54.7	47.9
20°		δ Urs. M.		151.4	147.1	129.6	124.7	105.4	100.0	79.3	73.5	51.7	45.7	23.3	17.0
		51 Cephei		159.4	164.8	146.8	152.9	131.8	138.5	114.7	121.8	95.6	103.1	75.0	82.7
		Polaris		23.6	18.3	31.6	25.8	39.1	32.8	46.0	39.2	52.1	45.0	57.3	50.1
30°		δ Urs. M.		165.8	161.1	142.0	136.7	115.1	109.8	87.1	80.8	56.9	50.2	25.6	18.7
		51 Cephei		171.0	178.0	158.3	165.0	142.0	149.2	123.4	131.1	102.8	110.9	80.6	88.8
		Polaris		25.5	19.8	34.2	27.9	42.3	35.5	49.8	42.5	56.4	48.7	62.0	54.2
40°		δ Urs. M.		189.6	184.3	162.6	156.6	132.6	125.9	99.9	92.7	65.3	57.7	29.4	21.5
		51 Cephei		193.3	200.1	177.7	185.2	159.2	167.3	138.2	146.8	112.4	121.2	90.1	99.3
		Polaris		28.7	22.3	38.5	31.4	47.7	39.9	56.1	47.8	63.5	54.9	69.9	61.1
50°		δ Urs. M.		229.4	223.1	197.2	190.0	161.1	153.0	121.6	112.8	79.5	70.3	35.8	26.2
		51 Cephei		228.5	236.6	209.6	218.5	187.5	197.1	162.5	172.7	135.1	145.7	105.7	116.6
		Polaris		34.0	26.4	45.6	37.2	56.5	47.4	66.5	56.8	75.4	65.2	83.0	72.6
55°		δ Urs. M.		259.8	252.7	223.7	215.5	182.9	173.7	138.1	128.2	90.4	79.9	40.7	29.8
		51 Cephei		254.6	263.7	233.3	243.3	208.4	219.2	180.5	191.9	150.0	161.8	117.3	129.3
		Polaris		37.9	29.5	50.9	41.5	63.1	52.9	74.3	63.4	84.2	72.9	92.8	81.2
60°		δ Urs. M.		302.0	293.9	260.5	251.1	213.3	202.7	161.4	149.8	105.7	93.5	47.6	34.9
		51 Cephei		290.0	300.4	265.3	276.8	236.7	249.0	204.8	217.7	170.0	183.4	132.9	146.5
		Polaris		43.3	33.7	58.1	47.4	72.1	60.4	84.9	72.5	96.3	83.3	106.2	92.9

Apparent altitude of pole = latitude + refraction correction.

FINDER ANGLES FOR NORTHERN CIRCUMPOLARS—Continued

TABLE 17.—*Simultaneous azimuths and angles of elevation above the pole*—Continued

(For 1932.0 and 1952.0, quantities in minutes, black face when negative)




Latitude	L. S. T. Year												
		1800 o'clock		1830 o'clock		1900 o'clock		1930 o'clock		2000 o'clock		2030 o'clock	
		1932	1952	1932	1952	1932	1952	1932	1952	1932	1952	1932	1952
Altitude above pole													
		/ /		/ /		/ /		/ /		/ /		/ /	
♂ Urs. M.		203.1	203.2	200.7	200.0	194.9	193.4	185.7	183.5	173.3	170.5	158.0	154.5
51 Cephei		162.8	162.5	168.0	168.7	170.4	172.0	169.8	172.4	166.3	169.8	160.0	164.4
Polaris		26.4	26.7	18.6	19.7	10.5	12.5	2.2	5.1	6.1	2.5	14.3	10.0
Azimuth													
		/ /		/ /		/ /		/ /		/ /		/ /	
10°	♂ Urs. M.	5.3	11.2	32.5	38.4	59.1	64.8	84.7	90.1	108.8	113.9	130.9	135.6
	51 Cephei	51.2	58.6	29.3	36.7	7.0	14.2	15.5	8.5	37.7	31.1	59.3	53.2
	Polaris	58.8	51.9	61.8	55.0	63.7	57.1	64.6	58.3	64.4	2.5	14.3	10.0
20°	♂ Urs. M.	5.6	11.9	34.5	40.7	62.6	68.7	89.7	95.4	115.1	120.4	138.4	143.3
	51 Cephei	53.2	60.9	30.4	38.2	7.2	14.8	16.1	8.8	39.1	32.3	61.6	60.5
	Polaris	61.5	54.3	64.7	57.5	66.8	59.8	67.7	61.1	67.5	61.3	66.1	60.5
30°	♂ Urs. M.	6.2	13.1	37.9	44.7	68.8	75.5	98.5	104.8	126.2	132.1	151.6	157.0
	51 Cephei	57.1	65.4	32.7	41.0	7.8	15.8	17.3	9.5	42.0	34.7	66.2	59.4
	Polaris	66.6	58.8	70.1	62.4	72.4	64.9	73.4	66.3	73.2	66.6	71.8	65.7
40°	♂ Urs. M.	7.1	15.0	43.5	51.3	79.0	86.6	112.9	120.2	144.7	151.4	173.6	179.6
	51 Cephei	63.8	73.1	36.5	45.7	8.7	17.7	19.3	10.6	47.0	38.7	73.9	66.3
	Polaris	75.2	66.3	79.1	70.4	81.8	73.3	83.0	74.9	82.9	75.3	81.3	74.3
50°	♂ Urs. M.	8.7	18.3	53.0	62.5	96.2	105.4	137.3	146.1	175.6	183.7	210.3	217.5
	51 Cephei	74.8	85.7	42.8	53.6	10.2	20.7	22.6	12.4	55.1	45.4	86.7	77.8
	Polaris	89.3	78.8	94.1	83.7	97.3	87.2	98.9	89.2	98.8	89.7	97.0	88.7
55°	♂ Urs. M.	9.9	20.9	60.3	71.1	109.3	119.9	156.0	165.9	199.3	208.4	238.4	246.5
	51 Cephei	83.0	95.0	47.4	59.4	11.3	23.0	25.1	13.8	61.0	50.3	96.2	86.2
	Polaris	99.9	88.2	105.3	93.7	109.0	97.6	110.8	99.9	110.8	100.5	108.8	99.4
60°	♂ Urs. M.	11.6	24.4	70.5	83.2	127.8	140.1	182.2	193.7	232.4	242.9	277.4	286.8
	51 Cephei	93.9	107.6	53.7	67.2	12.7	26.0	28.3	15.6	69.1	57.0	108.9	97.6
	Polaris	114.3	100.9	120.6	107.3	124.9	111.9	127.1	114.6	127.1	115.3	124.9	114.1

Apparent altitude of pole = latitude + refraction correction.

FINDER ANGLES FOR NORTHERN CIRCUMPOLARS—Continued

TABLE 17.—*Simultaneous azimuths and angles of elevation above the pole*—Continued

[For 1932.0 and 1952.0, quantities in minutes, black face when negative]

Latitude	L. S. T. Year												
		2100 o'clock 1932 1952	2130 o'clock 1932 1952	2200 o'clock 1932 1952	2230 o'clock 1932 1952	2300 o'clock 1932 1952	2330 o'clock 1932 1952						
		Altitude above pole											
	δ Urs. M.	140.0	135.9	119.5	115.0	97.1	92.1	72.8	67.6	47.6	42.0	21.4	15.7
	51 Cephei	151.0	156.1	139.3	145.1	125.4	131.6	109.2	115.9	91.2	98.3	71.6	78.9
	Polaris	22.2	17.3	29.9	24.3	37.0	31.0	43.4	37.0	49.1	42.5	54.0	47.2
		Azimuth											
10°	δ Urs. M.	150.7	154.9	167.9	171.5	182.2	185.1	193.3	195.5	201.1	202.6	205.4	206.1
	51 Cephei	79.8	74.4	99.1	94.3	116.7	112.7	132.3	129.2	145.7	143.4	156.6	155.3
	Polaris	60.6	55.9	57.2	53.2	52.7	49.5	47.4	45.0	41.2	39.7	34.4	33.7
20°	δ Urs. M.	159.2	163.5	177.2	180.9	192.0	195.0	203.4	205.7	211.3	212.8	215.5	216.2
	51 Cephei	83.0	77.3	103.1	98.1	121.4	117.3	137.8	134.5	151.9	149.5	163.4	162.1
	Polaris	63.6	58.6	60.0	55.8	55.4	51.9	49.8	47.2	43.3	41.7	36.1	35.4
30°	δ Urs. M.	174.2	179.0	193.6	197.6	209.5	212.7	221.7	224.1	229.9	231.4	234.1	234.8
	51 Cephei	89.2	83.1	110.9	105.5	130.7	126.2	148.5	144.9	163.9	161.3	176.6	175.0
	Polaris	69.1	63.7	65.2	60.6	60.2	56.5	54.2	51.4	47.1	45.3	39.3	38.5
40°	δ Urs. M.	199.1	204.4	220.9	225.4	238.6	242.2	251.9	254.5	260.7	262.4	265.0	265.6
	51 Cephei	99.7	92.8	124.0	118.0	146.4	141.3	166.5	162.4	184.0	180.9	198.5	196.6
	Polaris	78.3	72.1	73.9	68.7	68.3	64.0	61.4	58.2	53.5	51.4	44.6	43.7
50°	δ Urs. M.	240.7	247.1	266.5	271.7	287.1	291.2	302.3	305.2	312.0	313.8	316.2	316.8
	51 Cephei	117.1	109.0	145.8	138.6	172.3	166.2	196.2	191.3	217.2	213.5	234.7	232.4
	Polaris	93.5	86.1	88.4	82.0	81.7	76.5	73.5	69.7	64.1	61.5	53.5	52.4
55°	δ Urs. M.	272.4	279.5	301.1	306.9	323.7	328.3	340.3	343.5	350.6	352.4	354.6	355.1
	51 Cephei	129.9	120.9	161.9	153.8	191.5	184.6	218.3	212.7	241.8	237.6	261.7	258.9
	Polaris	104.9	96.6	99.2	92.1	91.8	85.9	82.7	78.3	72.1	69.2	60.2	58.9
60°	δ Urs. M.	316.5	324.5	348.9	355.5	374.4	379.4	392.6	396.0	403.4	405.3	407.1	407.5
	51 Cephei	147.2	136.9	183.5	174.3	217.3	209.5	248.0	241.5	275.1	270.2	298.2	294.9
	Polaris	120.6	111.0	114.1	105.8	105.6	98.8	95.2	90.1	83.0	79.7	69.4	67.8

Apparent altitude of pole = latitude + refraction correction.

SUN AZIMUTHS WITH A 1-MINUTE TRANSIT

Because of the indefinite nature of the targets presented by the sun's limbs, sun azimuths observed with a 1-minute transit, or even with a 10-second transit or other instrument of higher precision, are considered to be of the order of precision of 1 minute, more or less; and therefore






to be unfit for the orientation of triangulation. But they are suitable for checking the accumulation of azimuth error in long traverses, and in connection with magnetic work. There are two kinds—time azimuths and altitude azimuths.

Time azimuths imply a precise reception of known time, or else concurrent observations for time, in order to derive the sun's hour angle. They are usually inconvenient for checking traverses, except when the instrument is within sight of the ship. They are computed by the formulae—

$$\tan M = \frac{\tan \delta}{\cos t} \quad \text{and} \quad \tan Z = \frac{\tan t \cos M}{\sin (\phi - M)}, \quad (1)$$

in which t is the sun's hour angle, δ is the sun's declination, ϕ is the latitude, Z is the azimuth, and M is an auxiliary angle used to render the computation logarithmic.

Altitude azimuths do not require a precise value of the time, which enters only in taking the sun's declination from tables, but require a concurrent measurement of altitude. Unless the instrument is provided with cross wires in the form of a square about 32 minutes of arc on a side, for enclosing the sun's image, it is necessary to sight on the limbs instead of the center. To avoid the troublesome reductions to center, which for altitude take the form $\pm r$ (r being the sun's radius), and for the plate angle the form $\mp r \sec (h \pm r)$, in which h , is the observed altitude, the observations may be arranged as in the following scheme, by which three observations are reduced to a single observation. It will be noted that successive observations are in opposite quadrants of the reticule, and that the second observation is written twice to facilitate finding the mean of two means.

Observation number	Limbs	Watch time	Altitude of limb	Reduction to center	A-vernier reading, limb	Reduction to center
1		T_1	h_1	$+r$	A_1	$-r \sec (h_1 + r)$
2		T_2	h_2	$-r$	A_2	$+r \sec (h_2 - r)$
2		T_3	h_3	$-r$	A_3	$+r \sec (h_3 - r)$
3		T_4	h_4	$+r$	A_4	$-r \sec (h_4 + r)$
Mean		$\frac{1}{2}(T_1 + 2T_2 + T_3)$	$\frac{1}{2}(h_1 + 2h_2 + h_3)$		$\frac{1}{2}(A_1 + 2A_2 + A_3)$	

The columns for reduction to center are added merely for explanation.

The altitude reductions to center disappear in the mean. The azimuth reductions do not, but they may be disregarded for altitudes between 10° and 30° and for extreme altitude runs up to 4° , provided that the time intervals between successive observations are nearly equal. They will be nearly equal, in this program, if the second altitude interval is made $1^m 04^s$ less than the first in morning observations, or $1^m 04^s$ greater than the first in afternoon observations.

The usual procedure is to clamp the vertical circle and to read and record a set altitude ahead of time, thereafter manipulating only the horizontal circle tangent screw. When the sun reaches the desired position in the quadrant selected, the time is marked, the horizontal circle is read, and both observations are recorded in the line of the previously set altitude. Setting the next altitude for observations in the opposite quadrant, the program is continued as before.

The index error of the vertical circle should be determined immediately before or after the observations by taking a double altitude on a fixed object. Further, the mean of the observed altitudes must be corrected for refraction.

Let h be the true altitude of the sun's center, p the polar distance, Z the azimuth, and ϕ the latitude. Then either the first two formulae (2), adapted to all-logarithmic computation, or the third formula (3), not so adapted, may be employed to compute the azimuth.

$$\cos^2 \frac{1}{2} Z = \cos s \cos (s - p) \sec \phi \sec h \quad (2)$$

$$\cos Z = \sin \delta \sec \phi \sec h - \tan \phi \tan h \quad (3)$$

CHAPTER X

MAGNETIC MEASUREMENTS

The following is a brief outline of magnetic measurements such as are at times specified in United States Navy surveys, and is adapted to the instruments now available (1936). For an extended treatment of the subject see *Directions for Magnetic Measurements* published by the United States Coast and Geodetic Survey as Serial No. 166.

For an area under geodetic control it is unnecessary to make the observations for geographic position and for true azimuth described in the publication referred to, provided that the magnetic station is visible from a triangulation tower and from one other station. The forward azimuth from the tower to the magnetic station may be converted into a true azimuth at the station by means of the following table, the arguments of which may be derived by scaling.

TABLE 18.—*Convergence of meridians for an easting or westing of 10,000 meters*

Lat.	ΔZ	Lat.	ΔZ	Lat.	ΔZ	Lat.	ΔZ	Lat.	ΔZ	Lat.	ΔZ
°	'	°	'	°	'	°	'	°	'	°	'
1	6	11	63	21	124	31	194	41	281	51	392
2	11	12	69	22	131	32	202	42	291	52	413
3	17	13	75	23	137	33	210	43	301	53	428
4	23	14	81	24	144	34	218	44	312	54	444
5	28	15	87	25	151	35	226	45	323	55	461
6	34	16	93	26	158	36	235	46	334	56	478
7	40	17	99	27	165	37	243	47	346	57	497
8	45	18	105	28	172	38	252	48	358	58	516
9	51	19	111	29	179	39	262	49	371	59	537
10	57	20	118	30	187	40	271	50	385	60	559

Azimuths increase clockwise from the south point of the horizon as zero. First apply 180° to the forward azimuth. Next, in north latitudes, apply to the result $+\Delta Z$ if the forward azimuth is easterly, or $-\Delta Z$ if it is westerly. In south latitudes these signs of ΔZ are reversed. Round off the result to the nearest minute.

On account of the great importance in navigation of that magnetic element called variation, or more strictly **magnetic declination**, it should be measured at suitably distributed stations, even when complete magnetic measurements are not specified, by using a surveyor's compass or a transit fitted with a needle. When only one observation can be made at a station, it is best made at about 11 a. m., when as a rule the diurnal variation of the declination is a minimum. Either of the following plans will require but a small expenditure of time:

(a) When the principal signals for a sounding area have been erected, select for a magnetic station, covering that area, the site of a wooden tripod or "quadrupod" visible all the way to the ground from a triangulation tower, thus providing a means of deriving an initial azimuth, namely, the reverse azimuth tower-tripod corrected by table 18. At the center set up a surveyor's compass or a transit fitted with a needle. Observe every 15 minutes from 9 a. m. to 1 p. m., or from 8 a. m. to 2 p. m., as described below.

(b) At intervals of 10 to 20 miles, when sufficient land is available at a triangulation tower, drive a wooden peg at least 500 feet from the tower, if possible on line to a distant object visible from the ground. The triangulation will furnish the true azimuth of the distant object from the tower, and with sufficient accuracy what may

be taken as the same true azimuth from the peg. When a distant object is not available it will be necessary, because of the short sight to the tower and the movement of the head of the tower with the changing bearing of the sun, to establish as a foresight a pin point under the tower in the vertical plane of the common telescopic sights of the transit and the theodolite, one over the peg and the other mounted on the tower.

There is more likely to be sufficient room, under plan (a) than under plan (b), to lay out a line and to take reciprocal bearings for detecting local attraction. The observer, of course, must divest himself of iron articles, not forgetting the iron rivet of the reading glass, which is sometimes overlooked. With the instrument set up, shaded, adjusted, set to zero, and oriented on the foresight, and with the variation circle, if any, set back to zero, the observations consist simply in reading the horizontal circle when the telescope is pointed so that the north end of the needle reads zero; bringing the telescope into the magnetic meridian by means of the upper motion tangent screw, first from the left, then from the right; first with telescope direct, then with telescope inverted, in all four readings.

MAGNETIC ELEMENTS

The magnetic elements usually prescribed for field stations on land are the **magnetic declination**, D ; the **horizontal component**, H , of the earth's magnetic force; and the dip or **inclination**, I . The magnetic declination is the horizontal angle, positive if clockwise, negative if counterclockwise, from the astronomical meridian to the magnetic meridian. The inclination is the angle made by the lines of force with the plane of the horizon. The first two are measured with a magnetometer, the last with a dip circle, or with an earth inductor. The relation between the earth's **total magnetic force**, F , the **horizontal component** of that force, H , and the **inclination** of the same to the plane of the horizon, I , is

$$H = F \cos I$$

On charts and for navigational purposes magnetic declination is usually termed **magnetic variation**. It is considered **positive** when the earth's magnetism deflects the north end of the needle to **east** of true north, and **negative** when the deflection is to the **west**.

DECLINATION OBSERVATIONS

Assuming that the site of the magnetic station is free from local attraction, that magnetic conditions at the time are normal, and that the azimuth of the mark is known, the task is to measure the horizontal angle from the mark to the magnetic meridian, in which, of course, there is no object to point upon. Having turned the first pointing on the mark, reading both verniers (see the example), the alidade is turned until the telescope of the magnetometer points approximately south magnetic, and the upper motion is clamped. The remaining part of the desired angle, from the vertical plane of the telescope to the magnetic meridian, must be measured by means of the scale etched on glass in one end of the collimator magnet, which may be recognized by the scale and by the lens in the other end. When the magnet is suspended at the height of the center of the telescope, without torsion of the fibers, with the magnet house darkened by the wooden shutters except at the front where there is a small glass window for illuminating the scale, and when the vertical plane of the telescope is sufficiently near the magnetic meridian, the scale will be seen through the telescope (adjusted to sidereal focus) vibrating from side to side. Excessive arcs of vibration may be reduced by manipulating a short length of needle at the end of a match stick, not too near the magnet. This accomplished, the division of the scale on which the cross wire appears to rest at the end of each arc may be read. In determining what would be the position of the magnet if stationary, the mean of a set of readings taken in the order left end, right end, right end, left end, will be free of the error due to diminution of arc. In determining the position of the **magnetic axis** (for this, rather than the optical axis, lies in the magnetic meridian), the mean of sets of readings taken with magnet erect and magnet inverted, or according to appearance with scale erect and scale inverted, will be free of the error due to the angle of divergence between the magnetic and optical axes. It will be seen that the observations in the example are arranged to eliminate both of these errors. It follows that the small angle that must be added algebraically to the plate angle to bring the telescope into the magnetic meridian is the

value of the number of scale divisions between the mean of the readings and the reading of the center of the scale.

Magnetic observations—Declination

Station: Tekrit, Asiatic Turkey.
Instrument: Magnetometer 7.
Mark: Spot on house in Tekrit.
Magnet: 7L.

Date: Dec. 13, -----
Observer: W. H. S.
Chronometer: 257.
Line of detorsion: 13°.

Chronometer time		Magnet E or I	Scale readings			Horizontal circle readings		
			Left	Right	Mean	Vernier	Mark	Magnet
<i>h</i>	<i>m</i>		<i>d</i>	<i>d</i>	<i>d</i>		° ' "	° ' "
6	20	E	41.8	48.9	45.35	Before, A	273 58 00	111 05 00
	21	E	42.0	48.7	45.35	B	58 00	04 40
	23	I	48.8	52.8	50.80	After, A	273 58 00	111 04 40
	24	I	48.9	52.8	50.85	B	58 20	04 40
	25	I	49.0	52.8	50.90			
	26	I	49.2	52.7	50.95	Mean	273 58 05	111 04 45
	28	E	43.3	47.5	45.40			
	29	E	44.0	46.7	45.35			
						Magnet erect		
						Magnet inverted		
						Mean scale		
Center scale reading			<i>d</i>	Mean chronometer time			<i>h</i>	<i>m</i>
Mean scale			50.00	Correction to local mean time			6	24.5
			48.12				-2	52.1
Center minus mean			+1.88	Local mean time			9	17
Reduction to center			+ 2.8	Remarks				
Circle reading			111 04.8					
Magnetic south			111 07.6					
Mark reading			273 58.1					
Azimuth of mark (south=0°)			164 22.4					
Azimuth of magnetic south			109 35.7	Torsion weight suspended 8 minutes. Temperature 9° C. Clear. Northwest wind. 1 scale division, $d = 1.49'$.				
Magnetic declination, east			+1 31.9					

If the necessary value of one division of the scale is not furnished, it may be found by separate observations of correspondences between scale-reading and vernier-reading differences. It is well to use as a divisor a sizable number of scale differences, 20 or 30, symmetrical with respect to the center of the scale; and having read correspondences in the increasing order of scale numbers, to repeat them in the reverse order, to eliminate gradual changes in the declination occurring during the observations. These observations, and the main observations as well, are best undertaken near eastern elongation, to be expected between 7 and 9 o'clock in the morning, and near western elongation, between 1 and 2 o'clock in the afternoon, for at these times the declination is nearly stationary. The investigation of this phenomenon calls for short series of morning and afternoon observations at intervals of 10 or 15 minutes on three successive days.

The suspension of a magnet without torsion of the fibers requires skill and patience. To reduce the quantity of torsion to a minimum, as few fibers as possible must be used, and consequently spares will be needed. As the torsion varies with the moisture of the air, all fibers on hand should be kept in the same hygrometric condition. Silk fibers are commonly kept

soaked in glycerine. The torsion also depends on the length of the fibers and the weight that they support. The fibers are attached at the top to a graduated torsion head, which may be twisted in either direction. In modern practice a single phosphor bronze wire has replaced silk fibers. Remove the magnet from its stirrup and substitute the torsion weight, which is a nonmagnetic ball or cylinder intended to have the same mass as the magnet. Put on the glass sides of the magnet house, and when the weight has ceased or nearly ceased to rotate about its vertical axis, turn the torsion head until the fore-and-aft line on the weight, or the easily estimated mean position thereof, is parallel to the sides of the magnet house. Make repeated trials, reading the degrees of the torsion head each time, and having adopted a mean reading of



FIGURE 102.—Kew magnetometer set up for declination.

the torsion head as corresponding to the plane of detorsion, clamp the head on that reading. In substituting the magnet for the weight, avoid twisting the fibers or giving them too much slack.

ELEMENTARY DEFINITIONS AND PRINCIPLES

In the centimeter-gram-second, or C. G. S. system, the unit of force is the **dyne**, defined as the force that will impart to a gram mass an acceleration of one centimeter per second. A **unit pole**, which may be either positive or negative, is one that will repel an equal like pole, or attract an unlike pole of equal magnitude, at a distance of one centimeter with a force of one dyne. If d centimeters separates two poles having strengths of a and b units, the force, f , subsisting between them is

$$f = \frac{ab}{d^2} \text{ dynes.}$$

A portion of magnetic space or field has an **intensity** of one **gauss** if when a unit pole is placed therein the pole experiences a force of one dyne. The earth's magnetic field has an intensity less than a gauss, and to avoid decimals is sometimes expressed in a smaller unit, the **gamma**, which is the ten-thousandth part of a gauss. If the intensity of a field is F gaussess and the strength of a pole is m units, the force acting on the pole is Fm dynes. Magnetic intensity is a vector quantity, possessing direction as well as magnitude. The direction of a field is manifested by the direction in which a positive, or north-seeking, pole is urged. We may conceive of the intensity of a field as represented by a certain number of lines of force per unit of cross-sectional area, or as a single line of force equal to their resultant, per unit area. Under this conception a **magnet** may be defined as a bundle of magnetic filaments acted on by parallel lines of force in the earth's magnetic field. The **poles** of the magnet are two points, N and S, within or

without the physical magnet, at which the resultant of the parallel forces is conceived to act. The length of the magnet is the distance NS, and the magnetic axis of the magnet, if it is a bar magnet, lies in the straight line NS.

When a magnet of pole strength m is placed perpendicular to the lines of force in a unit field, a couple, $2ml$, is formed having the maximum arm l . The moment, M , of this couple, the greatest that the magnet can experience in a unit field, is called the **magnetic moment** of the magnet. If the magnet is suspended in the earth's magnetic field and is constrained by gravity to oscillate in a horizontal plane, it can manifest the effect only of H , the horizontal component of the intensity of the earth's magnetic force. At any instant during the oscillation when the magnetic axis crosses the lines of force at an angle θ , the moment of the couple exerted will be $HM \sin \theta$, the maximum value of which, corresponding to $\theta=90^\circ$, is HM . The magnetometer is designed to determine the value of HM by "observations of oscillations" and that of $H \div M$ by "observations of deflections", whence the value of H is derived by extracting the square root of the product of the results. For a brief and clear development of the theory of magnetic measurements with a magnetometer, see Professor Nipher's Theory of Magnetic Measurements.

HORIZONTAL INTENSITY OBSERVATIONS

The **moment of inertia** of a suspended magnet about the vertical axis defined by the suspension fibers is the sum of the products of each elementary mass multiplied by the square of its distance from the axis. The time of one oscillation over a small arc is that required for the magnet to traverse the arc from rest to rest, or to cross a line of sight twice in opposite directions. If T is the period of an oscillation in mean time seconds and K is the moment of inertia,

$$T = \pi \sqrt{\frac{K}{HM}}; \text{ whence } HM = \frac{\pi^2 K}{T^2}. \quad (1)$$

Horizontal intensity—Oscillations

Station: Tekrit, Asiatic Turkey.

Instrument: Magnetometer 7.

Chronometer: 257.

Daily rate: 0.4^s, gaining.

Date: Dec. 13, ----

Observer: W. H. S.

Magnet: 7L (E).

Oscillation number	Chronometer time	Temperature t'	Extreme scale readings		Period of 70 oscillations
	h m s	$^{\circ}C.$	d	d	
0.....	6 32 50.8	10.5	22	78	
5.....	33 08.8				
10.....	26.7				
15.....	44.7				
20.....	34 02.6				
25.....	20.5				
30.....	38.4				
35.....	56.4				
40.....	35 14.3				
45.....	32.2	10.7	27	73	
70.....	37 01.9				m s 4 11.1
75.....	19.8				11.0
80.....	37.7				11.0
85.....	55.7				11.0
90.....	38 13.7				11.0
95.....	31.5				11.0
100.....	49.4				11.0
105.....	39 07.4				11.0
110.....	25.3				11.0
115.....	43.2	10.9	31	69	11.0
Means.....	6 36 17.0	10.70			251.02

$$\text{Formulae: } T^2 = T'^2 \left(\frac{f+h}{f} \right) [1 - (t' - t)q] \left(\frac{M + \mu H}{M} \right); HM = \frac{\pi^2 K}{T^2}$$

Coefficient of torsion $d = 1.49'$					Observed, T_0	s = 3.58600
					Correction for rate	= -0.00002
Torsion circle	Scale		Mean	Difference	T''	= 3.5860
°					<i>Logarithms</i>	
13	48.2	51.8	50.00		T''	= 1.10922
103	44.8	49.1	46.95	3.05	$f+h = 5404.45$	= 0.00036
283	51.0	54.7	52.85	5.90	$f = 5400$	= 0.00029
373	48.6	51.1	49.86	3.00	$1 - (t' - t)q$	= 0.00083
					$\frac{M + \mu H}{M}$	= 0.00083
Mean = $2.99d = 4.45' = h$					3.73275	$T^2 = 1.11070$
$t' - t = -1.40''$					3.73239	$\pi^2 K = 3.37230$
Local mean time, 9 hours 28 minutes.					0.00036	$HM = 2.26160$

The forms in this chapter have been adapted from those of the Carnegie Institution of Washington. It will be noted in the example that the result of the observations after correction for the rate of the chronometer is an approximate period T' to which certain corrective factors differing but little from unity must be applied to derive T .

COEFFICIENT OF TORSION

Suppose that a magnet suspended without torsion of the fibers and at rest in the magnetic meridian is drawn aside and released. At any part of the arc of oscillation the moment of the earth's horizontal force acting to restore the magnet to the meridian will be assisted by the torsion of the fibers. For a small arc it is permissible to assume that both of these moments are proportional to θ , and to consider that in finding T'^2 by observation we have, in fact, used for MH in (1) a value of MH increased by a certain part of itself; in consequence of which, as MH occurs in the denominator of (1), the value of T'^2 is too small, and must be multiplied by unity plus this fraction, called the coefficient of torsion. In the example the coefficient of torsion is found by suspending the torsion weight and observing the mean effect of twisting the torsion head 90° , regarded as a unit of twist, first one unit to the right, then two units to the left, then one unit to the right.

TEMPERATURE CORRECTIONS

Changes in temperature affect K by changing the dimensions of the magnet, and M by changing the strength of the poles as well, an increase in temperature causing a decrease in M . The management of temperature in the field, and the determination of temperature corrections, present great difficulties. It is well to minimize changes in temperature by using a double tent or one with a large fly; and to rely on the coefficients in the standardization certificate rather than to attempt to find them by observation. The certificate furnishes a table of logarithms of $\pi^2 K$ and a temperature formula,

$$M = M_s [1 - (t - t_s)q - (t - t_s)^2 q'] \quad (2)$$

by which the moment M at any working temperature t may be found from a given moment M_s at a standard temperature t_s , and the given numerical values of q and q' , the latter very small. The factor in brackets, expressing the ratio between two magnetic moments when the difference in temperature is $t - t_s$, is employed in oscillations to correct for the excess of temperature $t' - t$ prevailing during oscillations over that prevailing during deflections (see examples), in order that M shall have the same value in both series, HM being required from one series and $H \div M$ from the other. Since the range of temperature between observations is small, the q' term is dropped in this use of the formula.

A different use of the formula is seen in deflections. Here the value of M obtained by dividing HM by $H \div M$ and extracting the square root is only a relative one, and must be corrected for the difference between the temperature of deflections and the standard temperature t_s in order to derive the standard value M_s . In this case the range of temperature may be great enough to require the use of the full formula.

CORRECTION FOR INDUCTION

Any length of magnetic substance, if placed with its axis parallel to the earth's field, thus offering a path of low resistance to the earth's magnetic flux, will tend to distort the field by borrowing lines of force from adjacent areas, thereby increasing the number of lines per square centimeter in a plane perpendicular to the field. Related to field distortion, as cause to effect or vice versa, is a tendency to magnetize the metal, or if it has been magnetized previously, to increase the strength of its poles and its magnetic moment.

Near the beginning of magnetization, that is, for weak magnets such as are employed in terrestrial measurements, the increase in the magnetic moment is proportional to the increase in the intensity. (For magnetization curves of various metals see Croft's Practical Electricity, part I, p. 165.)

Horizontal intensity—Deflections

Magnet	North end	Circle readings													
		I Distance $r=27.5$ cm.						II Distance $r=35.0$ cm.							
		No.	A			B	Mean	No.	A			B	Mean		
E	E	1	°	'	''	°	'	''	°	'	''	°	'	''	
	W	4	124	35	00	34	50	34	55	2	118	04	50	04	50
			99	39	40	39	20	39	30	3	106	07	30	07	25
	Mean	a	112	07	12					c	112	06	08		
	2u		24	55	25						11	57	25		
W	W	5	99	32	00	31	40	31	50	6	106	04	50	04	50
	E	8	124	35	50	35	30	35	40	7	118	05	30	05	10
	Mean	b	112	03	45					d	112	05	02		
	2u		25	03	50						12	00	35		
Magnetic meridian		a+b 2	112	05	28					c+d 2	112	05	35		

Mean magnetic meridian, south, $112^{\circ}05.5'$

Formulae: $\frac{H}{M} \left[\frac{2 \left(1 + \frac{P}{r^2} + \dots \right)}{r^2 \left(1 + \frac{2\mu}{r^2} \right)} \right] \cdot \frac{1}{\sin u} = \frac{C}{\sin u}; \log H = \frac{1}{2} \left(\log \frac{H}{M} + \log HM \right).$

I			II			Logarithms		I	II	
2u, mean,			°	'	''	°	'	''		
u			24	59	38	11	59	00	C	=5.99098
			12	29	49	5	59	30	sin u	=9.33523
			h	m		Temperature		11.0		
Began at			6	58		"		11.8	H	=6.65575
Ended at			7	17		"		13.5	M	
Mean			7	07.5		t=12.1			HM, from os-	
Correction			2	52.1					cillations	=2.26160
Local mean time			10	00						2.26160
H=0.28753, 0.28747 gauss.									H ²	8.91735
									H	=9.45868
										9.45860
									M ²	5.60585
									M	=2.80292
									1-(t-t ₀)q=	-0.00166
									M ₀	=2.80126
										2.80134
M _s =632.79, 632.91 gram-cm.										

If μ , called the **coefficient of induction**, is the increase in M , for any particular magnet, due to induction in a parallel unit field, the corresponding increase in a parallel field of intensity H will be μH . Since the value of T'^2 coming from the observations of oscillations corresponds to an augmented moment $M + \mu H$, which if substituted for M in the denominator of (1) would give too small a value of T'^2 , in passing from T'^2 to T^2 the factor required to compensate for induction is $(M + \mu H) \div M$.

It is impracticable to determine μ in the field.

REDUCTION TO INFINITESIMAL ARC

The fundamental formula of the oscillation series is based on the assumption that the semiarc of vibration is infinitely small, not differing sensibly from its chord. It is seldom necessary to

employ an arc greater than 1° . To correct from a finite arc to an infinitely small one, when such a correction becomes sensible, the observed period of one oscillation is to be multiplied by a factor slightly less than unity, the approximate value of which is $\left(1 - \frac{1}{64}a^2\right)$, a being the mean of the initial and terminal arcs expressed in radians.

When arc =	1°	2°	3°	4°	5°
$1 - \frac{1}{64}a^2$	= 1.00000	0.99998	0.99996	0.99992	0.99988

OBSERVATIONS OF DEFLECTIONS

In various textbooks the **collimator magnet** is also called the long magnet, the deflector magnet, the oscillating magnet, and the intensity magnet. Similarly, a second magnet, introduced in deflections, a short and relatively weak magnet bearing a mirror, is called the short magnet, the **mirror magnet**, the deflected magnet, and the suspended magnet. The first, by means of a collimating lens, focuses parallel rays of light on a glass scale in its south end. The second reflects a beam of light on a scale. In the Kew type of magnetometer the magnets differ in weight, and consequently a torsion weight is provided for each. In the Carnegie Institution type the magnets are of equal weight.

In the **tangent type of magnetometer**, so-called because for a given deflection distance the ratio of M to H is nearly proportional to the tangent of the angle of deflection, the telescope remains in the magnetic meridian, the bar bearing the collimator magnet remains in the magnetic prime vertical, and the deflection of the mirror magnet is read from the scale. It is necessary to correct the preliminary value of u , the angle of deflection, for torsion of the fibers, as in oscillations. If the magnets differ in weight, separate observations are required to find their torsion coefficients.

In the **sine type of magnetometer**, in which $M \div H$ is proportional to $\sin u$, nearly, the whole upper part of the instrument, bearing the telescope and the deflection bars, is rotated about the vertical axis to follow the deflection of the mirror magnet through the angle u , and the amount of deflection is obtained from circle readings. Torsion is eliminated in this method, being the same for all positions of the telescope when pointed at the center of the scale.

The magnets, at all times except when in position, should be kept more than a foot apart. They should be handled gently with dry hands, and kept always at a safe distance away from the bar magnets of the dip circle equipment. The collimator magnet should be brought into position gradually to prevent violent vibrations of the mirror magnet, meanwhile turning the instrument slowly in the direction of the expected deflection, which is clockwise when the north end of the magnet is east, and counterclockwise when it is west.

The use of the thermometer is to find the temperature of the collimator magnet, and incidentally that of the deflection bars, if they are of metal. In some instruments the bars are hollow near the ends for the accommodation of the thermometer. It requires considerable care to avoid fluctuations in temperature and to obtain the true temperature of the magnet. The tube in the magnet box should be plugged when the thermometer is removed.

The sine magnetometer is designed so that the axes of the two magnets make a horizontal right angle at the center of the mirror magnet when the instrument is in adjustment with the telescope pointed on the beam of light at the center of the scale. Assuming that this adjustment has been made, and that the collimator magnet has not yet been brought near the instrument, the telescope is directed to the approximate magnetic south, the mirror magnet is suspended, its vibration is damped, and the pointing is rectified by the slow-motion screw. It is not necessary to record this position of the azimuth circle. In the example it would be about 112° . Next the collimator magnet is brought up and placed on the deflection bar at one of the points predetermined in the standardization of the instrument—in the example at 27.5 centimeters east of the center of the mirror magnet, and with its north end east, thus subjecting both poles of the short mirror magnet to the opposing influences of the north and south poles of the collimator magnet. Since the south pole, the collimator magnet being long, is considerably nearer than the north pole, its influence will predominate, and the mirror magnet will be deflected

clockwise. On being followed by the alidade the mirror magnet will finally come to rest where the moment of the mutual action of the two magnets is balanced by the moment of the earth's couple, which beginning with zero when the magnet is in the meridian, increases as $\sin u$ increases until the point of equilibrium is attained. The second half of the observation, step 4, presents the north pole, in the same position, to the mirror magnet, deflecting the magnet through an angle $-u$. The difference of the circle readings gives one value of $2u$, and their mean gives one position of the magnetic meridian. Others are obtained as illustrated in the example, the symmetrical arrangement of which is designed to compensate for gradual changes in H and to meet the conditions of standardization.

Parts I and II, with both magnets erect, constitute a magnet-erect series. It is to be followed by a magnet-inverted series, to eliminate the divergence between the optical axis and the magnetic axis. Finally both series are to be followed by a second series of oscillations.

Let N and S be the poles of the collimator magnet; $2l = NS$, the length of the magnet (usually about five-sixths of the length of the bar); m , the pole strength; and $M = 2ml$, the moment of the magnet. The same notation, with primed letters, will serve for the mirror magnet. Let r be the distance between the magnets, center to center, and since l' is short in comparison with r , instead of the distances between poles (the hypotenuses of right triangles), use their approximate values, $r-l$ and $r+l$. The forces subsisting between the poles are proportional to the products of the pole strengths and inversely proportional to the squares of the distances. The corresponding moments have the common arm l' .

With the collimator magnet placed in the position west, north end east, its north pole exerts on the mirror magnet a clockwise turning moment of $2mm'l' \div (r-l)^2$, and its south pole a counterclockwise turning moment of $2mm'l' \div (r+l)^2$. The algebraic sum of the moments is $2MM'r \div (r^2 - l'^2)^2$, clockwise. In the position of equilibrium the resultant moment is balanced by the counterclockwise moment of the earth's field, which for a deflection angle u is $HM' \sin u$. Equating moments and dividing both members of the equation by $MM' \sin u$, we have, approximately,

$$\frac{H}{M} = \frac{2r}{\sin u} \cdot \frac{1}{(r^2 - l'^2)^2} = \frac{2}{r^3 \sin u} \left(1 + \frac{2l'^2}{r^2} + \frac{3l'^4}{r^4} + \dots \right),$$

or

$$\frac{H}{M} = \frac{2}{r^3 \sin u} \left(1 + \frac{P}{r^2} + \frac{Q}{r^4} \right), \quad (3)$$

in which P and Q , called the **distribution coefficients**, make allowance for unsymmetrical distribution of magnetism, and, as well as may be, for the errors committed by assuming approximate pole distances. Their values are supplied in the certificates of Kew instruments for stated deflection distances. In instruments of later types the second distribution coefficient is eliminated (see the example) by using magnets whose lengths are as 1.225 to 1. Equation (3), which for given values of r involves three unknown quantities $H \div M$, P , and Q , may be used to determine all of them (temperature and induction corrections applied) by series of deflections at three different distances. Ordinarily this would not be undertaken in the field, unless the results from numerous stations were available.

The ratio of H to M in (3) is that due to the permanent moment, M , of the collimator magnet. The true ratio is that corresponding to the augmented moment $M + \mu H \sin u$, due to induction when the magnet makes the angle u with the earth's horizontal field. Since M occurs in the denominator of (3), the preliminary value of the ratio must be divided by the induction factor,

$$\frac{M + \mu H \sin u}{M} = \frac{M + \mu H \cdot \frac{2M}{Hr^3}}{M} = 1 + \frac{2\mu}{r^3},$$

the elimination of $\sin u$ being effected by substituting its approximate value from (3) with the trinomial factor dropped.

Some numerical illustrations.—Kew magnetometer 147 was originally tested at Kew Observatory in 1902, and later at Cheltenham, in 1906 and in 1915. The following facts come from the original certificate.

(a) Deflection bar, lengths corrected for bending.

(b) Deflection apparatus, $d=58.1''$.

(c) Collimator magnet 147A, $d=1.92'$.

Temperature correction $= .00032 (t-t_0) - .00000102 (t-t_0)^2$.

Induction coefficient, $\mu=7.20$.

Inertia cylinder, length $l=9.682\text{cm}$, $d=1.026\text{cm}$, mass $M=68.0579\text{g}$. When the cylinder is used to redetermine K , the moment of inertia of the magnet, use the formula

$$K = M \left(\frac{l^2}{12} + \frac{d^2}{16} \right) \frac{T'^2}{T''^2 - T'^2},$$

in which T'' and T' are the periods of oscillation with and without the cylinder, corrected for torsion, temperature, etc.

Six tables are given, including corrections to T_0 for daily rates; corrections for initial and terminal arcs; for the effect of torsion per 90° of twist; values of the induction factor; temperature corrections for every degree from -5° to $+45^\circ \text{C}$.; values of $\log \pi^2 K$ for various temperatures; and values of $\log \frac{1}{2}$ for the same temperatures and for various deflection distances.

In 1906 the values of the distribution coefficients were determined as $P=6.67$ and $Q=-57.1$. The magnetic moment of 147A was found to be about 584 C. G. S. units, and the magnetic axis was placed at about the thirty-eighth division of the scale. The test of 1915 showed a decrease in the moment of inertia from 265.02 to 258.24 since 1902, and the temperature coefficient was found to be 0.00028. The distribution coefficients of 1906 were allowed to stand.

INCLINATION OBSERVATIONS

When a dip circle is used, the dip is measured in the plane of the magnetic meridian by means of a magnetized needle with its pivots resting on agate knife-edges. It is manipulated

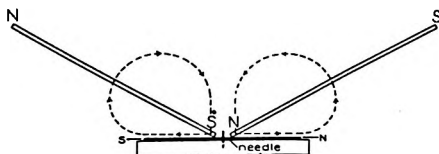


FIGURE 103.—Reversing polarity.

with a lifter. When it is in position, the axis of the pivots passes through the center of a graduated vertical circle read with the aid of microscopes at opposite verniers. Two needles are usually provided for dip observations, and two others for total intensity observations by Lloyd's method.

It is assumed that a dip needle loses its magnetism rapidly. The polarity of the two needles should be reversed at the beginning of observations. This is accomplished by stroking them with the bar magnets, presenting the north pole of one magnet to the north end of the needle, and the south pole of the other magnet to the south pole of the needle.

Fasten the needle in the reversing block, and with the bar magnets held inclined at about 120° with each other, like the sides of a wide funnel, place the lower poles in contact with the needle on either side of the pivot, then with a slow and uniform movement draw them toward and over the ends of the needle. The greatest inductive effort, tending to neutralize the existing polarity and to substitute the opposite kind, occurs as the poles of the magnet pass simultaneously over the ends of the needle. Return the magnets to the first position in vertical arcs well above the needle, clockwise in the left hand and counterclockwise in the right hand. This constitutes one stroke. Make four or five strokes, or the number specified in the directions; then interchange the magnets in the hands, turning each through 180° so as to preserve the association of like poles, and make an equal number of strokes. Now turn the needle over without reversing the ends, and magnetize the opposite face in the same way.

The polarity is again reversed in the middle of the observations. While observing, keep the magnets at least 30 feet away from the dip circle.

The bar magnets should be stowed with unlike poles at the same end of their metal box, and contacting the magnetic guards.

The example shows the order of operations for one needle. It is usual to observe with two needles, and in case of discordant results, with three. The needle magnetized first is used first.

For getting into the meridian some instruments are provided with a separate compass and peep sights. It is well to record the horizontal circle reading of the meridian and also of some distant object selected for a backsight. After use, remove the compass to a safe distance. This is a determination of the meridian only for purposes of observation. An even less accurate method must be resorted to when no peep sights are furnished, or in high magnetic latitudes where the indications of the compass needle are unreliable; taking advantage of the fact that the dip needle points vertically in the magnetic prime vertical. To find the plate circle reading of the magnetic prime vertical, satisfy this condition in four positions of the instrument, namely:

Face of circle south, face of needle south, then north.

Face of circle north, face of needle north, then south.

At a cost of doubling the observations, the trouble and uncertainty of getting into the magnetic meridian may be avoided by observing in two perpendicular vertical planes selected at random. For if x and y are the dips observed in two such planes, it may be shown that the true dip will be given by the relation

$$\cot^2 I = \cot^2 x + \cot^2 y \quad (4)$$

A lack of agreement between the results from north and south polarities indicates that the needle is not perfectly balanced. The true dip may be obtained from

$$\tan I = \frac{1}{2} (\tan I_n + \tan I_s) \quad (5)$$

Magnetic inclination

Station: Gracias a Dios, Nicaragua.
 Dip Circle: 128.
 Chronometer: Negus 1890.

Date: Mar. 16, 1911.
 Observer: X. Y. Z.
 Needle: 1.

Polarity of marked end, B.

Circle east				Circle west			
Face east		Face west		Face east		Face west	
S	N	S	N	S	N	S	N
$\begin{smallmatrix} \circ & f \\ 42 & 44 \\ 43 & 05 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 06 \\ 42 & 29 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 36 \\ 42 & 37 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 41 & 53 \\ 42 & 06 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 46 \\ 43 & 02 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 39 \\ 42 & 48 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 54 \\ 43 & 03 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 45 \\ 42 & 06 \end{smallmatrix}$
42 54.5	42 17.5	42 36.5	41 59.5	42 54.0	42 43.5	42 58.5	42 25.5

Mean: $x = 42^\circ 36.2'$
 $\tan x = .91966$

Polarity of marked end, A.

Circle west				Circle east			
Face west		Face east		Face west		Face east	
S	N	S	N	S	N	S	N
$\begin{smallmatrix} \circ & f \\ 43 & 01 \\ 43 & 06 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 46 \\ 42 & 52 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 43 & 13 \\ 43 & 16 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 59 \\ 43 & 04 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 46 \\ 42 & 43 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 14 \\ 42 & 11 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 43 & 05 \\ 43 & 02 \end{smallmatrix}$	$\begin{smallmatrix} \circ & f \\ 42 & 32 \\ 42 & 31 \end{smallmatrix}$
43 03.5	42 49.0	43 14.5	43 01.5	42 44.5	42 12.5	43 03.5	42 31.5

Mean: $y = 42^\circ 50.1'$
 $\tan y = .92714$

$\tan I = .92340$

Dip = $42^\circ 43.2'$

	h	m
Local time of beginning.....	15	24
Local time of ending.....	16	25
Mean of local times.....	15	54
	\circ	f
Magnetic meridian reads.....	17	51
	197	51

	Circle in magnetic P. V.	\circ	f
Circle S. Needle S.....		17	36
Circle S. Needle N.....		18	02
Circle N. Needle N.....		198	15
Circle N. Needle S.....		197	31
Mean.....		17	51

Polarities reversed by strokes of magnet on each face.

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CHAPTER XI

LATITUDE

For the determination of latitude with instruments of precision, see the U. S. Naval Observatory's Manual of Field Astronomy.

The following methods of determining latitude are for use with a **sextant and artificial horizon** or with a **10-second transit-theodolite of the repeating type**; and are described in the expectation of their occasional use on ships detailed to make preliminary surveys of no great extent, remote from the control of any coastal triangulation, and with no usable geographic coordinates available for an origin.

For this purpose the usual navigation methods are inadequate, and it is assumed that there is no astronomical transit available for the more precise determination of latitude by the Horrebow-Talcott method. On the other hand, the determinations in mind are above the grade of those obtainable by observations of the sun. Assuming that the ship is equipped with wireless and chronometers, not necessarily sidereal chronometers, the observations for local time may be made, with the same theodolite, by the horizontal angle method of determining local sidereal time described in the chapter on **Azimuth and Time**.

Considering the difficulties arising from lack of precise equipment and often of time for extended work, only three methods will be described in detail, one for the sextant and two for the theodolite, but other methods will be mentioned.

The selection of a station site is of some importance. It is well to avoid station error or displacement of the zenith due to local attraction of the plumb line.

The first quality of a latitude instrument is stability. Though the instruments mentioned are lacking in this, much may be done to minimize basic errors by building piers, even for sextants; by providing shelters from the wind; and by making arrangements for observing in a comfortable posture.

Let α , δ , t , A be the right ascension, declination, hour angle, and azimuth of a star; ϕ the latitude; and T the chronometer time of an observation, corrected to the sidereal rate from an initial epoch T_0 whenever the chronometer used is not a sidereal chronometer.

TWO-STAR EQUAL-ALTITUDE METHOD

This method for the **sextant**, due to Cañete del Pinar, "Observations of Precision with the Sextant," consists in noting the times, indicated by a rated chronometer, when an invariable altitude set on the sextant is satisfied twice by each of two stars, first east, then west of the meridian. It is not necessary to know the error of the clock. The hour angles are half of the respective intervals. The pairs should be about equally north and south. The subscript 0 applies to one of the stars, most conveniently that of the earlier observation, and the subscript 1 to the other star. The quantities B , C , D , m , and n are auxiliaries for adapting the basic formulae to logarithmic reduction.

$$D \sin B = \sin \frac{1}{2}(t_1 - t_0) \cot \frac{1}{2}(\delta_0 - \delta_1) \quad (1)$$

$$D \cos B = \cos \frac{1}{2}(t_1 - t_0) \tan \frac{1}{2}(\delta_0 + \delta_1) \quad (2)$$

$$C = B + \frac{1}{2}(t_1 - t_0) \quad (3)$$

$$\tan \phi = D \cos (t_0 + C) \quad (4)$$

The following alternative equations may be used for checks:

$$\text{Assume} \quad m = \cos \delta_1 \cos t_1 \quad (5)$$

$$\text{and} \quad n = \cos \delta_0 \cos t_0 \quad (6)$$

$$\text{Then} \quad \tan \phi = \frac{1}{2}(m - n) \sec \frac{1}{2}(\delta_0 + \delta_1) \csc \frac{1}{2}(\delta_0 - \delta_1) \quad (7)$$

This method is practically the same as the "seventh method", described in Chauvenet's *Spherical and Practical Astronomy*, volume 1, page 277. It is an excellent method for the sextant, and a fairly good one for the transit or the theodolite. Any kind of a sextant may be used for one pair of stars per night, provided that its arc is long enough to measure fairly large double altitudes. The essential qualities are planeness and good silvering of mirrors, permanence of adjustments, and tightness of clamps and tangent screw nuts. It is well, also, to make provision to aid the observer in holding the sextant steady in a truly vertical position, by cementing a vertical bolt in the upper face of the pier, or by suitably shaping the upper part of the pier.

Transits as near the meridian and as high as is comfortable are favorable to precision. If 130° is adopted as the highest practicable double altitude to be measured by using an artificial horizon, the declination of the north star should not exceed $\phi + 25^\circ$ by more than 10° to 20° ; nor should that of the south star fall below $\phi - 25^\circ$ by more than the same amount. It is better to multiply observations on a few of the most favorable pairs of stars than to observe on less favorable pairs. For example, to observe on three pairs of stars in one night (the same or any other three could be used the next night, if desired), it is suggested that nine sextants be used to give a fairly full program. Clamp sextant no. 1 on the altitude of the earliest observed and higher star of the pair and lay the sextant aside to be used later on the other star of the pair. Allowing some little time to elapse, pick up sextant no. 2, clamp it on the altitude now attained by the first star, and lay it aside. Do the same with sextant no. 3. This is one-quarter of the observations on that pair of stars. Use the sextants, still clamped in their first star positions, in the same order on the second star of the pair. There will now be a considerable wait until the stars of that pair come to their desired positions west of the meridian, when the sextants will be used in the reverse order. During this wait there may be time to make observations with sextants 4, 5, and 6 on a second pair of stars, and so on.

The mean, in this case, of the six rated clock times on one star will give a rated clock correction at the time of meridian transit and a better determination of the three hour angles east and west than that obtainable by using only one sextant.

In a program like that suggested above it is most unfortunate to miss any of the observations on the second of a pair of stars. Therefore it is well to avoid hurry during the observations on the first of the pair.

HORIZONTAL ANGLE METHOD

This method is designed for the **repeating theodolite**.

Like the method of the same name for finding local time, it utilizes the highly developed precision of the theodolite in the measurement of horizontal angles, and the possibility of refining observations by repetitions and reversals of the telescope when the star is nearly stationary, near the time of either elongation.

The term "theodolite" is used here, out of its primary sense, to mean a 5- or 10-second transit-theodolite with a low, wide, three-screw base, mounted on a pier or other solid support, stability being the indispensable requirement for dependable results. Instruments of the micrometer type are unsatisfactory under field conditions, if they are light. A four-screw base is unsatisfactory because of the strains set up by changes of temperature. The instrument should not be tied down nor rigidly attached to the support. The telescope should be balanced perfectly while in sidereal focus, and the friction bearings should be adjusted to avoid the necessity of clamping, at least during the moment of observation. The caps at the ends of the horizontal axis should be loosened or removed. The instrument should be protected from wind and care should be exercised in bringing lights near the levels. The telescope should be adjusted once for all to sidereal focus (for which reason the mark should be distant at least 3 miles), and all preliminary adjustments, especially of the horizontal axis, should be perfected after night-fall, when the instrument is cool.

Focus on a star, not on the mark; but the mark should be fairly distinct. Often a lighthouse with a fixed light may be used, but in general the best target is a vertical slit in a light box, subtending about 2 seconds of arc at the station. A small light or other easily visible object should be placed near the instrument nearly on range, horizontally and vertically, to the mark,

to enable the latter to be found quickly. If the mark is low, it may be necessary to consider the range of tide as well as the curvature of the earth. As a precaution against picking up the wrong light, read and record its angle of elevation.

A useful rule of thumb for computing the width of the slit is that at 3 nautical miles 1 inch subtends 1 second of arc, approximately.

The altitudes of observation stars will vary from 45° to 60° or more, and their azimuths, reckoned both ways from the meridian, will be near the value of the colatitude, say 10° on either side. With these facts in view it is well to build the pier and benches (one for each side of the pier) of heights suitable for comfortable observing. The heights will vary with the observer and with the instrument. A good rule is that when the observer is seated at the instrument the telescope axis should be 2 inches above his height of eye. For a man of average stature, seated on a chair 18 inches high, and using a theodolite 12 inches high, the height of the pier would be close to 36 inches. The top of the pier should be a little larger than the base of the instrument, but the sides should be cut away to give leg room for the observer when seated and facing in the directions mentioned. If the instrument is of the altazimuth type, that is, constructed principally for measuring vertical angles (not a good type), it will probably have a prismatic eyepiece set in a tube several inches long, which will require different arrangements than those suggested.

For a theodolite, a prismatic eyepiece, if used, should be thin, so as not to interfere with transiting. If it is not thin enough, the telescope caps may be removed and the telescope may be lifted out of the standards for reversal, as in the original type of theodolite, thus avoiding screwing the eyepiece on and off. On the whole it is better to use instead of such an eyepiece, at least for the higher altitudes, a sextant mirror on a small wedge-shaped block, thin and flat on the bottom, so as not to rock. With this, stars having altitudes up to about 75° may be utilized. When observing a star, the mirror and block are simply laid on the glass cover of the upper motion, under and in front of the eyepiece, in the small circle of illumination; and when observing the mark they are pushed aside enough to permit transiting the telescope when required. With a comfortable seat there is no great difficulty in bringing the image of the vertical wire to that of the star.

For the use of the stride level, see **Azimuth and Time**. Assuming that six-time angles are observed from mark to star, the reversal of the telescope after the third pointing on the star eliminates, theoretically, most of the error of the inclination of the horizontal axis. But to correct the outstanding error the stride level should be used before and after the observations on each star. The inclination error is the error most to be guarded against. The collimation error, on the other hand, is completely eliminated by the reversal of the telescope.

Since the field of view is small, the use of a second instrument, which may be a small transit, is almost indispensable for finding stars ahead of time, and for directing pointings of the theodolite, thus saving time and avoiding sweeping and useless manipulation. To direct pointings in azimuth it must be oriented parallel with the theodolite. It may be set up on the line theodolite-to-mark a little below the line of sight; or if the distance to the mark is known, D , and if the transit is placed a distance d from the theodolite perpendicularly to the line of sight, the setting on the mark, when the theodolite setting is zero, will be the angle whose tangent is $d \div D$; or, with the zeros of the theodolite on the mark, the two instruments may be made to read the same when pointing simultaneously on a pole star. Immediately afterward the alidade of the transit should be turned to the mark, and the plate angle read and recorded for a "backsight", for reorienting the transit from time to time.

Another important use of the transit consists in the opportunity afforded to watch each star approach, pass through, and recede from elongation. First, this will insure the identity of the star as a star of a certain declination and as one having an elongation. Second, since the stars utilized in this method, unlike circumpolars, have an easily perceptible motion near elongation, a rough value of the clock time of elongation may be obtained from the interval between the two passages across the stationary vertical wire. This, combined with the times of the several pointings of the theodolite on the star, will give the intervals from elongation required for computing their reductions to elongation or for scaling them from the graphs appended.

Even if the time thus obtained were in error as much as a minute, the error of reduction to elongation would be only 1 percent of the 10-minute value, at most about 2 seconds.

The reductions to elongation, when scaled for a 10-minute interval, may be converted into the actual reduction for any less interval, x , minutes, by means of the proportion

$$\text{Reduction for } x \text{ minutes: Reduction for 10 minutes} = x^2:100, \text{ or } x^2 \text{ percent.}$$

For as precise a determination of the latitude as may be obtained with a theodolite, the azimuth of the mark, the clock error before and after latitude observations, and the rate of the clock should be determined, as described in **Azimuth and Time**, horizontal angle method. Otherwise, with a tolerance of about 2 seconds, and with a second instrument to give time as described above, special time observations may be omitted. The direction of the meridian with reference to the mark may be obtained in the course of the latitude observations by taking the mean of the directions of one or more of the stars at both elongations. For by this latitude method, time enters in the computation of latitude only in the reduction of the separate pointings to elongation.

Special observations for azimuth and time, besides improving the latitude determination, will shorten the period of observations, for with these elements known all elongations that come along, indifferently east or west, may be observed in succession.

FORMULAE RELATED TO THE HORIZONTAL ANGLE METHOD

At this point it seems useful to collect formulae related to this method. Some of them may be needed to extend the tables that follow, but usually only the last formula, that for computing the latitude, will be required in the field. To the usual notation, ϕ , δ , t , n , A , and Δ as general symbols for latitude, declination, hour angle, altitude, azimuth, and change or error, are added special symbols with the subscript e , referring to the time of elongation, and 10 referring to a time 10 sidereal minutes later than elongation.

General formulae.—

$$\left[\begin{array}{l} -\tan A = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t} \\ -\tan A = \sin x \sec(\phi + x) \tan t, \text{ when } x = \cos t \cot \delta \end{array} \right. \quad \begin{array}{l} (1) \\ (2) \end{array}$$

Relating to elongation.—

$$\left[\begin{array}{l} \sin h_e = \frac{\sin \phi}{\sin \delta} = \sin \phi \cdot \csc \delta \end{array} \right. \quad (3)$$

$$\left[\begin{array}{l} \sin A_e = \frac{\sec \phi}{\sec \delta} = \sec \phi \cdot \cos \delta \end{array} \right. \quad (4)$$

$$\left[\begin{array}{l} \cos t_e = \frac{\tan \phi}{\tan \delta} = \tan \phi \cdot \cot \delta = \sin h_e \sin A_e \end{array} \right. \quad (5)$$

Relating to errors and precision.—

$$\Delta A'' = A_e - A_{10} = \csc t_{10} \sec \phi \cdot \sin 2\delta [1.991, 9309] \cdot 1'' \quad (6)$$

$$\left[\begin{array}{l} R = \frac{\Delta \phi}{\Delta A_e} = \cot A_e \cot \phi \end{array} \right. \quad (7)$$

$$\left[\begin{array}{l} R < 1, \text{ and } \Delta \phi < \Delta A_e, \text{ when } A_e > 90 - \phi \end{array} \right. \quad (8)$$

$$\left[\begin{array}{l} R < 1, \text{ and } \Delta \phi < \Delta A_e, \text{ when } \cos \delta < \cos^2 \phi \end{array} \right. \quad (9)$$

The latitude formula.—

$$\cos \phi = \frac{\cos \delta}{\sin A_e} \quad (10)$$

(1) is the general formula for azimuth, clockwise from north.

(2) adapts (1) to all-logarithmic computation.

(3) to (5) express relations at elongation. Remember them as quotients of like functions of latitude (the lesser) and declination (the greater), as follows:

Sine of altitude (height) = quotient of sines,
 Sine of azimuth (direction) = quotient of secants,
 Cosine of hour angle = quotient of tangents,

always less than 1.

Note that (5) is the product of (3) and (4). The finder angle tables are computed from these three formulae.

(6) expresses the numerical decrease of azimuth from elongation to 10 minutes later than elongation. The number in brackets is a logarithm. The formula is approximate, giving results about one-tenth percent too large. If preferred, the decrease of azimuth in 10 minutes may be scaled from the graphs. This quantity is one of small effect on the latitude.

(7) to (9) express relations between latitude error and azimuth error, the latter made in measuring the angle from mark to star; and are useful in selecting stars so as to minimize latitude error. (8) shows that the azimuth must exceed the colatitude. (9) shows that the upper limit of declinations for a favorable choice of stars is the angle whose cosine is the square of the cosine of the latitude. Favorable choices correspond to values of the ratio R less than 1.

(10) gives the latitude from the declination and the azimuth at elongation.

SELECTION OF STARS FOR OBSERVING AT ELONGATION

The declinations must exceed the latitude (else elongations cannot occur). It is well to avoid declinations within about 3° of the latitude, however, as associated with uncomfortably high altitudes. These declinations occur in the left-hand columns of the appended tables, and to avoid them the tables have been stopped at altitudes of just above 60° , one greater value being given for interpolation purposes.

The error ratio R , formulae (7) to (9), is small in the left-hand part of each table, and approaches the value 1 in the right-hand part. The value of the declination corresponding to $R=1$ at any station, the approximate latitude being assumed to be known, may well be taken as the upper limit of declinations of stars suitable for observation at that station. This limit may be found in two ways:

(a) *Mathematically.*—Square the cosine of the latitude, and find the angle whose cosine is this squared cosine. This is the limiting declination required.

(b) *Graphically.*—On the graph of decreases of azimuth in 10 minutes from elongation draw an interpolated latitude curve for the station. The upper limit of declinations will be at the intersection of this curve and the curve of upper limits for all latitudes. See pages 216, 217.

The stars available, therefore, are those in the American Ephemeris:

(a) Having declinations between the two limits mentioned.

(b) Of brightness down to about the fifth magnitude.

(c) Of right ascensions beginning with that corresponding to local time beginning 3 to 4 hours before the end of twilight, when stars that have transited may be observed at western elongation; and ending with that corresponding to local time 3 to 4 hours after the beginning of dawn, when stars that have not yet come to the meridian may be observed at eastern elongation. In the middle of this period a certain number of stars will be available for both elongations.

To avoid frequent references to the tables a short sectional table may be made for the approximate latitude of observation and written at the head of the program sheet. The sectional table may be formed from the general tables by interpolation or computed by formulae (3) to (5) without much labor. Thus, for latitude $19^\circ 50'$:

$$\text{Log. cos } 19^\circ 50' = 9.946,8870 = \text{log cos } 27^\circ 46'. \quad 27^\circ 46' = \text{upper limit} = \phi + 8^\circ, \text{ nearly.}$$

$\delta - \phi =$	3°	4°	5°	6°	7°	8°
$t_s =$	h m s 2 04 15	h m s 2 21 05	h m s 2 35 12	h m s 2 47 23	h m s 2 58 06	h m s 3 07 39
$h_s =$	60 58	57 06	53 53	51 08	48 44	46 37
$A_s =$	78 27	76 30	74 45	73 06	71 33	70 04

LATITUDE

Use declinations 23° to 28° ; first choice $22\frac{1}{2}^{\circ}$ to 23° , and second choice 28° to $29\frac{1}{2}^{\circ}$.

In the illustration just given, suppose the local sidereal times of the end of twilight and of the beginning of dawn are, respectively:

Applying the maximum interval, $\begin{matrix} h & m & s \\ 4 & 25 & 47 \text{ and} \\ -3 & 07 & 39 \text{ and} \end{matrix}$ $\begin{matrix} h & m & s \\ 14 & 33 & 27 \\ +3 & 07 & 39 \end{matrix}$

we have

1 18 08 and 17 41 06

for the limits of the right ascensions of available stars. In this particular case there are 25 "first-choice" stars and 7 "second-choice" stars.

After a list of all available stars has been made, noting right ascensions, declinations, and magnitudes, the next step is to weed out the least desirable ones. Next, the hour angles of elongation are taken from the table and the local sidereal times of observation are found. The final list may now be written on the program sheet in the chronological order of observations. The following headings are suggested:

[illegible]

TABLE 19.—Finder altitudes at elongation, in latitudes -55° to $+55^{\circ}$

Latitude north or south	Excess of declination, north $\delta - \phi$, south $-(\delta - \phi)$									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
1	30 00									
2	41 49									
3	48 37	36 54								
4	53 10	41 52								
5	56 29	45 40								
6	59 04	48 41	41 56							
7	61 07	51 10	44 34							
8	62 50	53 16	46 50	42 01						
9	64 16	55 04	48 48	44 04						
10	65 31	56 38	50 32	45 52	42 08					
11	66 36	58 01	52 04	47 30	43 48					
12	67 33	59 15	53 27	48 58	45 20					
13		60 22	54 42	50 18	46 43	43 42				
14		61 22	55 50	51 32	48 00	45 01				
15		62 17	56 53	52 39	49 11	46 14	43 42			
16		63 07	57 51	53 42	50 17	47 22	44 52			
17		63 54	58 45	54 40	51 18	48 26	45 58			
18		64 37	59 34	55 35	52 16	49 27	46 59	44 49		
19			60 21	56 26	53 10	50 23	47 57	45 49		
20			61 05	57 14	54 01	51 17	48 53	46 46		
21			61 46	58 00	54 50	52 08	49 46	47 40	45 47	
22			62 25	58 43	55 36	52 56	50 36	48 31	46 40	
23			63 02	59 23	56 20	53 42	51 24	49 21	47 30	45 50
24				60 02	57 02	54 26	52 10	50 08	48 19	46 40
25				60 40	57 42	55 08	52 54	50 54	49 06	47 28
26		4°	5°	6°	7°	8°	9°	10°	11°	12°
27	61 15	58 20	55 49	53 36	51 37	49 51	48 14	46 45		
28	61 49	58 57	56 28	54 17	52 20	50 34	48 58	47 31		
29	62 22	59 32	57 06	54 56	53 00	51 16	49 41	48 15		
30		60 07	57 42	55 34	53 40	51 57	50 23	48 57	47 35	
31		60 40	58 17	56 11	54 18	52 37	51 04	49 39	48 21	
32		61 11	58 51	56 47	54 55	53 15	51 44	50 20	49 03	
33		61 42	59 24	57 22	55 32	53 53	52 22	50 59	49 43	48 32
34		62 12	59 56	57 55	56 07	54 29	53 00	51 38	50 22	49 13
35			60 27	58 28	56 41	55 05	53 37	52 16	51 01	49 52
			60 58	59 00	57 15	55 40	54 12	52 53	51 39	50 31
36		6°	7°	8°	9°	10°	11°	12°	13°	14°
37	61 27	59 32	57 48	56 14	54 48	53 29	52 16	50 9	48 51	47 47
38		60 02	58 20	56 47	55 22	54 05	52 53	51 47	50 45	49 43
39		60 32	58 51	57 20	55 58	54 40	53 29	52 24	51 23	50 22
40		61 02	59 22	57 52	56 30	55 14	54 04	53 00	52 00	51 00
41		61 31	59 53	58 24	57 03	55 48	54 40	53 36	52 37	51 37
42			60 22	58 55	57 35	56 22	55 14	54 11	53 13	52 19
43			60 52	59 26	58 07	56 55	55 48	54 48	53 49	52 56
44			61 21	59 56	58 39	57 27	56 22	55 21	54 25	53 32
45				60 26	59 10	58 00	56 55	55 55	55 00	54 08
				60 56	59 41	58 32	57 28	56 30	55 35	54 44
46		10°	11°	12°	13°	14°	15°	16°		
47	60 12	59 04	58 01	57 03	56 10	55 20	54 33	53 10		
48	60 42	59 35	58 34	57 37	56 44	55 56	55 13	54 13		
49		60 06	59 06	58 11	57 19	56 31	55 46	54 56		
50		60 38	59 39	58 44	57 53	57 06	56 23	55 32		
51			60 11	59 17	58 28	57 42	56 59	56 12		
52			60 43	59 51	59 02	58 17	57 36	56 49		
53				60 24	59 36	58 53	58 12	57 25		
54					60 11	59 28	58 49	58 12		
55						60 04	59 25	58 49		
						60 40	60 02	59 25		

The correction for refraction, to the nearest minute, is $+1'$ for all tabular altitudes.
 The heavy vertical lines mark the same limits as in the table of finder azimuths.

TABLE 20.—Finder azimuths at elongation, in latitudes -55° to $+55^{\circ}$

Latitude, north or south	Excess of declination, north $\delta - \phi$, south $-(\delta - \phi)$									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
1.....	88 16									
2.....	87 46									
3.....	87 21	86 00								
4.....	87 00	85 31								
5.....	86 41	85 06								
6.....	86 23	84 42	83 17			Azimuth unfavorable				
7.....	86 07	84 20	82 50							
8.....	85 52	83 59	82 26	81 02						
9.....	85 37	83 39	82 02	80 35						
10.....	85 24	83 20	81 39	80 09	78 46					
11.....	85 10	83 02	81 17	79 44	78 19					
12.....	84 58	82 44	80 56	79 19	77 52					
13.....		82 27	80 35	78 57	77 26	76 01				
14.....		82 11	80 16	78 34	77 01	75 34				
15.....		81 54	79 56	78 12	76 37	75 08	73 43			
16.....		81 39	79 37	77 50	76 13	74 42	73 15			
17.....		81 23	79 18	77 29	75 49	74 16	72 48			
18.....		81 08	79 00	77 08	75 26	73 51	72 21	70 55		
19.....			78 42	76 47	75 03	73 26	71 55	70 27		
20.....	Altitude		78 24	76 27	74 41	73 02	71 29	69 59		
21.....	unfavorable		78 07	76 07	74 19	72 38	71 03	69 32	68 04	
22.....			77 49	75 47	73 57	72 14	70 37	69 04	67 35	
23.....			77 32	75 27	73 35	71 50	70 11	68 37	67 07	65 39
24.....				75 08	73 13	71 26	69 46	68 10	66 38	65 10
25.....				74 48	72 51	71 03	69 21	67 43	66 10	64 40
26.....		4°	5°	6°	7°	8°	9°	10°	11°	12°
27.....	74 29	72 30	70 39	68 55	67 17	65 42	64 10	62 42		
28.....	74 10	72 08	70 16	68 30	66 50	65 14	63 41	62 11		
29.....	73 50	71 47	69 52	68 05	66 23	64 46	63 11	61 40		
30.....		71 25	69 29	67 40	65 56	64 17	62 42	61 09	59 39	
31.....		71 04	69 06	67 15	65 30	63 49	62 12	60 38	59 06	
32.....		70 42	68 42	66 50	65 03	63 20	61 42	60 07	58 34	
33.....		70 21	68 19	66 24	64 36	62 52	61 12	59 35	58 01	56 29
34.....		69 59	67 55	65 59	64 09	62 23	60 42	59 04	57 28	55 55
35.....			67 31	65 33	63 41	61 54	60 11	58 32	56 55	55 21
			67 07	65 07	63 14	61 25	59 41	58 00	56 22	54 46
36.....	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°
37.....	66 43	64 41	62 46	60 56	59 10	57 27	55 48	54 11	52 37	
38.....		64 15	62 18	60 26	58 39	56 55	55 14	53 36	52 00	
39.....		63 49	61 50	59 56	58 07	56 22	54 39	53 00	51 33	
40.....		63 22	61 21	59 26	57 35	55 48	54 04	52 24	50 45	
41.....		62 55	60 51	58 55	57 03	55 14	53 29	51 47	50 07	48 29
42.....			60 23	58 24	56 30	54 40	52 53	51 09	49 28	47 50
43.....			59 53	57 42	55 56	54 05	52 16	50 31	48 48	47 08
44.....			59 22	57 20	55 22	53 29	51 39	49 52	48 08	46 26
45.....				56 47	54 48	52 53	51 01	49 13	47 27	45 43
				56 18	54 14	52 16	50 23	48 32	46 45	45 00
46.....	10°	11°	12°	13°	14°	15°	16°			
47.....	53 27	51 38	49 43	47 51	46 02	44 16	42 31			
48.....	53 00	50 59	49 03	47 09	45 18	43 20	41 44			
49.....		50 20	48 21	46 26	44 33	42 43	40 56			
50.....		49 39	47 39	45 42	43 47	41 56	40 10			
51.....			46 55	44 56	43 00	41 06	39 15	Azimuth unfavorable		
52.....	Altitude		46 10	44 09	42 11	40 16	38 23			
53.....	unfavorable			43 21	41 21	39 24	37 29			
54.....					40 29	38 30	36 33			
55.....						37 34	35 35			
						36 37	34 35			

The azimuths are reckoned eastward or westward from the upper arc of the meridian.

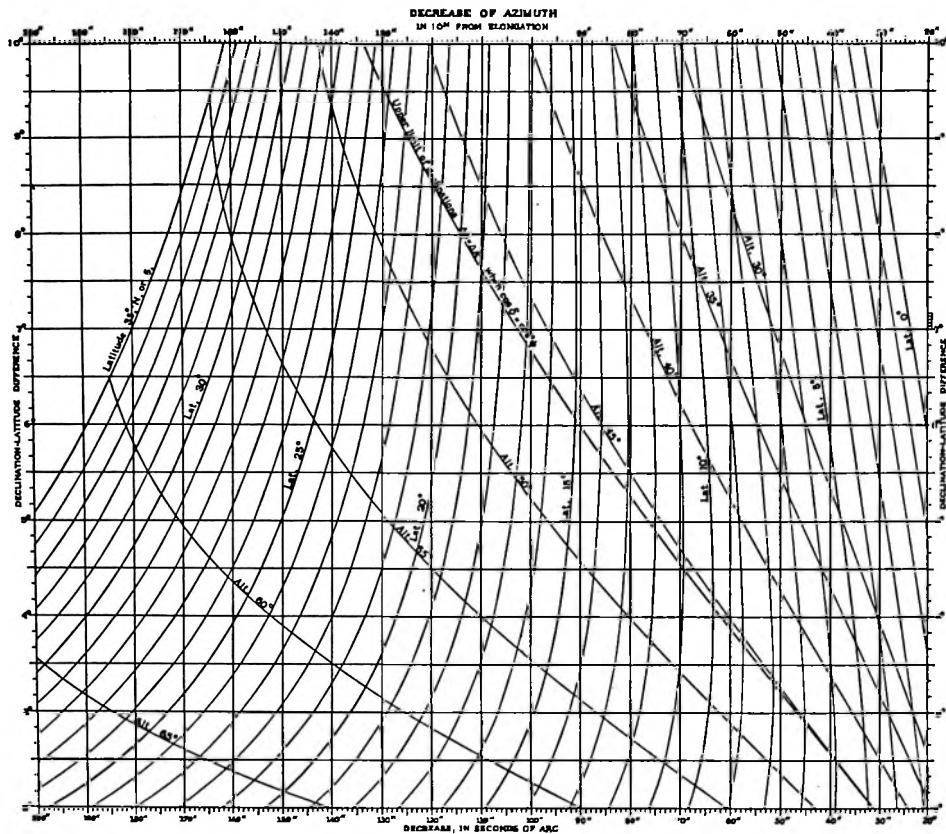
The heavy vertical lines mark the limits $h=60^{\circ}$ and $A=90^{\circ}-\phi$. On any horizontal line all azimuths except the first correspond to altitudes less than 60° , and all azimuths except the last are greater than the colatitude.

TABLE 21.—Finder hour angles at elongation, latitudes -55° to $+55^{\circ}$

Latitude north or south	Excess of declination, north $\delta - \phi$, south $-(\delta - \phi)$									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
Latitude north or south	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s
1.	4 00 02									
2.	3 12 52									
3.	2 45 49	3 32 48								
4.	2 27 45	3 13 11								
5.	2 14 37	2 58 14								
6.	2 04 31	2 46 23	3 13 42							
7.	1 56 27	2 36 42	3 03 28							
8.	1 49 50	2 28 36	2 54 47	3 14 26						
9.	1 44 17	2 21 43	2 47 19	3 06 44						
10.	1 39 33	2 15 47	2 40 49	2 59 58	3 15 23					
11.	1 35 28	2 10 37	2 35 06	2 53 59	3 09 17					
12.		2 06 03	2 30 02	2 48 38	3 03 49					
13.		2 02 00	2 25 30	2 43 51	2 58 53	3 11 35				
14.		1 58 24	2 21 27	2 39 32	2 54 25	3 07 03				
15.		1 55 09	2 09 47	2 35 37	2 50 22	3 02 55	3 13 49			
16.		1 52 13	2 14 28	2 32 04	2 46 41	2 59 09	3 10 01			
17.		1 49 33	2 11 27	2 28 50	2 43 22	2 55 42	3 06 32			
18.		1 47 08	2 08 42	2 25 52	2 40 12	2 52 32	3 03 23	3 12 54		
19.			2 06 10	2 23 09	2 37 22	2 49 37	3 00 22	3 09 56		
20.			2 03 52	2 20 40	2 34 46	2 46 56	2 57 39	3 07 12		
21.			2 01 45	2 18 22	2 32 22	2 44 28	2 55 08	3 04 41	3 13 19	
22.			1 59 49	2 16 16	2 30 09	2 42 11	2 52 50	3 02 21	3 10 59	
23.			1 58 01	2 14 20	2 28 07	2 40 06	2 50 42	3 00 13	3 08 51	3 16 44
24.				2 12 33	2 26 15	2 38 10	2 48 44	2 58 14	3 06 52	3 14 47
25.				2 10 55	2 24 31	2 36 24	2 46 56	2 56 25	3 05 03	3 12 59
26.		4°	5°	6°	7°	8°	9°	10°	11°	12°
27.		2 09 24	2 22 56	2 34 46	2 45 17	2 54 45	3 03 24	3 11 20	3 18 40	
28.		2 08 01	2 21 29	2 33 16	2 43 45	2 53 14	3 01 52	3 09 49	3 17 07	
29.		2 06 45	2 20 09	2 31 54	2 42 22	2 51 50	3 00 29	3 08 27	3 15 50	
30.			2 18 56	2 30 39	2 41 06	2 50 34	2 59 14	3 07 13	3 14 37	3 21 32
31.			2 17 50	2 29 31	2 39 57	2 49 25	2 58 06	3 06 06	3 13 32	3 20 28
32.			2 16 50	2 28 29	2 38 55	2 48 23	2 57 04	3 05 06	3 12 33	3 19 32
33.			2 15 55	2 27 33	2 37 59	2 47 28	2 56 10	3 04 13	3 11 42	3 18 43
34.			2 15 07	2 26 44	2 37 10	2 46 39	2 55 22	3 03 27	3 10 58	3 18 01
35.				2 26 00	2 36 26	2 45 57	2 54 41	3 02 47	3 10 20	3 17 25
				2 25 22	2 35 49	2 45 20	2 54 06	3 02 14	3 09 49	3 16 56
36.		6°	7°	8°	9°	10°	11°	12°	13°	14°
37.		2 24 49	2 35 17	2 44 49	2 53 37	3 01 46	3 09 24	3 16 34	3 23 20	3 29 45
38.			2 34 50	2 44 24	2 53 13	3 01 25	3 09 06	3 16 18	3 23 07	3 29 35
39.			2 34 29	2 44 05	2 52 56	3 01 10	3 08 53	3 16 09	3 23 01	3 29 31
40.			2 34 13	2 43 51	2 52 45	3 01 02	3 08 47	3 16 06	3 23 01	3 29 35
41.			2 34 03	2 43 43	2 52 39	3 00 59	3 08 47	3 16 09	2 23 07	3 29 45
42.				2 43 40	2 52 39	3 01 02	3 08 53	3 16 18	3 23 20	3 30 01
43.				2 43 43	2 52 45	3 01 10	3 09 06	3 16 34	3 23 40	3 30 25
44.				2 43 51	2 52 56	3 01 25	3 09 24	3 16 56	3 24 06	3 30 55
45.					2 53 13	3 01 46	3 09 49	3 17 25	3 24 39	3 31 32
					2 53 37	3 02 14	3 10 20	3 18 01	3 25 18	3 32 16
46.		10°	11°	12°	13°	14°	15°	16°		
47.		3 02 47	3 10 58	3 38 43	3 26 05	3 33 08	3 39 53	3 46 22		
48.		3 03 27	3 11 42	3 19 32	3 26 59	3 34 07	3 40 57	3 47 31		
49.			3 12 33	3 20 28	3 28 01	3 35 13	3 42 09	3 48 48		
50.			3 13 32	3 21 32	3 29 10	3 36 28	3 43 29	3 50 14		
51.				3 22 43	3 30 27	3 37 51	3 44 57	3 51 49		
52.				3 24 02	3 31 48	3 39 22	3 46 35	3 53 33		
53.					3 33 25	3 41 02	3 48 22	3 55 26		
54.						3 42 52	3 50 19	3 57 30		
55.							3 52 26	3 59 58		
							3 54 43	4 02 11		

The hour angles are reckoned eastward or westward from the upper arc of the meridian.

In agreement with the preceding tables, hour angles corresponding to altitudes greater than 60° , and to azimuths less than the colatitudes, are omitted, on the left and right, respectively, of the heavy vertical rules.

FIGURE 104.—Decrease of azimuth in 10^m from elongation, declination excess 2° to 10°.*Illustrations of the use of the graph and the altitude table*

Let the latitude be 25° N. and the declination 30° N.; to find the correction to a pointing taken 2 minutes either before or after elongation.

The correction will be negative; and since the correction varies as the square of the interval, it will amount to 4 percent of the correction for 10 minutes. Find the intersection of the side argument 5° and the latitude curve for 25°.

1. The azimuth correction is 4 percent of 130.5" or 6.02".

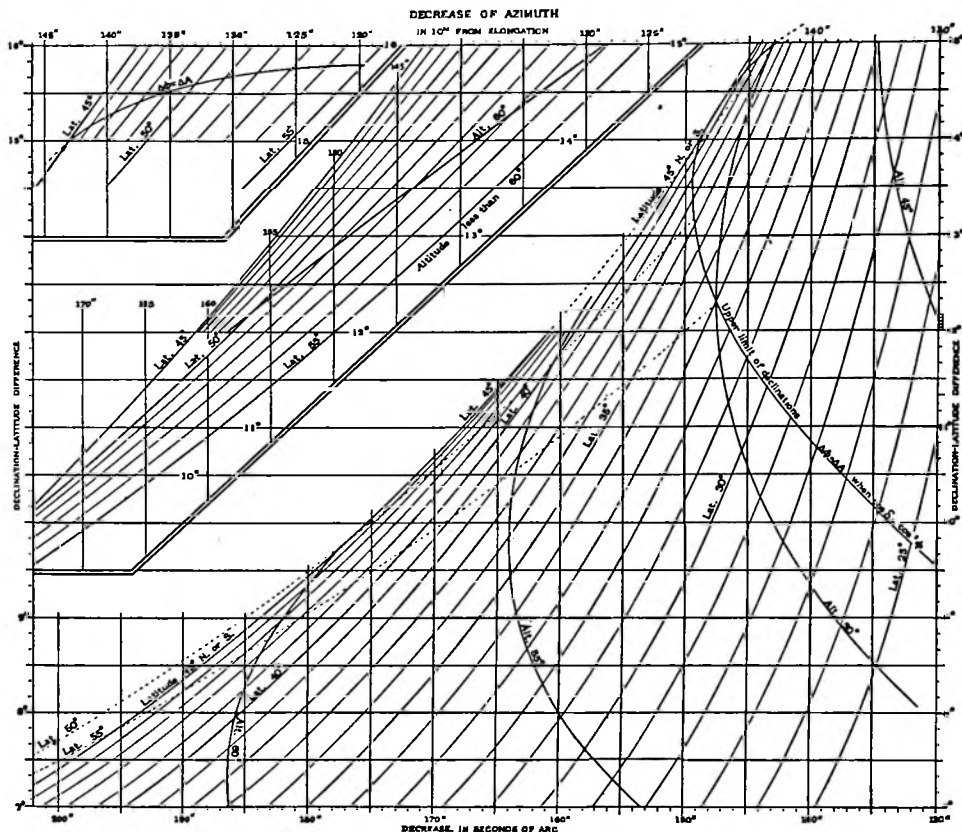
The altitude is about 57½°.

The altitude by table is 57°42' (column 6).

2. Following the curve for latitude 25° in a direction to find a more favorable altitude, it is seen that for an altitude of 55°, 52½°, 50°, etc., the declination would have to exceed the latitude by about 6°05', 7°10', and 8°25', respectively. Similar results may be found from the table of *finder altitudes*. Referring to the graph again, it is seen that it will be safe to use declinations up to about 9½° beyond the latitude, or to 34½°.

The formulae and the tabular values are to be applied in either hemisphere with respect to the elevated pole. The quantities ϕ and δ are always on the same side of the equator, and δ is always numerically greater than ϕ , else elongation could not occur. The quantities t and A may be reckoned either way from the meridian, but are always on the same side of it.

The correction to any observed direction for inclination of the horizontal axis is equal to the value of one division of the bubble in seconds of arc, multiplied by half the travel of the bubble between the direct and reversed positions of the stride level, and by the tangent of the apparent altitude. Numerically expressed, the two positions of the bubble are the means of the end readings. When the higher end of the axis is on the left when the observer is facing the target, the correction is clockwise (for angles of elevation).

FIGURE 105.—Decrease of azimuth in $10''$ from elongation, declination excess 7° to 15° .

VERTICAL CIRCLE NEAR THE PRIME VERTICAL METHOD

1. Both transits of a star over the prime vertical (Bessel).

Of the various modifications of Bessel's method that are described in Chauvenet's Spherical and Practical Astronomy, Vol. II, beginning at page 238, the one to which a theodolite could be most easily adapted, page 242 and formula (170), is the method of transits of the **same star both east and west** of the meridian, with the axis in the same position for both; followed by observations of the same star on another night with the axis in the reversed position. It is a method of high precision, but usually the choice of stars is very limited, if excessively high transits and extended periods of observation are to be avoided.

2. Transits of two stars, or of the same star twice, over a vertical circle near the prime vertical.

This may be considered as a modification of Chauvenet's eleventh method or of Döllén's method.

It is necessary to know the azimuth of the vertical circle or else the clock correction.

Assuming the azimuth to be unknown, it is first necessary to determine the clock correction, once if the rate is known with sufficient precision, twice if the rate also must be determined. The clock correction may be found by the horizontal angle method described in **Azimuth and Time**, or by the following special case of it, illustrated by the figure. The quantities determined are

the clock correction and the azimuth error, a , of the instrument set in a false meridian $N'S'$ near the true meridian NS . The figure, drawn for latitude $30^\circ N.$, illustrates the use of β Ursae Minoris and α Librae, which culminate only a few minutes apart. It will be assumed for the moment that the observed transits are free of collimation and inclination error, the former having been removed by daytime adjustments, and corrections for the latter being provided for by stride level readings.

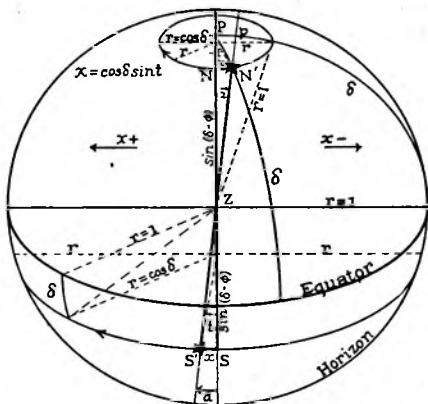


FIGURE 106.—Time and azimuth errors of a meridian transit circle.

From the similar triangles ZNN' and ZSS' .

$$\frac{x_n}{x_s} = \frac{\cos \delta_n \sin t_n}{\cos \delta_s \sin t_s} = \frac{\sin (\delta_n - \phi)}{\sin (\delta_s - \phi)}.$$

$$\sin t_n = \frac{\cos \delta_s \sin (\delta_n - \phi)}{\cos \delta_n \sin (\delta_s - \phi)} \sin t_s = k \sin t_s.$$

The known ratio k is negative if the north star is above the pole (northern hemisphere). Replacing $\sin t_n$ and $\sin t_s$, which are small, by their arcs,

$$t_n = kt_s. \quad (11)$$

If T_n, T_s , are the chronometer times, corrected for inclination, of transiting the false meridian, and if ΔT is the actual clock correction, the rate being neglected for the short time between the two observations, then the times of arrival at the true meridian are

$$a_n = T_n + \Delta T + kt_s \text{ and } a_s = T_s + \Delta T - t_s.$$

Subtracting, substituting (11), and solving for t_s ,

$$t_s = \frac{(T_s - T_n) - (a_s - a_n)}{1 + k}, \quad (12)$$

which gives the clock correction; and from triangle ZSS' we have

$$\tan a = \frac{\cos \delta_s \sin t_s}{\sin (\delta_s - \phi)}, \quad (13)$$

which gives the azimuth error, not required, but useful as a check.

An approximate value of ϕ may be used in the solutions. The azimuth error may be regarded as constant while the instrument remains in the same position. Also in work of this class the instrumental error involved in turning 90° from the supposed meridian to the supposed prime vertical may be ignored.

Supposing that the graduations of the stride level are numbered from 0 at one end to 60 (or some other number) at the other end, then the correction of each transit for inclination takes the form

$$Bb = \cos \zeta \sec \delta \cdot \frac{1}{2} d(p - p'), \quad (14)$$

in which d is the value of one division of the level in seconds of time, p is the sum of the end readings of the level when the zero end is east, and p' is the sum when the zero end is west. The sign of the B -factor is positive if the star is above-pole, negative if it is below-pole, while that of the b -factor depends upon the end readings.

Having made these corrections, and having derived the hour angles, t and t' when two stars of declination δ and δ' transit the supposed prime vertical, we are in a position to make use of Chauvenet's formulae numbered (357), Vol. I, page 295, in which M is an auxiliary angle and A is an angle differing from 90° or 270° by the value of a in (13). They are:

$$\tan M = \tan \frac{1}{2} (t' - t) \frac{\sin (\delta' + \delta)}{\sin (\delta' - \delta)} \quad (15)$$

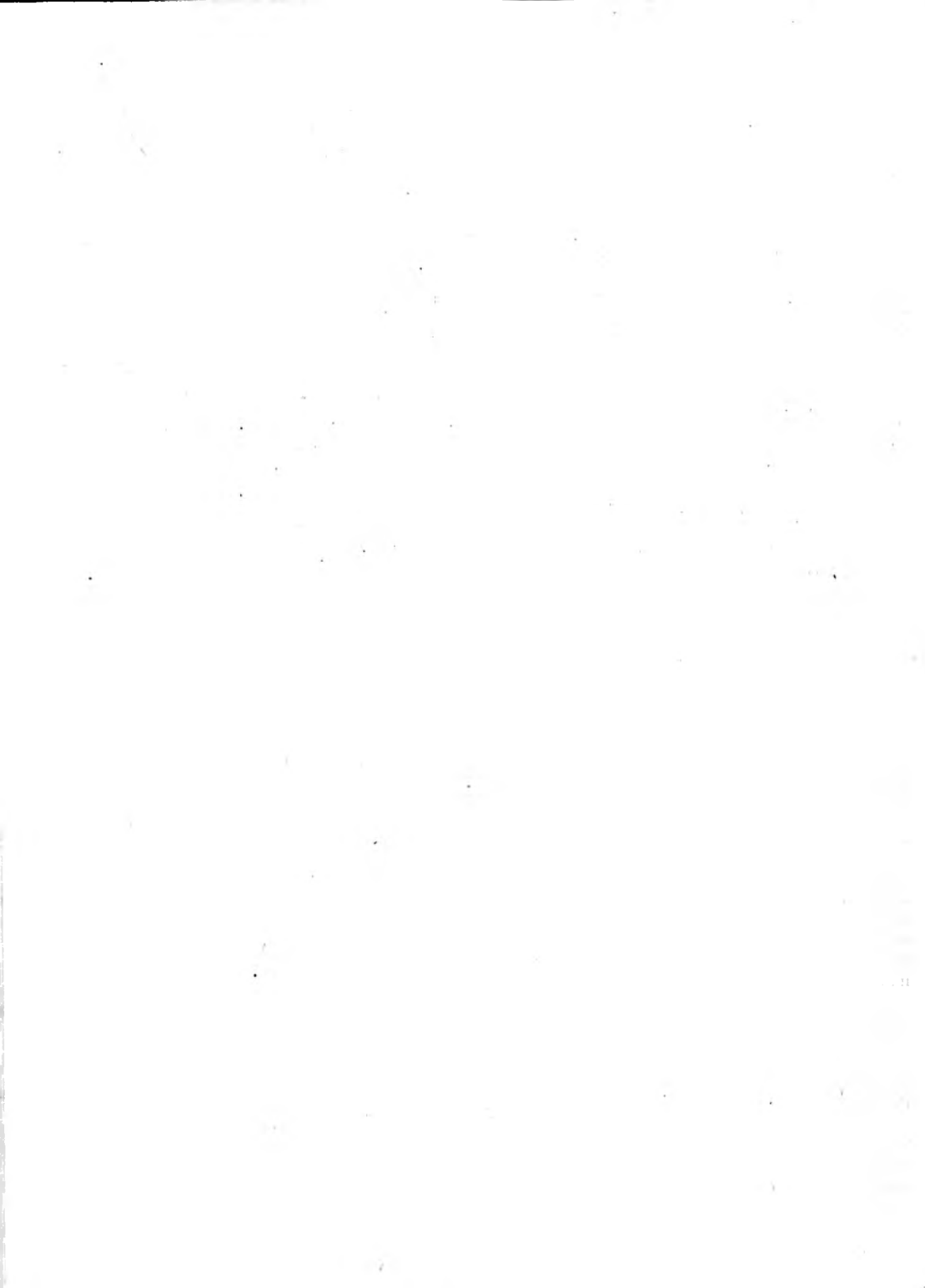
$$\tan \phi = \tan \delta \frac{\sin [\frac{1}{2} (t' + t) - M]}{\sin [\frac{1}{2} (t' - t) - M]} \quad (16)$$

$$\tan A = \frac{\tan [\frac{1}{2} (t' + t) - M]}{\sin \phi} \quad (17)$$

The latitude given by (16) is still to be corrected for the inclination of the horizontal axis, for though t' and t are the true hour angles, they correspond, not to ϕ , but to a latitude less than ϕ , if the north end of the axis is high, or to a latitude greater than ϕ , if the south end is high. Hence if the vertical circle is sufficiently near the prime vertical, the latitude coming from (16) is to be corrected by adding to it the inclination of the axis when the higher end is north, or by subtracting the inclination when the higher end is south.

Taking D for the value of one division of the level in seconds of arc, P for the sum of the end readings when the zero end is south, and P' for the sum when the zero end is north, the correction to the latitude given by (16) is

$$\Delta\phi = \frac{1}{2} D (P - P') \quad (18)$$



CHAPTER XII

PLOTING

All sheets, smooth or rough, should be polyconic projections, in ink, with grids commensurate with their scales. Uniformity in this respect is most desirable for transferring and for joining areas in the process of chart making. The following system is in general use in United States Navy surveys:

LATITUDE AND LONGITUDE GRIDS

Scale	Minutes divisible by—	Latitude and longitude interval
1: 5,000 to 1: 25,000.....		0 1
1: 30,000.....	2	2
1: 40,000 to 1: 60,000.....	5	5
1: 80,000 to 1: 200,000.....	10	10

SIZE AND QUALITY OF PAPER FOR SMOOTH SHEETS

Considering the contradictory requirements of the size of files, filing space, the proper scale of development, and the economical, in many cases the inevitable, distances between towers, it is inadvisable to specify a standard size for smooth sheets. The trimmed size of 41 inches by 58 inches should not be exceeded as sheets of larger dimensions cannot be filed flat in archive cabinets.

Larger sheets can usually be avoided by properly planning the field sheets, having in mind when they are laid out the particular rectangular areas into which they are designed to fit when smooth plotted; by keeping sounding signals in their own smooth areas; by providing more signals for middle distances; by keeping off the field sheets signals involving excessive distances, notably triangulation towers which are at times over-decorated for sounding purposes; and by computing more plotting distances, even when the signals involved are floaters.

The source of the difficulty is the disparity in the scale of development of hydrography and the distances easily sighted over and often necessary to obtain good triangulation control, which demands three cuts on intersection stations. The water signals used by the Navy can be seen 14 nautical miles from the towers, and when islands are far apart the greatest distance is often necessary to obtain the third cut. Not uncommonly the smooth sheet, unless made unduly large, shows only two of the towers, rarely only one of them, making plotting by intersections impossible. There should be no hesitation in plotting such a station by two cuts and two distances or even by one cut and one distance, provided that the computations of sides show a proper agreement on a common side.

This difficulty does not often arise in the field because of the large size of the master sheets, but presents itself when smooth plotting begins. In anticipation of this, and for the sake of providing numerical checks, all long sides should receive a double computation in the field, and the computation should be revised before smooth plotting.

Drag work is about the only kind of work that must be plotted in the field in a final form. Other smooth plotting should not be attempted there unless it can be prosecuted rapidly and completed. It is not only the change in climate from field to office that tends to vitiate plotting begun in one place and completed in the other, but the amount of salt absorbed by the paper.

The paper on which surveys costing thousands of dollars per sheet are to be plotted, in a final permanent form, should be the best obtainable. The sheets should be mounted on muslin and manufactured flat, not rolled thereafter, nor cut from rolls. In packing for shipment by the makers the sheets are to be enclosed in a heavy wrapping paper in units of three, and the whole packed in a flat, air-tight, tin container, well crated against buckling during transportation. In shipping or storing by the field parties smooth sheets may be rolled with backing outward to a diameter not less than about 10 inches, then kept in metal tubes or cases away from unfavorable conditions of temperature and moisture, and in as safe a location as possible.

COMPLETENESS AND FINALITY OF SMOOTH SHEETS

In the course of adjusting surveys as a whole to previous surveys it is sometimes necessary to shift meridians and parallels of smooth sheets before photographing them to chart scale. In all other respects, however, smooth sheets submitted by field parties are expected to be complete and final, needing no preliminary examination in the way of assembling or checking. This requires:

Standard projections made in a northern climate.

Stations plotted immediately after making the projections.

Neat, but not beautiful execution. Informative details not suppressed for the sake of appearance.

Careful checking at every stage.

Correct colors of ink for photographing distinctively.

Matching of adjacent sheets at edges, of the same season and of successive seasons (e. g., fathom lines, soundings).

Assembly on each sheet of all features in the area, including soundings, swept areas, shore line, elevations and contours, beacons, buoys, and landmarks, track and bearing lines, notes on landmarks and appearance of bottom, and so on.

Tidal and current stations, current roses or arrows, tidal and current summaries.

Recording, on each sheet, of the planes of reference used for heights and depths.

Emphasis in the delineation of dangers and shoals. The recommendations of the Federal Board of Surveys and Maps, applying to charts but not to survey sheets, are not to be followed when they interfere with the distinctive and complete delineation of peculiar local features.

References by number to all field sheets, photographs, shore line sheets, sounding books, angle books, computation books, tide and current books, town, cadastral, and drainage maps, sheets of previous seasons, if used, and, in short, records of all kinds pertaining to the area.

Soundings of previous seasons, P.D. and E.D. spots, in ink of a contrasting color.

Recording, on each sheet, the latitude and longitude of the origin.

The number of the sheet and the year, in the title and on all four corners of the back (for easy finding and restoring to place in the files).

These requirements, corresponding to past and present usage, are set down by way of a complete memorandum.

TRIANGULATION DIAGRAMS

Diagrams of final triangulation values should be drawn on white mounted paper not exceeding the size of 24 by 36 inches. The azimuths, forward and back, and distances and logs, may be written along the observed directions. Geodetic positions are conveniently shown in tabulated form. The latitude and longitude of the observation spot, or origin for the season, and connection with the main scheme, should be entered.

PREPARATION OF DATA FOR EXPEDITING PLOTTING

On the score of accuracy in plotting it is not advisable to begin the projections until all of the necessary computations of geodetic positions have been completed for figure-adjusted sides; nor before the necessary subordinate distances have been computed; nor before all angle books have been indexed for stations occupied, so that cuts may be found quickly. When there are several plotters it will pay to prepare, from the chronological book of daily angles, kept by the draftsman in the field, one or more alphabetically arranged books of cuts only, with all cuts col-

lected for the several stations, and all computed distances added. The regular angle books, containing triangulation angles and computations, elevations, and a great body of tangents too numerous to copy, will not always be available, being in use in the field or needed for constant reference by the chief computer and by the plotter of the photographic prints. If all cuts are copied on loose-leaf sheets the names of stations may be kept in alphabetical order. Each observer should copy his own cuts on the loose sheets and turn them in daily to the drafting room.

Experiments made in the Hydrographic Office indicate that most of these difficulties may be avoided by using zinc sheets having a certain white coating. They are relatively inexpensive.

The principal scale, which as soon as laid down becomes the final unit of measure for the projection and for all distances on the sheet, is best placed parallel to an element of the cylinder into which the sheet is habitually rolled, and not too near the edge. It is of little consequence whether it lies across the hydrography or is in the clear. The scale blank is commonly made one meter long with an extra centimeter space divided diagonally. To enable the plotter to compute shrinkages it is well to add several radiating pencil lines with the identical span of beam compass used in laying down the standard meter. The scale is not inked until after the stations have been plotted. It may be inked in fine black, brick red, or orange lines, so as to appear in photographs.

The projection, like the scale, is made in pencil, to facilitate accurate measurements, and is later inked.

Geodetic stations are plotted in pencil, first as pencil crosses; next as intersections of arcs struck with radii equal to the sides of triangles, modified to their shrinkage values; and finally as the intersection of azimuth lines. For laying down the latter it is advisable to use a table of chords or tangents rather than a protractor. In deciding on the final position of a station, the geodetic plusses are given the greatest weight and the azimuth lines the least weight. Exact agreement cannot be expected near the corners of the sheet, if the lines are long, but if substantial agreement is not obtained it may be necessary to make a new projection.

The basis of representation of signals on a smooth sheet is not their character as control but as navigation marks. For example, beacons and buoys should be shown by small circles, for precise location, surmounted by distinctive forms in black ink, for conventional description, supplemented by a verbal description, as "bk. bn.", "red nun by.", and the like; while ordinary survey marks, not permanent in nature, are shown by red circles, that they may be inconspicuous in photographs. It is desirable to delineate triangulation stations, the precise location of which on the sheet may be needed later for making measurements or for laying down channel ranges, in a way to indicate the center without obscuring nearby points of land. An equilateral triangle with inscribed circle and name in capitals, all in red ink, is suggested for triangulation signals; or the appropriate beacon sign if a permanent visible mark has been left.

PROTRACTORS

The ordinary three-arm steel protractor is not good enough for plotting forward cuts unless calibrated. After setting one vernier at zero when two arms are together, and the other vernier at 180° when the edges of two arms lie in a straight line, the errors of graduation should be found at every tenth degree and tabulated, first for points near the ends of the arms, then for points near the ends of the extensions. For this purpose compare the nominal angles with true angles constructed by employing a table of natural tangents or table 23, page 237.

Long forward cuts required in checking the geodetic plotting of triangulation stations or in laying down lines to distant peaks should be plotted by using a table of chords, or, more conveniently, with a triangulation protractor like that described in the chapter on **Equipment**.

CHECKING AND INSPECTION

Checking is a continuous operation beginning in the field and ending when the sheets and records are shipped. Proper checking is not a duplication of the work of one person by another, but rather an examination by each person of each part of his own work as it progresses, to guard against committing errors that he is prone to make, and an exercise of judgment in interpreting

results. This is followed by the more general inspection that is a part of superintendence. Plotting, like computation, becomes largely self-checking, when the results are judged by their reasonableness. There is economy in inspection at the source.

The matter of crooked sounding lines is sometimes troublesome. The ship's lines on course should be straight, and due credit should be given her and the helmsman for going as directed. The problem is complicated by the interference of stacks, masts, and lifeboats, which sometimes force the angle observers to occupy opposite ends of the bridge. On a scale of 1:30,000, with good observers and a good sheet, it is easily possible to spot every position within 30 feet, if the observers stand together. The uncertainties introduced by wandering anglers on a bridge 60 feet wide may make the plotted positions difficult to reconcile with the notes. In general, when ship's lines are excessively crooked, judged by the recorded courses, they should be straightened somewhat by apportioning part of the error to the anglers and part to the ship. In long reaches over flat bottom it is permissible to reject positions freely for the sake of straightening the lines and evening the intervals. On channel lines it is necessary to take account of port and starboard soundings. It is understood that plotters' notes added to the original records should be in colored pencil or ink, as subject to review, and initialed.

Ship's sounding lines that systematically press toward one side of the pencil lines give an indication and even a measurement of current. If they belly in opposite directions they may indicate compass error. Cases of local attraction will doubtless be investigated in the field.

The final check on the hydrography of a smooth sheet should be systematic, consisting of comparisons, first with the sounding books, page by page, and then with the boat sheets, to insure that all the shoalest soundings have been plotted, and that no rocks, shoals, or reefs have been overlooked. This includes an examination of the right-hand pages of the sounding books, where dangers are noted. The notes of transit observers and others should be gone through for additional details such as extra soundings, boat channels, and openings in reefs.

It is the business of the plotter, rather than of the engraver, to harmonize sheets where they join, leading fathom lines and shore contours smoothly across the adjacent margins. For this purpose the soundings along the edges of sheets that border on new territory should be copied for use the next season. When fathom lines run nearly parallel to the edge of a sheet, and when the period of tidal control is too short, it may happen that adjacent and overlapping lines, run at different seasons of the year, make the positions of fathom lines uncertain. In such cases the old soundings should be added in color, and the fathom lines should be reconciled as well as possible with both systems, adding full notes on the sheet in the localities affected. Soundings inconsistent with the adopted fathom lines should be kept (unless the space is needed for shoaler soundings considered better) but canceled by drawing short ink lines through them.

SHORE LINE CONTROL

To whatever extent aerial photography is employed in a survey, the location of all land features that can possibly be used by ships for bearings should be based on instrumental, not on merely graphical angles, to the end that fixes by bearings or angles, at whatever point taken, shall be interchangeable and above suspicion.

It often happens that an unmarked channel, used only by small craft, is deep enough for large vessels, and would be easily negotiable but for the erroneous location or the lack of detail in the shapes of the islands in sight. One of the purposes of the survey being the development of such possibilities, it is in order to conduct the field operations in such a way, and to bring the smooth sheets to such a state of completion, that turning points for the best available channel shall be defined by **natural ranges in numerical terms**, that is, in degrees and minutes. A natural range, so defined, becomes a starting point in the subsequent buoyage of the channel, for it provides orientation for a transit. This degree of completeness demands not only observing tangents from triangulation stations, but the occupation of the more conspicuous salients to complete their definite location. In brief, these important points of **ultimate control**, whether or not marked by signals for sounding, should be located instrumentally.

The control afforded by precise location of conspicuous salients, however, is not sufficient for the aerial work. The following method of gaining more intimate control for aerial photog-

raphy, and at the same time turning the results to advantage in another way, was developed by the U. S. S. *Nokomis*:

(a) Near the end of a survey season the photography for the next year is done, completing, as well, anything missed the previous season, and reef lines.

(b) The results are assembled, under little or no control, on celluloid sheets, and after reduction to a smaller scale yield a reconnaissance sheet convenient for laying out triangulation extensions and sounding areas for the next season.

(c) The next season, accompanying the work of signaling out sounding areas, the prints are taken into the field, and a large number of points recognizable on the prints are occupied with sextant or transit, or else are marked to be cut on from towers. (See U. S. Naval Institute Proceedings, June 1929.)

(d) In the smooth plotting season the land forms furnished by the prints are again assembled and reduced to the smooth sheet scale, but this time they are adjusted in scale and in orientation to fit the control points especially provided and those furnished by the triangulation. The triangulation control is largely by *tangents*. The plotting of control and the adjustments are generally done on templates of the smooth sheets, which are thus released for plotting hydrography; but the final results are transferred to the smooth sheets.

MANGROVE SHORES, HEADLANDS, AND LAND FORMS

By convention the shore line of coasts and islands is the high-water line. But this convention often reduces to an absurdity in the case of low flat mangrove shores, for under a strict application of the high-water criterion thousands of tropical islands, having no high-water line, would have to be erased from charts, and coast lines would have to be moved back miles into the interior. Therefore, with the needs of navigation in view, it is customary to plot the visible or apparent shore line, using a solid line bordered by the culture.

The habit of the common red mangrove (not the white mangrove nor the black mangrove, which are trees) is to begin its growth as a sprouting pod lodged on a shoal awash. It thrives mightily as long as its roots are bathed with tidal streams. In the course of time, with the accretion of floating snags, sand, and earth washed up by wave action, or sometimes suddenly during violent storms, a beach or other portion of solid land appears. Behind this, if it becomes a high continuous barrier, cutting off the supply of flowing water, the mangrove dies and is replaced by other forms of vegetation, grass, trees, etc. Also lagoons are formed. Meanwhile on the front the shore-forming process continues, though interrupted by storms and scouring currents, so that large islands often have several distinct shore lines, crescent-shaped beaches, arranged spoon-fashion. Only two of these are important, the outer line of visible vegetation, and if there are continuous or fragmentary beaches outside of that, the high-water lines on the latter. The proper delineation for a shore of this kind is a solid line for the visible shore and sanding with appropriate legends outside, provided that the range of tide is small.

When the range of tide is great there may be a distinct continuous high-water line a considerable distance in front of the line of vegetation. In this case the high-water line should be shown solid and the visible culture line should be shown in its actual form, not solid, but with a delineation dense enough to show bights and salients in their actual location.

Photographic prints will take care of the vegetation line, but not always of a high-water line a considerable distance in front, unless it should happen that the photographs were taken at a time of high water. In such cases, the photographed shore line has to be corrected by supplementary instrumental work or observations, by triangulation observers, shore-line parties, and sounding parties.

The task confronting field parties and smooth plotters is the gathering of exact information and the production of sheets that will indicate to the mariner, as well as may be, the **apparent location and form of the shore at any stage of the tide.**

Headlands and clusters of high trees, tree lines on capes, etc., when possessing a definite form, at a distance appear to the mariner as islands. It is well to locate their outlines by tangents, and by way of emphasizing them as **land forms**, to surround them by dotted lines indicating their extent, with appropriate notes concerning their height and appearance.

Land forms, especially the summits of hills and mountains, should be delineated by using sketch contours. See **Trigonometric Leveling.**

On the smooth sheet it is not sufficient to leave indefinite the positions of the summits of peaks by merely enclosing them with form lines, unless the locations and elevations are, in fact,

indefinite, as in the case of sextant work, in which case the elevations also should be indicated as approximate. Whenever the location is precise, depending on transit or theodolite cuts, the precise position of the summits should be indicated by a dot and circle, with the elevation written alongside. This is primarily for the purpose of furnishing a precise position to which mariners may take bearings. And in case the terrain is photographed, it gives a precise control point for the compilation of prints.

AERIAL PHOTOGRAPHY

Survey ships of the U. S. Navy have employed vertical photography regularly, and oblique photography in a supplementary way, in hydrographic surveys, since 1921. The photographs serve a double purpose, ultimately in supplying abundant details for charts, but first in furnishing advance information useful in selecting tower sites (see **Reconnaissance**) and in planning surveys. The preliminary compilations for this purpose are made at the Hydrographic Office.

The technical requirements considered applicable to any particular survey, such as instruments and equipment, flight layouts, flying altitudes and scales of development, percentage of overlaps, etc.—in brief all special and variable factors—will usually be described in the specifications of the survey. It will suffice here to mention a few general principles and requirements applicable to the field work of all aerial photography used in connection with coast and harbor surveys.

The photographs are needed for the two purposes mentioned. Besides, details not immediately or fully exploited in the construction of charts of certain scales may meet later needs of larger-scale charts and of local developments, such as harbor improvement and the construction of roads.

Photographs lacking proper horizontal and vertical control have but little value in the compilation of coast charts.

The incidental *horizontal control* furnished without extra effort and cost by the triangulation is seldom sufficient for developing the full chart value of the photographs. It should be supplemented by measurements and cuts from stations to objects of natural control recognizable in the photographs, for instance, islets, sharp points of land, isolated sand patches, solitary wide-spreading trees, and the crossings of roads, trails, and streams. This passing from artificial to natural control may be done in advance of the photography; but it is better to select control points first on the prints, and afterwards to place hydrographic signals in locations favorable to tying in the selected control points. To mention several ways of doing this:

(a) Having marked points A, B, C, . . . on the prints, examine the prints to find where signals may be placed within sight of as many of the points as possible without detracting from their value as sounding signals. Then occupy as many of the points as is practicable, obtain three-point fixes, and mark the spots with flags, that they may be cut upon from the stations in sight.

(b) If A is a natural control point not intervisible with stations, but is near some station, locate it from that station by traverse, or by an intercept method, or by any method adequate for plotting.

(c) Instead of locating natural objects from stations (on the ground) as in (a) and (b), the opposite procedure, locating stations from natural objects (on the photographs) may be employed. Since the shore line stations constitute, in fact, the horizontal control connecting all flights, it is best, for the purpose stated, to have a plane make a circuit or meander of all shore line stations in a manner to photograph each in passing, at a low altitude and nearly vertically.

Provided with these unconnected large-scale photographs, properly marked for the identification of signals, and with the regular small-scale continuous-flight pictures, the plotter will be enabled to spot the site of each signal on the latter by means of its revealed position among near-by objects common to both large-scale and small-scale photographs. By beginning with easily recognizable objects, though at some distance, and gradually narrowing the circle of reference points, the spotting of the site will attain the necessary precision on the working scale.

The *vertical control* furnished by water lines and elevations computed trigonometrically will usually suffice if the coast is flat, but may fail if the coast is hilly. Ordinarily the charts require form lines of hills close to the coast, but merely the apparent sketched forms of inland peaks,

which are beyond the photographed areas and offer no vertical control. For the coast hills, therefore, a special effort should be made to obtain a number of top and middle elevations, well distributed.

Photographic prints should be marked in the field to insure proper *interpretation*, by office compilers, who may be unfamiliar with the terrain, of culture and noteworthy features that might otherwise prove to be difficult to identify. The following features often require a brief description:

High and low water lines on wide sandy beaches.

Black mud areas nearly awash, resembling islands.

White mud areas, sometimes mistaken for sand.

Culture areas, with approximate heights of trees.

Cliffs, bluffs, and headlands.

Reefs and rocks; heights, depths, and appearance.

Buildings, especially lighthouses, and all conspicuous objects.

The final compilations are made at the Hydrographic Office, using the control afforded by the smooth sheets and the recorded angles. Some allowance should be made for the loss of control points due to bad pictures or unsuitable locations, usually about 25 percent. It is for this reason that the amount of control asked for above is more than appears to be absolutely necessary.

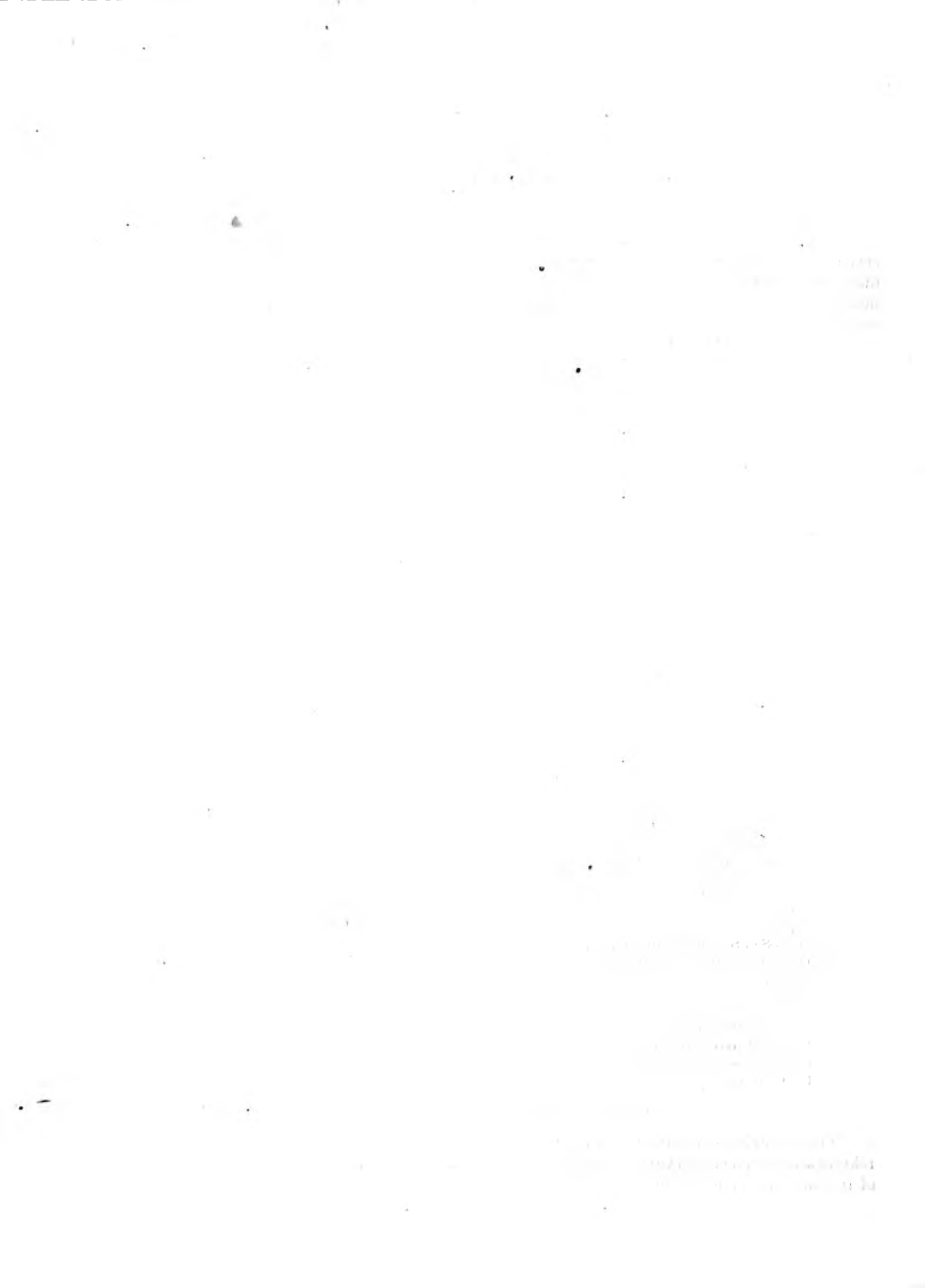
SAILING DIRECTIONS

Sailing directions are written systematically, day to day, preferably in the form of one or more sheets showing track lines, distances, and bearings at turning points; accompanied by a smooth notebook, sketches, and photographs (see chapter on *Equipment*), and a detailed yet terse written description of the courses shown on the sheets; with some account of winds, currents, character of the bottom, anchorages, availability of water, fuel, and supplies, and of communications and other matters of interest to mariners. It is obvious that without continuous study as the survey progresses and without forethought in the matter of recording in a book devoted to that sole purpose such facts as may be observed from time to time on the spot, the final notebook may be incomplete; whereas with such notes at hand and in view of accumulated experience gained by repeated voyages through the surveyed area, the final account will constitute a worthy summary of the valuable features of the survey. An officer should be assigned in charge of compiling these sailing directions, which are to be submitted in smooth form at the end of the field season as part of the survey records. More complete instructions are contained in specifications issued for individual surveys.

POLYCONIC PROJECTIONS

In the polyconic projection each parallel—or in practice the parallels at the top, middle, and bottom of the sheet—is the development of the base of the cone tangent to the reference spheroid at the particular latitude. Formulae for the radii of meridians and parallels may be found in Johnson's *Surveying* and in other texts. As a rule they are unsuitable for computing sheets of less than continental dimensions. Instead, use is made of tables in which points on successive parallels are defined by x and y coordinates. The intersection of the central meridian, which is a straight line, and a straight line perpendicular thereto, tangent to the central parallel, is taken as the origin of coordinates. In practice, according to the latitude, to the scale, and to the extent of the sheet, a grid of meridians and parallels is formed of straight lines, instead of arcs of circles, joining what is deemed a sufficient number of the points thus defined by rectilinear coordinates. Projection tables are published by the United States Geological Survey and by the United States Coast and Geodetic Survey.

For much detailed information on the subject of plotting small areas on a system of plane rectangular coordinates, see Breed and Hosmer's *Elementary Surveying*, Vol. I, Chapter XVI, and *Higher Surveying*, Vol. II, Part II. The latter book, in Part IV, gives the construction of polyconic projections and a brief account of other projections of less interest to the surveyor.



APPENDICES

APPENDIX I.—REFERENCE LIBRARY

A supply of standard works on surveying and allied subjects, to be considered survey property as distinct from the ship's library, and to be kept as a part of the equipment of any particular survey regardless of changes in the assignment of different ships to that survey, is recommended for every extensive hydrographic survey. The following list is intended to be suggestive rather than final or complete:

- Theory and Practice of Surveying, Johnson.
- Topographical, Trigonometric, and Geodetic Surveying, Wilson.
- Elementary Surveying, Vol. I, Breed and Hosmer.
- Higher Surveying, Vol. II, Breed and Hosmer.
- Elements of Precise Surveying and Geodesy, Merriman.
- Text-book on Geodesy and Least Squares, Crandall.
- Elementos de Hidrografia, Hartley y Hartley, Ministro de Marina, Madrid.
- Plane and Spherical Trigonometry and Tables, Crockett.
- Seven-place Logarithms, Bruhns'.
- Elements of Practical Astronomy, Campbell.
- Spherical and Practical Astronomy, Chauvenet.
- Compendium of Spherical Astronomy, Newcomb.
- American Practical Navigator, Bowditch.
- American Ephemeris and Nautical Almanac, for the current year.
- Manual of Field Astronomy (U. S. Naval Observatory).
- Navigation, Dutton.
- Photographic Methods and Instruments, App. 10, C. & G. S. Report 1897.
- Photographic Surveying, E. Deville, Ottawa.
- Manual of Tides, Part 5, App. 6, C. & G. S. Report, 1907.
- Notes on Self-registering Tide Gages, App. 7, C. & G. S. Report, 1897.
- Methods and Results of Survey of West Coast of Lower California, U. S. S. *Ranger*.
- Determination of Time, Longitude, Latitude, and Azimuth, C. & G. S. Sp. Pub. 14.
- Formulae and Tables for the Computation of Geodetic Positions, C. & G. S. Sp. Pub. 8.
- Tables for a Polyconic Projection of Maps, C. & G. S. Sp. Pub. 5.
- Seamanship, Knight.
- Observations of Currents with Direction Current Meter, C. & G. S. Report, 1891, Part 2, App. 10.
- Construction and Operation of Wire Drag, C. & G. S. Sp. Pub. 56, Serial 107.
- Wireless Longitude, C. & G. S. Sp. Pub. 109, Serial 281.
- General Instructions for Precise and Secondary Traverse, C. & G. S. Sp. Pub. 58, Serial 111.
- Some Elementary Examples of Least Squares, Adams, Serial 250.
- Pocket Companion, Carnegie Steel Co., Pittsburgh.
- Mechanical Engineers' Pocket-Book, Kent.
- Methods and Results, Directions for Measurement of Terrestrial Magnetism, Hazard C. & G. S. App. 8, Report 1881.
- Dictionaries, foreign language.
- Deep-Sea Sounding and Dredging, Sigsbee.
- Deep-Sea Exploration, Tanner.
- Submarine Cables, H. O. Publication 103.
- Invar and Related Nickel-steels, B. S. Circular 58, 1923.
- Manual of Map Reading, Photo Reading, and Field Sketching, 1929, United States War Office.
- Geodesy, Pub. 11, Geodetic Survey of Canada, 1929.
- Errors of Astronomical Positions, Pub. 13, Geodetic Survey of Canada, 1925.
- Bulletin Geodesique, No. 12, Oct.-Dec. 1926, International Ellipsoid, formulae and tables.
- Hydrographic Review, periodical of the International Hydrographic Bureau.

APPENDIX II.—SAMPLE LIST OF EQUIPAGE

The following list is given in detail with the idea of being of service to ships about to undertake extensive coast surveys. Many of the articles, as instruments, will last for the duration of the survey. The list of paper and supplies covers many necessities for the smooth plotting

season as well as the field season. The quantities appear to be ample except in a few items, such as sounding books, angle books, sounding leads, magnetic forms, black canvas, photographic supplies, and medical supplies for an increased complement. Most expeditions will require more transits. Most observers on the bridge prefer quintants with large telescopes to the ordinary boat sextants. There should be two 10-inch sextants for cutting in distant signals. The number of three-arm steel protractors is possibly ample for field work, but these cannot be used afterwards for smooth plotting.

FOR FIELD MEASUREMENTS

Current meters.—See **Tides and Currents**.

Dip circles, Barrows type, with tripod and spare needles (1).

Levels, Y-, 20-inch telescope, complete with sunshade, adjusting tools, and gossamer; with Federal Board of Specification interchangeable tripod, 8 threads per inch (1).

Levels, hand, Locke type (2).

Level trier (1) for astronomical work.

Magnetometer, Kew type, complete with tripod, 3 spools of bronze suspension fibers, and certificate of constants (1).

Quintants with spring clamp and endless worm tangent screw, large diameter limb with vernier reading to 30 seconds, monocular or binocular type; in carrying case (5).

Spare index mirrors (10), spare horizon mirrors (10), spare index mirror frames with screws (2), spare horizon mirror frames with screws (2).

Sextants, boat.—Standard Navy, limb graduated to 20 minutes, vernier reading to minutes, preferably, erecting telescope of 4-power, object glass 1½-inch diameter; with carrying case (25). Spares as follows: index mirrors (75); horizon mirrors (100); index mirror frames (5); horizon mirror frames (5); tangent screws (5); adjusting tools (5).

Sextants, navigator type, 10-second (for intersection work) (2).

Tallying machines, hand-operated, registering to 9999, for counting paces (1).

Tapes, base measuring, invar, 100-meter marked at 25-meter intervals, on wooden reel not less than 20 inches in diameter; accompanied by United States Bureau of Standards certificate for support throughout entire length and also for suspension at 25-meter intervals with thermometers in place, tension 15 kilograms (1).

Tapes, base measuring, similar to the foregoing except in length, 50 meters (2).

Tape thermometers for the foregoing with flat mercury chambers for tape contact. Centigrade graduations, United States Bureau of Standards certificate, identifying number, protective case (4)

Tape spring balances, registering 0 to 15 kilograms, nickel-plated, with tension handle. To be standardized, in the position of use with a tape, by the United States Bureau of Standards, and in that position to have the 15-kilogram tension point marked on the dial (2).

Tapes, steel, 50 feet long, of stainless steel, graduated in feet, tenths, and hundredths; in leather case (6).

Tide gages.—C. & G. S. type automatic recording, with float, counterweights, and all accessories (2). Recording paper for same (24 rolls).

Transits, 1-minute, as per specifications of the Federal Board of Specifications; with standard-thread interchangeable tripod and all accessories including spare cocoon spider web cross wires and balsam.

Transit-theodolites (2)—

Base and support, 3-screw leveling base with shifting center, the screws resting in grooves of a trivet attached to the tripod head. *Tripod* with detachable legs.

Horizontal circle, 8 inches, limb silver, graduated to 10 minutes with degrees numbered 0 to 360 clockwise only. Two opposite verniers, under mounted microscopes, reading to 10 seconds.

Vertical circle, full, diameter 6 inches, quadrants graduated 0° to 90°, limb with 20-minute spaces, read to 20 seconds by two opposite verniers under mounted microscopes.

Focusing design.—Internal.

Telescope, inverting. Reticule with one horizontal and one vertical cross wire, platinum preferred, otherwise spider web.

Electric illumination.—Telescope cross wires to have direct illumination, with rheostat control, and verniers of horizontal and vertical circles to have electric reading lamps with shades. Unit switches to be compactly secured to U frame of instrument.

Attachments.—Two eyepieces, 20 and 30 diameters, each with small shade glass; prismatic attachment for high altitudes, 28 diameter, with shade glass; level for vertical circle; stride level of sensitiveness about 10 seconds per division, with the tested value inscribed on the frame; plumb bob and extra cord; object glass shade; adjusting tools; gossamer; oil can; spare level vials, tangent screws, and reticule.

Wire, piano, for precise traverse tapes, 1 coil of about 500 fathoms.

FOR SIGNAL CONSTRUCTION

Anchors, chain, and shackles for 4-drum floating signals, wire drags, and spares for launches.

Bags, sugar, for transporting sand, gravel, and camping gear (100).

Boiler tubes, used, about 5 feet long, for station centers, guy stays, and weights.

Bolts and washers.—See *Survey Signals*.

Cement in 94-pound paper bags, for foundations and center marks.

Cloth for signal dressings—

800 yards black canvas (cotton duck), 50 inches wide.

800 yards white, canvas (cotton duck), 72 inches wide.

1,500 yards bleached sheeting, 72 inches wide.

600 yards black signal cloth, sheeting, 36 inches wide.

600 yards red signal cloth, sheeting, 36 inches wide.

600 yards orange signal cloth, sheeting, 36 inches wide (yellow is unsatisfactory).

Gin poles (6) for erecting towers, 3-inch diameter, 30 feet long, dressed, varnished.

Gin pole tail blocks (2), brass, sheave diameter about 2½ inches for 3-inch rope; sheave axle fitted in section of 2½-inch brass pipe 6 inches long.

Gin pole tail blocks (6), wood, with becketts, for 3-inch line.

Gravel in bags, for foundation work.

Grommets and dies, for fashioning signal dressings:

10 gross no. 4 spur grommets, with washers.

2 sets dies, inserting, eyelet grommets, with no. 4 punches.

Nails, steel wire and zinc, all sizes up to 40-penny, principally 4-penny and 10-penny.

Rope, white line, marline, and spun yarn, for hoisting, for edging, for lashing, etc.; *manila rope,* 3-strand, 6-, 9-, and 18-thread, and 2-inch.

Rules, 6-foot, folding, wooden, yellow enamel (6).

Rules, 2-foot, folding, carpenter's (6).

Sewing machine, electric, heavy-duty, for canvas, with motor and foot control for ship's current. With ½ gross of needles and 12 tubes or cones of white flax thread, 2 to 3 pounds.

Sewing machine, electric, for duck and sheeting, with foot control, and with motor for ship's current. Universal motor for A. C. or D. C., if possible. With 24 cones of white linen thread, ½ gross of needles and spare set of small parts.

Tacks, carpet, tinned, ½-inch (50 pounds).

Tacks, copper, ½-inch (5 pounds).

Tools:

Axes, 4-inch blade, single-bit (18), with spare handles (12).

Bits, wood-boring, ⅞-inch, solid-center, single-twist (18).

Braces, ratchet, 8-inch sweep (12).

Chisels, cold, ½ by 6 inches (12); ¾ by 6½ inches (12).

Chisels, wood, ¾-inch (6).

Drills, rock, 1-inch, star-point (6).

Emery wheels, hand-power (1).

Hammers, claw, 1-pound (36); with spare handles (12).

Tools—Continued.

Hammers, sledge, 10-pound (12); with spare handles, 30-inch (12).

Hack-saw blades (6 dozen).

Hack-saw frames (6).

Hatchets, shingling, 4-inch edge (18); with spare handles (18).

Levels, carpenter's, 12-inch (3).

Machetes, 24-inch blade, Panama Canal type, bone handles (24).

Oil cans (6).

Picks (6); with spare handles (6).

Pliers, 8-inch, side-cutting, flat-nose (18).

Pliers, 6-inch, combination slip-joint (6).

Saws, crosscut, 20-inch (18).

Saws, crosscut, 2-man, 6-foot (1).

Saws, crosscut, 2-man, 7-foot (1).

Saws, key-hole (3).

Saws, rip, 20-inch (6).

Shovels, short D handle, round-point (6).

Shovels, short D handle, square-point (6).

Squares, carpenter's, steel (4).

Spades, short D handle (6).

Try-squares, 8-inch blade (2).

Tapes, woven, 50-foot (6).

Wrenches, monkey, 8-inch (12).

Wrenches, monkey, 10-inch (12).

Wrenches, structural, open at one end, pointed at the other; for $\frac{3}{8}$ -inch bolts (10), for $\frac{1}{2}$ -inch bolts (20), for $\frac{3}{4}$ -inch bolts (20).

Wrenches, Stillson, 12-inch (4); 8-inch (4).

Towers, steel, Navy standard sectional 4-post, for triangulation, usual sizes 60-, 80-, and 100-foot. All metal parts galvanized. Complete, with 5-foot platform, cast-steel footplates, bolts, and nuts (15 towers).

Spare sets of bolts and nuts (10), footplates (4), and tie rods (4).

Wire, iron, galvanized, 0.135-inch diameter, in 50-pound coils (20 coils), for guys.

Wire, seizing, galvanized, 0.125-inch diameter, in 1,500-foot coils (2 coils).

LUMBER

It is difficult to specify the quantity of lumber required, unless the area and character of the region to be charted is known in advance. The following are some of the more important pieces generally used:

3 by 3 inches by 30 feet, 4 by 4 inches by 40 feet, $2\frac{1}{2}$ by $2\frac{1}{2}$ inches by 18 feet; select Douglas fir, rough, for legs and flagpoles of signal tripods.

1 by 4 inches by 16 feet, 1 by 12 inches by 16 feet; select Douglas fir, rough, for tripod braces, tower foundations and general purposes.

2 by 4 inches by 16 feet, 3 by 6 inches by 16 feet; select Douglas fir, rough, for floating signals.

$\frac{3}{4}$ by $1\frac{1}{2}$ inches by 4 feet; laths for securing signal dressing.

$\frac{1}{2}$ by 6 inches by 12 feet; white pine, dressed four sides, for signal targets.

Men landing structural steel and erecting towers on low muddy shores will need an extra quantity of wide boards for approaches and working platforms.

FOR FIELD PLOTTING

Divider sets, boat.—Wooden case containing one 5-inch pair of dividers, one bow-spring pair of dividers with replaceable steel points, and a complete extra set of steel points (6).

Drawing boards for boats, $\frac{3}{4}$ by 31 by 43 inches, white pine, both sides flush, mortised cleat at each end, shellacked over-all (6).

Parallel rulers, 18-inch (2).

Plane table.—One complete outfit, with alidade, drawing board, tripod, compass, and adjusting tools. Cases for instrument and drawing board. Umbrella.

Protractors, 3-arm metal.—Diameter of limb 6 inches; graduations, every half degree on silver, degrees numbered 0 to 180 both ways from the zero arm on the silver ring and 360 to 180 both ways from the zero arm on the brass ring of smaller diameter adjoining, every tenth degree numbered. Two adjustable verniers graduated on silver flush with the limb to read to minutes, without division marks outside of those for 0 and 30 minutes. Graduations to be easily discernible to the naked eye. Attached reading glass pivoting at center. Center well not less than $\frac{1}{8}$ inch in diameter. Length of arms 18 inches; of extensions, $13\frac{1}{2}$ inches. Punch center, cross-wire center, and pencil-point center, screw driver, small reading glass. Fitted in substantial box with brass hinges and fittings. Average quantity 16, 8 left-hand and 8 right-hand.

Protractors, 3-arm xylonite.—Fixed center arm and two movable arms fitted with clamps; length of arms, $13\frac{1}{2}$ inches. Limb graduated in degrees, every tenth degree numbered from 0 to 180 both ways from the zero arm, and the left-hand semicircle also from 180 to 360. Verniers reading to 2 minutes. In rigid cloth-covered cases (16).

CLOCKS

Alarm clocks for boats and camping parties (12).

Chronometer.—Navy standard ship's, with small rate, for survey use distant from the ship (1).

Wrist watches, for sounding and field service; 7 jewels, hand and dial for seconds, Arabic numerals, unbreakable crystal, leather wrist bands (18).

FOR OFFICE PLOTTING

Adhesives.—Glue, 6 bottles, about 2-ounce.

Scotch drafting tape, 1 inch wide, 6 rolls, 72 yards per roll, paper.

Scotch drafting tape, $\frac{3}{4}$ inch wide, 12 rolls, 10 yards per roll, transparent cellulose.

White rubber cement, in 1-pint cans (3).

Beam compasses.—Tubular bar 10 inches long in leather case, with micrometer slow-motion screw and two tapering steel points. Extra attachments, a pencil-lead holder, a line pen, and a needle-point holder. In leather case (2).

Beam compass sets for wooden beams, with micrometer slow-motion screw and two tapering steel points, also extra holders for pencil leads, line pen, and replaceable steel points. In leather case (2).

Beams, wooden, to fit the foregoing, lengths 24, 36, 48, and 60 inches.

Blueprint frame with felt pad and glass, 24 by 36 inches.

Blueprint paper, four rolls, each 10 yards, 24 inches wide, sensitized for 1-minute sun printing. For tropical use, arrangements should be made for partial orders by mail.

Blueprint tube.—Tin, about $2\frac{1}{2}$ inches in diameter and 26 inches long, for holding blueprint paper in rolls 24 inches by 10 yards.

Brushes for dusting drawings (2).

Cellulose acetate sheets.—Each 40 by 50 inches, 0.008 inch thick, transparent, one side dull, the other smooth (30).

Cross-section paper, 1 roll, 50 yards, 20 inches wide, 10 by 10 green rules to the inch.

Curves, irregular, 1 set.

Drawing boards, for ship and for smooth plotting; of white pine, first quality, $1\frac{1}{2}$ by 48 by 72 inches, compounded of strips not wider than 6 inches securely glued, with expansion cleats at both ends, shellacked over-all (6). Trestles for the foregoing, rigidly constructed, 38 inches long, 36 inches high.

Drawing instrument sets (3), precision quality for drafting, with replaceable steel points. In leather case to contain the following:

One 5-inch hairspring dividers, replaceable points.

One $5\frac{1}{2}$ -inch compass with lengthening bar and pen attachment.

One each, bow dividers, bow pen, bow pencil, $4\frac{1}{2}$ inches over-all, center-wheel adjustment for spacing, replaceable steel points.

Two ruling pens, 4-inch and 5-inch.

One drop-spring pen with pencil attachment.

One complete spare set of steel points to be furnished with each set.

Drawing paper, mounted, 36 sheets, extra heavy, best obtainable quality, for projections, size 42 by 58 inches, packed flat (not rolled) in tin case with waterproof soldered joints, and securely crated for shipment.

Drawing paper, mounted, cream, in 20-yard rolls, 42 inches wide (4 rolls).

Drawing paper.—One roll, 20 yards, 30 inches wide, medium weight, smooth surface.

Eraser heads, for pencils, pointed ends, soft red rubber (3 gross).

Erasers, gum, for cleaning, about $1\frac{1}{4}$ by $1\frac{1}{4}$ by $2\frac{1}{2}$ inches (3 dozen).

Erasers, ink, soft, about $2\frac{3}{4}$ by $1\frac{1}{8}$ by $1\frac{1}{8}$ inches (3 dozen).

Erasers, pencil, soft, about 2 by 1 by $\frac{3}{4}$ inches (3 dozen).

Erasing shields, transparent celluloid, $2\frac{3}{8}$ by $3\frac{3}{4}$ inches (6).

Ink holders, for holding ink bottles; with lever, pivot, and bottle alined (6).

Inks, waterproof drawing, in $\frac{3}{4}$ -ounce bottles:

Black, 4 dozen; blue, 2 dozen; brown, 2 dozen; carmine, 2 dozen; green, 2 dozen; orange, 1 dozen.

Lettering sets, templates for vertical and slanting letters and numerals as follows:

Capitals and numerals for heights 0.5, 0.425, 0.35, 0.29, 0.24, 0.2, 0.175, 0.14, 0.12, and 0.09 inch.

Lower case letters and numerals, for heights 0.25 and 0.185 inch, corresponding capitals to be included (1 complete set).

Pens for the foregoing, 2 complete sets.

Memorandum pads, plain, 3 by 5 inches, white, 100 sheets each (12).

Memorandum pads, ruled, 8 by $10\frac{1}{2}$ inches, white, each 100 sheets (12).

Oil, for instruments, sewing machine, and typewriter, 4 bottles.

Oilstones, Arkansas. Two about 1 by 3 inches in wooden cases; two knife-blade.

Oil, for removing corrosion, 4 quarts.

Pantograph, precision, metal arms graduated in millimeters for setting at any decimal ratio, suspension arm clamp for attaching to drawing board, pivot-standard, weight, suspension wires, pulley cord, level, prick point and pencil points in special spring-release attachments.

Pencils, drawing—

1H, 2 gross, ordinary quality, for recording.

2H, 2 gross, ordinary quality, for recording.

2H, 3 dozen, best quality, for plotting.

3H, 6 dozen, best quality, for plotting.

4H, 6 dozen, best quality, for plotting.

6H, 4 dozen, best quality, for plotting.

8H, 3 dozen, best quality, for plotting.

Pencils, china and glass-marking, black, paper-covered, 6 dozen.

Pencils, mechanical, Navy standard, 5 dozen. Leads for the same, 3 dozen each of red, green, yellow, blue, and 2H black.

Pencil pointers, sandpaper on wooden handles, $1\frac{1}{4}$ by 4 inches, 3 dozen.

Pencil sharpeners, rotary, 2.

Pen points, of fineness equal to Gillott's numbered as follows: no. 170, 3 gross; no. 303, 1 gross; no. 404, 1 gross; no. 290, 1 gross.

Pens, crow-quill, short-point, steel, and holder, 6 cards.

Penholders, cork-tip, tapered, 6 dozen.

Planimeter, polar, for measuring areas.

Pounce, in 5-ounce cans, 3 cans.

Proportional dividers (3), German silver, 10-inch, graduations and vernier, rack adjustment; in wooden case, with table of ratios.

Protractors, circular, diameter 8 inches, xylonite, graduated to half degrees, numbered 0 to 360 clockwise (4).

Protractor,¹ triangulation, having one arm, a single piece of metal 36 inches long; limb divided to 10 minutes on silver, and degrees numbered clockwise 0 to 360; vernier flush with limb, reading to 10 seconds.

Scale, architect's, 12 inches long, boxwood, graduations on white celluloid, various scales.

Scales, diagonal, metric, of German silver, blank 1 meter long in addition to the diagonally divided part, section $\frac{3}{4}$ by $2\frac{1}{2}$ inches, edges squared and straight. One each of the following:

1 : 5000 and 1 : 15000 scales on opposite faces.

1 : 10000 and 1 : 20000 on opposite faces.

1 : 30000 and 1 : 60000 on opposite faces.

Scale, diagonal, centimeters, of German silver, 1:10000, consisting of 26 centimeters, of which the left-hand centimeter is divided diagonally; on one face only; for base-line work.

Shears, office, 12-inch, japanned handle (2).

Spacing dividers, eleven-point, minimum space 0.1 inch (6).

Spline weights, felt-covered on contact surface (10).

Stereoscope, portable magnifying type, compact and improved design, effective stereoscope base approximately 10 inches.

Straightedges, of stainless steel, polished smooth, squared edges, with $\frac{3}{8}$ -inch hole drilled on center line 3 inches from each end, in hinged wooden case fastened with hook and eye at each end, (5) as follows:

Two $\frac{3}{4}$ by 3 by 72 inches.

One $\frac{3}{4}$ by 3 by 48 inches.

Two $\frac{3}{4}$ by $2\frac{1}{2}$ by 42 inches.

Stools, draftsman's, cane seat and back (6).

Thumbtacks, nickel-silver, $\frac{1}{2}$ -inch, 4 gross.

Tracing cloth, dull-back, 36-inch, 1 roll, 24 yards.

Tracing cloth, dull-back, 42-inch, 1 roll, 42 yards.

Tracing paper, extra transparent, thin, and tough, 42-inch, 6 rolls, each 24 yards.

Triangles, steel, stainless, polished surfaces, no buttons, squared edges, open centers. Two 8-inch, 30 by 60 degrees, two 10-inch, 30 by 60 degrees.

Triangles, zylonite:

30 by 60 degrees: twelve 4-inch, four 6-inch, two 8-inch, and two 16-inch.

45 degrees: two 4-inch, two 6-inch, two 8-inch, and two 10-inch.

Water-color outfits (2) with 12 colors including Chinese white, and camel's hair brushes numbers 1, 3, 5, 7, and 10.

FOR SOUNDING WORK

Binoculars, prismatic, 7 by 50, Navy standard, in leather case (12).

Bunting, wool, 19 inches wide, for marking lead lines; 10 yards each of white, Navy blue, red, green, and orange.

Compasses, magnetic, Navy standard $7\frac{1}{4}$ -inch liquid, in case with gimbals and removable top (10).

Drawing boards. See **Field Plotting**.

Lead lines, 1,500 fathoms tiller rope in coils of 60 or 120 fathoms, no. 8 cotton braided, $\frac{3}{4}$ inch in diameter, with phosphor bronze core, stranded, waterproof.

Lead line shackles, 100 shackles, $\frac{3}{8}$ -inch, galvanized.

Lead line thimbles, 100 thimbles, $\frac{1}{8}$ -inch, galvanized.

Leather, rigging, light, $\frac{3}{4}$ - to $\frac{1}{2}$ -inch, for marking lead lines (20 pounds).

Lunch kits, light metal box with handle, with pint thermos bottle (12). Thermos jars, 1-gallon, unbreakable type.

Needles, wax, and twine, for lead lines.

12 packages sail needles no. 16, 3 pounds beeswax, 6 balls sail twine, flax.

Sounding leads, hand, with cupped ends for bottom specimens.

40 9-pound and 100 14-pound.

Sounding leads, deep-sea, with inset rods and cupped ends, 30- to 50-pound (25).

Sounding machines, hand, standard Navy type (2).

¹ 3-arm protractors are listed under **Field Plotting**.

Sounding machine, power, standard Navy type.

Sounding tubes, pressure, ground glass (60).

Receptacles for same (25).

Driers for same, electric hair drier or bicycle pump.

Scales for same, reading to 100 fathoms (4).

Sounding wire, steel, galvanized, in 1,800-foot coils, 10 coils.

Tallow, for arming leads.

Whistles, of celluloid or bakelite, police type, for recorders' use in sounding.

Wire drags, at least one short drag and one long drag, all parts provided and stowed. See drawing and descriptions. See **Sounding**.

CAMPING ACCESSORIES

Blankets, rubber, of duck, rubberized on one side, 4 by 8 feet, with 3 grommets on each edge (24).

Canteens, Navy standard landing force canteens (30).

Clothing, assorted sizes for crew:

Boots, hip rubber, 30 pairs.

Shirts, khaki, Marine Corps type, 150.

Sweat shirts, 100.

Shoes, Marine Corps type, 150 pairs.

Trousers, khaki, Marine Corps type, 150 pairs.

Cots, canvas, folding, 40.

Lights, *lumber*, and *medicines* from the general supply.

Mosquito nets, for cots, Marine Corps type, 50.

Stoves, gas; also compressed gas in tanks (6).

Tents, two, complete with flies, 6 feet 11 inches by 8 feet 4 inches by 6 feet 10 inches high.

Tools for clearing and trenching, from general supply. See **Survey Signals**.

RECORD BOOKS AND FORMS

Level books, leather-covered, $4\frac{1}{2}$ by $6\frac{1}{2}$ inches, 18 lines and 6 columns to the page (4 dozen).

LMZ or geodetic evaluation forms (200).

Loose-leaf sheets, 500 price-book sheets, $7\frac{1}{2}$ by $4\frac{1}{2}$ inches, 34 horizontal rules to the page, 3 holes for the following binders:

3 stiff canvas loose-leaf binders.

300 loose-leaf sheets, 11 by $8\frac{1}{2}$ inches, cross sectioned in blue 5 to the inch on both sides, muslin strip at binder edge, 3 holes to fit the following:

3 stiff canvas loose-leaf binders.

Sounding books, 200, Hydrographic Office standard.

Sounding book labels, 200.

Tide gage record paper, in rolls $13\frac{1}{2}$ inches by 66 feet, 24 rolls.

Tide records, 200 tide record forms.

Transit books, leather-covered, $4\frac{1}{2}$ by $7\frac{1}{2}$ inches, 26 ruled lines to the page, 1 vertical rule down the center of the page or 5 vertical rules equally spaced. Right-hand page may be like left-hand page or cross sectioned (6).

Wire drag sheets, 600; with 6 binders for the same.

MISCELLANEOUS

Cameras, roll-film, $2\frac{1}{2}$ by $4\frac{1}{2}$ inches, F6.3 anastigmat lens, bulb release shutter, in leather carrying case (2).

24 rolls of film, 6-exposure, for the foregoing, each roll sealed in airtight metal case, for use in tropics.

Outboard motors, 8 horsepower, with 3 spare screws and other spares as may be advisable (2).

Pocket compasses, *magnetic*, with sight vane and reflecting prism, reading 0° to 360° ; in leather case (2).

Typewriter, for use at the working base.

APPENDIX III.—MISCELLANEOUS TABLES

TABLE 22.—Chords, radius 10,000

	0°	1°	2°	3°	4°	5°	6°	7°
00	0.00	174.53	349.05	523.54	697.99	872.39	1,046.72	1,220.97
01	2.91	177.44	351.95	526.44	700.80	875.29	1,049.62	1,223.87
02	5.82	180.35	354.86	529.35	703.80	878.20	1,052.52	1,226.77
03	8.73	183.26	357.77	532.26	706.71	881.10	1,055.43	1,229.68
04	11.64	186.17	360.68	535.17	709.62	884.01	1,058.33	1,232.58
05	14.55	189.07	363.59	538.08	712.52	886.91	1,061.24	1,235.48
06	17.45	191.98	366.50	540.98	715.43	889.82	1,064.14	1,238.39
07	20.37	194.89	369.41	543.89	718.34	892.73	1,067.05	1,241.29
08	23.27	197.80	372.31	546.80	721.26	895.63	1,069.95	1,244.19
09	26.18	200.70	375.22	549.70	724.15	898.54	1,072.86	1,247.10
10	29.08	203.61	378.13	552.61	727.06	901.44	1,075.76	1,250.00
11	31.99	206.52	381.04	555.52	729.96	904.35	1,078.67	1,252.92
12	34.90	209.43	383.95	558.43	732.87	907.26	1,081.57	1,255.81
13	37.81	212.34	386.85	561.34	735.78	910.16	1,084.48	1,258.71
14	40.72	215.25	389.76	564.25	738.69	913.07	1,087.38	1,261.61
15	43.63	218.16	392.67	567.15	741.59	915.97	1,090.29	1,264.52
16	46.54	221.07	395.58	570.06	744.50	918.88	1,093.19	1,267.42
17	49.45	223.98	398.49	572.97	747.41	921.78	1,096.10	1,270.32
18	52.36	226.89	401.40	575.88	750.32	924.69	1,099.00	1,273.23
19	55.27	229.79	404.30	578.78	753.22	927.59	1,101.90	1,276.13
20	58.18	232.70	407.21	581.69	756.13	930.50	1,104.81	1,279.03
21	61.09	235.61	410.12	584.60	759.03	933.41	1,107.71	1,281.94
22	64.00	238.52	413.03	587.51	761.94	936.31	1,110.62	1,284.84
23	66.91	241.43	415.94	590.42	764.85	939.22	1,113.52	1,287.74
24	69.82	244.34	418.84	593.33	767.75	942.12	1,116.43	1,290.64
25	72.72	247.25	421.75	596.23	770.66	945.03	1,119.33	1,293.55
26	75.63	250.16	424.66	599.14	773.56	947.94	1,122.23	1,296.45
27	78.54	253.06	427.57	602.05	776.47	950.84	1,125.14	1,299.35
28	81.45	255.97	430.48	604.95	779.38	953.75	1,128.04	1,302.25
29	84.36	258.88	433.39	607.86	782.29	956.65	1,130.95	1,305.15
30	87.27	261.79	436.29	610.77	785.19	959.56	1,133.85	1,308.06
31	90.17	264.70	439.20	613.68	788.10	962.46	1,136.76	1,310.96
32	93.08	267.61	442.11	616.58	791.01	965.37	1,139.66	1,313.87
33	95.99	270.52	445.02	619.49	793.92	968.27	1,142.57	1,316.77
34	98.90	273.43	447.93	622.40	796.82	971.18	1,145.47	1,319.67
35	101.81	276.33	450.84	625.31	799.73	974.09	1,148.37	1,322.57
36	104.72	279.24	453.74	628.21	802.64	976.99	1,151.28	1,325.47
37	107.63	282.15	456.65	631.12	805.54	979.90	1,154.18	1,328.38
38	110.54	285.06	459.56	634.03	808.45	982.80	1,157.09	1,331.28
39	113.44	287.97	462.47	636.94	811.35	985.71	1,159.99	1,334.18
40	116.35	290.88	465.38	639.84	814.26	988.61	1,162.89	1,337.09
41	119.26	293.78	468.28	642.75	817.17	991.52	1,165.80	1,339.99
42	122.17	296.69	471.19	645.66	820.07	994.42	1,168.70	1,342.89
43	125.08	299.60	474.10	648.57	822.97	997.33	1,171.60	1,345.79
44	127.99	302.52	477.01	651.47	825.88	1,000.23	1,174.51	1,348.69
45	130.90	305.42	479.92	654.38	828.79	1,003.14	1,177.41	1,351.60
46	133.81	308.33	482.83	657.28	831.70	1,006.05	1,180.31	1,354.50
47	136.72	311.23	485.74	660.19	834.61	1,008.95	1,183.22	1,357.40
48	139.63	314.14	488.64	663.10	837.51	1,011.85	1,186.12	1,360.30
49	141.53	317.05	491.55	666.01	840.42	1,014.76	1,189.03	1,363.21
50	145.44	319.96	494.46	668.92	843.32	1,017.66	1,191.93	1,366.11
51	148.35	322.87	497.37	671.82	846.23	1,020.57	1,194.83	1,369.01
52	151.26	325.78	500.27	674.73	849.13	1,023.48	1,197.74	1,371.91
53	154.17	328.69	503.18	677.64	852.04	1,026.38	1,200.64	1,374.81
54	157.08	331.60	506.09	680.55	854.95	1,029.29	1,203.54	1,377.71
55	159.99	334.50	509.00	683.45	857.85	1,032.19	1,206.45	1,380.62
56	162.90	337.41	511.91	686.36	860.76	1,035.10	1,209.35	1,383.52
57	165.80	340.32	514.81	689.27	863.67	1,038.00	1,212.26	1,386.42
58	168.71	343.23	517.72	692.17	866.57	1,040.90	1,215.16	1,389.32
59	171.62	346.14	520.63	695.08	869.48	1,043.81	1,218.06	1,392.22

10'' = 0.48, 20'' = 0.97, 30'' = 1.45.

TABLE 22.—*Chords, radius 10,000—Continued*

	8°	9°	10°	11°	12°	13°	14°	15°
00	1, 395. 13	1, 569. 18	1, 743. 11	1, 916. 91	2, 090. 57	2, 264. 07	2, 437. 38	2, 610. 53
01	98. 03	72. 08	46. 01	19. 80	93. 46	66. 96	40. 26	13. 41
02	1, 400. 93	74. 88	48. 91	22. 70	96. 35	69. 85	43. 15	16. 29
03	03. 83	77. 88	51. 81	25. 60	99. 24	72. 74	46. 04	19. 17
04	06. 73	80. 78	54. 70	28. 49	2, 102. 14	75. 63	48. 92	22. 06
05	09. 64	83. 68	57. 60	31. 39	05. 03	78. 52	51. 81	24. 94
06	12. 54	86. 58	60. 50	34. 28	07. 92	81. 41	54. 70	27. 82
07	15. 44	89. 48	63. 40	37. 18	10. 82	84. 30	57. 58	30. 71
08	18. 34	92. 38	66. 30	40. 08	13. 71	87. 19	60. 47	33. 59
09	21. 24	95. 28	69. 19	42. 97	16. 60	90. 08	63. 36	36. 47
10	24. 15	98. 18	72. 09	45. 87	19. 50	92. 97	66. 25	39. 36
11	1, 427. 05	1, 601. 08	1, 774. 99	1, 948. 76	2, 122. 39	2, 295. 86	2, 469. 13	2, 642. 24
12	29. 95	03. 98	77. 89	51. 66	25. 28	98. 75	72. 02	45. 12
13	32. 85	06. 88	80. 78	54. 55	28. 17	2, 301. 64	74. 91	48. 00
14	35. 75	09. 78	83. 68	57. 45	31. 06	04. 53	77. 79	50. 89
15	38. 66	12. 67	86. 58	60. 34	33. 96	07. 42	80. 68	53. 77
16	41. 56	15. 57	89. 47	63. 24	36. 85	10. 31	83. 57	56. 65
17	44. 46	18. 47	92. 37	66. 13	39. 74	13. 20	86. 45	59. 54
18	47. 36	21. 37	95. 27	69. 03	42. 63	16. 09	89. 34	62. 42
19	50. 26	24. 27	98. 16	71. 92	45. 52	18. 98	92. 23	65. 30
20	53. 16	27. 17	1, 801. 06	74. 82	48. 42	21. 87	95. 12	68. 19
21	1, 456. 07	1, 630. 07	1, 803. 96	1, 977. 71	2, 151. 31	2, 324. 75	2, 498. 00	2, 671. 07
22	58. 97	32. 97	06. 86	80. 60	54. 20	27. 64	2, 500. 89	73. 95
23	61. 87	35. 87	09. 75	83. 50	57. 09	30. 53	03. 77	76. 83
24	64. 77	38. 76	12. 65	86. 39	59. 98	33. 42	06. 66	79. 71
25	67. 67	41. 66	15. 55	89. 29	62. 88	36. 31	09. 55	82. 60
26	70. 57	45. 56	18. 44	92. 18	65. 77	39. 19	12. 43	85. 48
27	73. 47	47. 46	21. 34	95. 07	68. 66	42. 08	15. 32	88. 36
28	76. 37	50. 36	24. 24	97. 97	71. 55	44. 97	18. 21	91. 24
29	79. 27	53. 26	27. 13	2, 000. 86	74. 44	47. 86	21. 10	94. 12
30	82. 17	56. 16	30. 03	03. 76	77. 34	50. 75	23. 99	97. 01
31	1, 485. 07	1, 659. 06	1, 832. 92	2, 006. 65	2, 180. 23	2, 353. 63	2, 526. 87	2, 699. 89
32	87. 97	61. 96	35. 82	09. 54	83. 12	56. 52	29. 76	2, 702. 77
33	90. 88	64. 86	38. 72	12. 44	86. 01	59. 41	32. 64	05. 65
34	93. 78	67. 76	41. 61	15. 33	88. 90	62. 30	35. 53	08. 53
35	96. 68	70. 65	44. 51	18. 23	91. 79	65. 19	38. 41	11. 42
36	99. 58	73. 55	47. 40	21. 12	94. 68	68. 07	41. 30	14. 30
37	1, 502. 48	76. 45	50. 30	24. 01	97. 57	70. 96	44. 18	17. 18
38	05. 38	79. 35	53. 20	26. 91	2, 200. 46	73. 85	47. 07	20. 06
39	08. 28	82. 25	56. 09	29. 80	03. 35	76. 74	49. 95	22. 94
40	11. 18	85. 15	58. 99	32. 70	06. 25	79. 63	52. 84	25. 83
41	1, 514. 08	1, 688. 05	1, 861. 88	2, 035. 59	2, 209. 14	2, 382. 51	2, 555. 72	2, 728. 71
42	16. 98	90. 95	64. 78	38. 48	12. 03	85. 40	58. 61	31. 59
43	19. 88	93. 84	67. 68	41. 38	14. 92	88. 29	61. 49	34. 47
44	22. 78	96. 74	70. 58	44. 27	17. 81	91. 18	64. 38	37. 35
45	25. 68	99. 64	73. 47	47. 17	20. 70	94. 07	67. 26	40. 24
46	28. 58	1, 702. 54	76. 37	50. 06	23. 59	96. 95	70. 15	43. 12
47	31. 48	05. 44	79. 26	52. 95	26. 48	99. 84	73. 03	46. 00
48	34. 38	08. 33	82. 16	55. 85	29. 37	2, 402. 73	75. 92	48. 88
49	37. 28	11. 23	85. 06	58. 74	32. 26	05. 62	78. 80	51. 76
50	40. 18	14. 13	87. 96	61. 14	35. 16	08. 51	81. 69	54. 65
51	1, 543. 08	1, 717. 03	1, 890. 85	2, 064. 53	2, 238. 05	2, 411. 39	2, 584. 57	2, 757. 53
52	45. 98	19. 93	93. 75	67. 42	40. 94	14. 28	87. 45	60. 41
53	48. 88	22. 83	96. 64	70. 31	43. 83	17. 17	90. 34	63. 29
54	51. 78	25. 72	99. 54	73. 21	46. 72	20. 05	93. 22	66. 17
55	54. 68	28. 62	1, 902. 43	76. 10	49. 61	22. 94	96. 11	69. 05
56	57. 58	31. 52	05. 33	78. 99	52. 50	25. 83	98. 99	71. 93
57	60. 48	34. 42	08. 22	81. 89	55. 39	28. 71	2, 601. 87	74. 81
58	63. 38	37. 32	11. 12	84. 78	58. 28	31. 60	04. 76	77. 69
59	66. 28	40. 21	14. 01	87. 67	61. 17	34. 49	07. 64	80. 57

10'' = 0.48, 20'' = 0.96, 30'' = 1.44.

TABLE 22.—*Chords, radius 10,000—Continued*

	16°	17°	18°	19°	20°	21°	22°	23°
00.....	2, 783. 46	2, 956. 19	3, 128. 68	3, 300. 95	3, 472. 96	3, 644. 71	3, 816. 18	3, 987. 35
01.....	86. 34	59. 06	31. 55	03. 82	75. 82	47. 57	19. 04	90. 20
02.....	89. 22	61. 94	34. 42	06. 69	78. 09	50. 43	21. 89	93. 05
03.....	92. 10	64. 82	37. 29	09. 55	81. 55	53. 29	24. 75	95. 90
04.....	94. 98	67. 69	40. 17	12. 42	84. 41	56. 15	27. 60	98. 75
05.....	97. 86	70. 57	43. 04	15. 29	87. 28	59. 01	30. 46	4, 000. 60
06.....	2, 800. 74	73. 45	45. 91	18. 16	90. 14	61. 87	33. 31	04. 45
07.....	03. 62	76. 32	48. 79	21. 03	93. 01	64. 73	36. 17	07. 30
08.....	06. 51	79. 20	51. 66	23. 90	95. 87	67. 59	39. 02	10. 15
09.....	09. 39	82. 08	54. 53	26. 77	98. 73	70. 45	41. 88	13. 00
10.....	12. 27	84. 96	57. 41	29. 64	3, 501. 60	73. 31	44. 73	15. 85
11.....	2, 815. 15	2, 987. 83	3, 160. 28	3, 332. 51	3, 504. 46	3, 676. 17	3, 847. 58	4, 018. 70
12.....	18. 03	90. 71	63. 15	35. 37	07. 32	79. 03	50. 44	21. 55
13.....	20. 91	93. 58	66. 02	38. 24	10. 19	81. 89	53. 29	24. 40
14.....	23. 79	96. 46	68. 89	41. 11	13. 05	84. 75	56. 15	27. 25
15.....	26. 67	99. 34	71. 77	43. 98	15. 91	87. 61	59. 00	30. 10
16.....	29. 55	3, 002. 21	74. 64	46. 85	18. 78	90. 46	61. 85	32. 94
17.....	32. 43	05. 09	77. 51	49. 71	21. 64	93. 32	64. 71	35. 79
18.....	35. 31	07. 96	80. 38	52. 58	24. 50	96. 18	67. 56	38. 64
19.....	38. 19	10. 84	83. 25	55. 45	27. 36	99. 04	70. 42	41. 49
20.....	41. 07	13. 72	86. 13	58. 32	30. 23	3, 701. 90	73. 27	44. 34
21.....	2, 843. 94	3, 016. 59	3, 189. 00	3, 361. 18	3, 533. 09	3, 704. 76	3, 876. 12	4, 047. 19
22.....	46. 82	19. 47	91. 87	64. 05	35. 95	07. 62	78. 98	50. 04
23.....	49. 70	22. 34	94. 74	66. 92	38. 82	10. 47	81. 83	52. 89
24.....	52. 58	25. 22	97. 62	69. 79	41. 68	13. 33	84. 68	55. 74
25.....	55. 46	28. 09	3, 200. 49	72. 65	44. 54	16. 19	87. 54	58. 59
26.....	58. 34	30. 97	03. 36	75. 52	47. 41	19. 05	90. 39	61. 43
27.....	61. 22	33. 84	06. 23	78. 39	50. 27	21. 91	93. 24	64. 28
28.....	64. 10	36. 72	09. 10	81. 25	53. 13	24. 76	96. 09	67. 13
29.....	66. 98	39. 59	11. 98	84. 12	56. 00	27. 62	98. 95	69. 98
30.....	69. 86	42. 47	14. 85	86. 99	58. 86	30. 48	3, 901. 80	72. 83
31.....	2, 872. 73	3, 045. 34	3, 217. 72	3, 389. 85	3, 561. 72	3, 733. 34	3, 904. 65	4, 075. 68
32.....	75. 61	48. 21	60. 59	92. 72	64. 58	36. 19	07. 50	78. 53
33.....	78. 49	51. 09	23. 46	95. 59	67. 45	39. 05	10. 36	81. 37
34.....	81. 37	53. 96	26. 33	98. 45	70. 31	41. 91	13. 21	84. 22
35.....	84. 25	56. 84	29. 20	3, 401. 32	73. 17	44. 77	16. 06	87. 07
36.....	87. 12	59. 51	32. 07	04. 18	76. 03	47. 62	18. 91	89. 92
37.....	90. 00	62. 58	34. 94	07. 05	78. 89	50. 48	21. 76	92. 77
38.....	92. 88	65. 46	37. 82	09. 92	81. 76	53. 34	24. 62	95. 61
39.....	95. 76	68. 33	40. 69	12. 78	84. 62	56. 19	27. 47	98. 46
40.....	98. 64	71. 21	43. 56	15. 65	87. 48	59. 05	30. 32	4, 101. 31
41.....	2, 901. 51	3, 074. 08	3, 246. 43	3, 418. 51	3, 590. 34	3, 761. 91	3, 933. 17	4, 104. 16
42.....	04. 39	76. 95	49. 30	21. 38	83. 20	64. 76	36. 02	07. 00
43.....	07. 27	79. 83	52. 17	24. 25	96. 07	67. 62	38. 88	09. 85
44.....	10. 15	82. 70	55. 04	27. 11	98. 93	70. 48	41. 73	12. 70
45.....	13. 03	85. 58	57. 91	29. 98	3, 601. 79	73. 34	44. 58	15. 55
46.....	15. 90	88. 45	60. 78	32. 84	04. 65	76. 19	47. 43	18. 39
47.....	18. 78	91. 32	63. 65	35. 71	07. 52	79. 05	50. 28	21. 24
48.....	21. 66	94. 20	66. 52	38. 58	10. 38	81. 91	53. 14	24. 09
49.....	24. 54	97. 07	69. 39	41. 44	13. 24	84. 76	55. 99	26. 93
50.....	27. 42	99. 95	72. 26	44. 31	16. 10	87. 62	58. 84	29. 78
51.....	2, 930. 29	3, 102. 82	3, 275. 13	3, 447. 17	3, 618. 96	3, 790. 48	3, 961. 69	4, 132. 63
52.....	33. 17	05. 69	78. 00	50. 04	21. 82	93. 33	64. 54	35. 47
53.....	36. 05	08. 56	80. 86	52. 90	24. 68	96. 19	67. 39	38. 32
54.....	38. 92	11. 44	83. 73	55. 77	27. 54	99. 04	70. 24	41. 16
55.....	41. 80	14. 31	86. 60	58. 63	30. 41	2, 801. 90	73. 10	44. 01
56.....	44. 68	17. 18	89. 47	61. 50	33. 27	04. 76	75. 95	46. 86
57.....	47. 55	20. 06	92. 34	64. 36	36. 13	07. 61	78. 80	49. 70
58.....	50. 43	22. 93	95. 21	67. 23	38. 99	10. 47	81. 65	52. 55
59.....	53. 31	25. 80	98. 08	70. 09	41. 85	13. 22	84. 50	55. 39

10'' = 0.48, 20'' = 0.96, 30'' = 1.43.

TABLE 22.—*Chords, radius 10,000—Continued*

	24°	25°	26°	27°	28°	29°	30°	31°
00.....	4, 158. 24	4, 328. 79	4, 499. 01	4, 668. 90	4, 838. 43	5, 007. 60	5, 176. 38	5, 344. 76
01.....	61. 09	31. 63	4, 501. 84	71. 73	41. 25	10. 42	79. 19	47. 57
02.....	63. 93	34. 47	04. 68	74. 56	44. 07	13. 23	82. 00	50. 38
03.....	66. 78	37. 30	07. 51	77. 38	46. 90	16. 05	84. 81	53. 18
04.....	69. 62	40. 15	10. 35	80. 21	49. 72	18. 86	87. 62	55. 98
05.....	72. 47	42. 98	13. 18	83. 04	52. 54	21. 68	90. 43	58. 78
06.....	75. 31	45. 82	16. 01	85. 87	55. 36	24. 50	93. 24	61. 58
07.....	78. 16	48. 66	18. 85	88. 70	58. 18	27. 31	96. 05	64. 38
08.....	81. 00	51. 50	21. 68	91. 52	61. 01	30. 13	98. 86	67. 18
09.....	83. 85	54. 34	24. 52	94. 35	63. 83	32. 94	5, 201. 67	69. 99
10.....	86. 69	57. 18	27. 35	97. 18	66. 65	35. 76	04. 48	72. 80
11.....	4, 189. 53	4, 360. 02	4, 530. 18	4, 700. 01	4, 869. 47	5, 038. 57	5, 207. 29	5, 375. 60
12.....	92. 38	62. 86	33. 02	02. 83	72. 29	41. 39	10. 10	78. 40
13.....	95. 22	65. 69	35. 85	05. 66	75. 11	44. 20	12. 90	81. 20
14.....	98. 07	68. 53	38. 68	08. 49	77. 93	47. 02	15. 70	84. 00
15.....	4, 200. 91	71. 37	41. 52	11. 13	80. 75	49. 83	18. 51	86. 80
16.....	03. 75	74. 21	44. 35	14. 14	83. 58	52. 65	21. 32	89. 60
17.....	06. 60	77. 05	47. 18	16. 97	86. 40	55. 46	24. 13	92. 40
18.....	09. 44	79. 88	50. 01	19. 80	89. 22	58. 28	26. 94	95. 20
19.....	12. 29	82. 72	52. 85	22. 62	92. 04	61. 09	29. 75	98. 00
20.....	15. 13	85. 56	55. 68	25. 45	94. 86	63. 91	32. 56	5, 400. 80
21.....	4, 217. 97	4, 388. 40	4, 558. 51	4, 728. 28	4, 897. 69	5, 066. 72	5, 235. 37	5, 403. 60
22.....	20. 82	91. 23	61. 34	31. 10	4, 900. 50	69. 54	38. 18	06. 40
23.....	23. 66	94. 07	64. 18	33. 93	03. 32	72. 35	40. 98	09. 20
24.....	26. 50	96. 91	67. 01	36. 75	06. 14	75. 16	43. 78	12. 00
25.....	29. 35	99. 75	69. 84	39. 58	08. 96	77. 97	46. 59	14. 81
26.....	32. 19	4, 402. 58	72. 67	42. 41	11. 78	80. 79	49. 40	17. 62
27.....	35. 03	05. 42	75. 50	45. 23	14. 60	83. 60	52. 21	20. 42
28.....	37. 87	08. 26	78. 34	48. 06	17. 42	86. 41	55. 02	23. 22
29.....	40. 72	11. 09	81. 17	50. 88	20. 24	89. 23	57. 82	26. 01
30.....	43. 56	13. 93	84. 00	53. 71	23. 06	92. 04	60. 62	28. 80
31.....	4, 246. 40	4, 416. 77	4, 586. 83	4, 756. 53	4, 925. 88	5, 094. 85	5, 263. 43	5, 431. 60
32.....	49. 24	19. 60	89. 66	59. 36	28. 70	97. 66	66. 24	34. 40
33.....	52. 09	22. 44	92. 49	62. 18	31. 52	5, 100. 48	69. 05	37. 20
34.....	54. 93	25. 28	95. 32	65. 01	34. 34	03. 29	71. 86	40. 00
35.....	57. 77	28. 12	98. 16	67. 83	37. 15	06. 10	74. 66	42. 80
36.....	60. 61	30. 95	4, 600. 99	70. 66	39. 97	08. 91	77. 46	45. 60
37.....	63. 45	33. 79	03. 82	73. 48	42. 79	11. 72	80. 27	48. 40
38.....	66. 30	36. 63	06. 65	76. 31	45. 61	14. 54	83. 08	51. 20
39.....	69. 14	39. 46	09. 48	79. 13	48. 43	17. 35	85. 88	54. 00
40.....	71. 98	42. 30	12. 31	81. 96	51. 25	20. 16	88. 68	56. 80
41.....	4, 274. 82	4, 445. 14	4, 615. 14	4, 784. 78	4, 954. 07	5, 122. 97	5, 291. 49	5, 459. 60
42.....	77. 66	47. 97	17. 97	87. 61	56. 89	25. 78	94. 30	62. 40
43.....	80. 50	50. 80	20. 80	90. 43	59. 70	28. 59	97. 10	65. 20
44.....	83. 34	53. 63	23. 63	93. 26	62. 52	31. 40	99. 90	68. 00
45.....	86. 19	56. 47	26. 46	96. 08	65. 34	34. 21	5, 302. 71	70. 79
46.....	89. 03	59. 31	29. 29	98. 90	68. 16	37. 03	05. 52	73. 58
47.....	91. 87	62. 14	32. 12	4, 801. 73	70. 98	39. 84	08. 32	76. 38
48.....	94. 71	64. 98	34. 95	04. 55	73. 79	42. 65	11. 12	79. 18
49.....	97. 55	67. 81	37. 78	07. 38	76. 61	45. 46	13. 93	81. 98
50.....	4, 360. 39	70. 65	40. 61	10. 20	79. 43	48. 27	16. 74	84. 78
51.....	4, 302. 23	4, 473. 49	4, 643. 44	4, 813. 02	4, 982. 25	5, 151. 08	5, 319. 54	5, 487. 58
52.....	06. 07	76. 32	46. 27	15. 85	85. 06	53. 89	22. 34	90. 38
53.....	08. 91	79. 16	49. 10	18. 67	87. 88	56. 70	25. 14	93. 17
54.....	11. 75	81. 99	51. 93	21. 49	90. 70	59. 51	27. 94	95. 96
55.....	14. 59	84. 83	54. 75	24. 31	93. 51	62. 32	30. 75	98. 76
56.....	17. 43	87. 67	57. 58	27. 13	96. 33	65. 13	33. 56	5, 501. 56
57.....	20. 27	90. 50	60. 41	29. 96	99. 15	67. 94	36. 36	04. 36
58.....	23. 11	93. 34	63. 24	32. 78	5, 001. 97	70. 75	39. 16	07. 16
59.....	25. 95	96. 17	66. 07	35. 61	04. 78	73. 56	41. 96	09. 95

10''=0.47, 20''=0.95, 30''=1.42, first part; 10''=0.47, 20''=0.93, 30''=1.40, second part.

TABLE 22.—*Chords, radius 10,000—Continued*

	32°	33°	34°	35°	36°	37°	38°	39°
00	5, 512. 74	5, 680. 30	5, 847. 44	6, 014. 12	6, 180. 34	6, 346. 10	6, 511. 36	6, 676. 14
01	15. 54	83. 09	50. 22	16. 89	83. 11	48. 86	14. 11	78. 88
02	18. 34	85. 88	53. 00	19. 66	85. 88	51. 62	16. 86	81. 62
03	21. 14	88. 67	55. 78	22. 44	88. 64	54. 37	19. 61	84. 36
04	23. 94	91. 46	58. 56	25. 22	91. 40	57. 12	22. 36	87. 10
05	26. 73	94. 25	61. 34	27. 99	94. 17	59. 88	25. 11	89. 84
06	29. 52	97. 04	64. 12	30. 76	96. 94	62. 64	27. 86	92. 58
07	32. 32	99. 83	66. 90	33. 53	99. 70	65. 40	30. 61	95. 32
08	35. 12	5, 702. 62	69. 68	36. 30	6, 202. 46	68. 16	33. 36	98. 06
09	37. 91	05. 41	72. 46	39. 08	05. 23	70. 92	36. 11	6, 700. 81
10	40. 70	08. 20	75. 24	41. 86	08. 00	73. 68	38. 86	03. 56
11	5, 543. 50	5, 710. 98	5, 878. 02	6, 044. 63	6, 210. 76	6, 376. 43	6, 541. 61	6, 706. 30
12	46. 30	13. 76	80. 80	47. 40	13. 52	79. 18	44. 36	09. 04
13	49. 09	16. 55	83. 58	50. 17	16. 29	81. 94	47. 11	11. 78
14	51. 88	19. 34	86. 36	52. 94	19. 06	84. 70	49. 86	14. 52
15	54. 68	22. 13	89. 14	55. 71	21. 82	87. 46	52. 61	17. 26
16	57. 48	24. 92	91. 92	58. 48	24. 58	90. 22	55. 36	20. 00
17	60. 27	27. 71	94. 70	61. 26	27. 35	92. 97	58. 10	22. 74
18	63. 06	30. 50	97. 48	64. 04	30. 12	95. 72	60. 84	25. 48
19	65. 85	33. 28	5, 900. 26	66. 81	32. 88	98. 48	63. 59	28. 22
20	68. 64	36. 06	03. 04	69. 58	35. 64	6, 401. 24	66. 34	30. 96
21	5, 571. 44	5, 738. 85	5, 905. 82	6, 072. 35	6, 238. 41	6, 403. 99	6, 569. 09	6, 733. 69
22	74. 24	41. 64	08. 60	75. 12	41. 18	06. 74	71. 84	36. 42
23	77. 03	44. 43	11. 38	77. 89	43. 94	09. 50	74. 59	39. 16
24	79. 82	47. 22	14. 16	80. 66	46. 70	12. 26	77. 34	41. 90
25	82. 61	50. 00	16. 94	83. 43	49. 46	15. 02	80. 08	44. 64
26	85. 40	52. 78	19. 72	86. 20	52. 22	17. 78	82. 82	47. 38
27	88. 20	55. 57	22. 50	88. 97	54. 99	20. 53	85. 57	50. 12
28	91. 00	58. 36	25. 28	91. 74	57. 76	23. 28	88. 32	52. 86
29	93. 79	61. 14	28. 06	94. 51	60. 52	26. 04	91. 07	55. 60
30	96. 58	63. 92	30. 84	97. 28	63. 28	28. 80	93. 82	58. 34
31	5, 599. 37	5, 766. 71	5, 933. 61	6, 100. 05	6, 266. 04	6, 431. 55	6, 596. 56	6, 761. 08
32	5, 602. 16	69. 50	36. 38	02. 82	68. 80	34. 30	99. 30	63. 82
33	04. 96	72. 28	39. 16	05. 59	71. 56	37. 05	6, 602. 05	66. 55
34	07. 76	75. 06	41. 94	08. 36	74. 32	39. 80	04. 80	69. 28
35	10. 55	77. 85	44. 72	11. 13	77. 09	42. 56	07. 54	72. 02
36	13. 34	80. 64	47. 50	13. 90	79. 86	45. 32	10. 28	74. 76
37	16. 13	83. 42	50. 28	16. 77	82. 62	48. 07	13. 03	77. 50
38	18. 92	86. 20	53. 06	19. 44	85. 38	50. 82	15. 78	80. 24
39	21. 71	88. 99	55. 83	22. 21	88. 14	53. 57	18. 52	82. 97
40	24. 50	91. 78	58. 60	24. 98	90. 90	56. 32	21. 26	85. 70
41	5, 627. 29	5, 794. 56	5, 961. 38	6, 127. 75	6, 293. 66	6, 459. 08	6, 624. 01	6, 788. 44
42	30. 08	97. 34	64. 16	30. 52	96. 42	61. 84	26. 76	91. 18
43	32. 87	5, 800. 13	66. 94	33. 29	99. 18	64. 59	29. 50	93. 92
44	35. 66	02. 92	69. 72	36. 06	6, 301. 94	67. 34	32. 24	96. 66
45	38. 45	05. 70	72. 49	38. 83	04. 70	70. 09	34. 99	99. 39
46	41. 24	08. 48	75. 26	41. 60	07. 46	72. 84	37. 74	6, 802. 12
47	44. 04	11. 26	78. 04	44. 37	10. 22	75. 59	40. 48	04. 86
48	46. 84	14. 04	80. 82	47. 14	12. 98	78. 34	43. 22	07. 60
49	49. 63	16. 83	83. 59	49. 90	15. 74	81. 10	45. 97	10. 33
50	52. 42	19. 62	86. 36	52. 66	18. 50	83. 86	48. 72	13. 06
51	5, 655. 21	5, 822. 40	5, 989. 14	6, 155. 43	6, 321. 26	6, 486. 61	6, 651. 46	6, 815. 80
52	58. 00	25. 18	91. 92	58. 20	24. 02	89. 36	54. 20	18. 54
53	60. 79	27. 96	94. 69	60. 97	26. 78	92. 11	56. 94	21. 27
54	63. 58	30. 74	97. 46	63. 74	29. 54	94. 86	59. 68	24. 00
55	66. 37	33. 52	6, 000. 24	66. 51	32. 30	97. 61	62. 42	26. 73
56	69. 16	36. 30	03. 02	69. 28	35. 06	6, 500. 36	65. 16	29. 46
57	71. 94	39. 09	05. 79	72. 04	37. 82	03. 11	67. 91	32. 20
58	74. 72	41. 88	08. 56	74. 80	40. 58	05. 86	70. 66	34. 94
59	77. 51	44. 66	11. 34	77. 57	43. 34	08. 61	73. 40	37. 67

10''=0.46, 20''=0.93, 30''=1.39, first part; 10''=0.46, 20''=0.92, 30''=1.38, second part.

TABLE 22.—Chords, radius 10,000—Continued

	40°	41°	42°	43°	44°	45°	46°	47°
00	6,840.40	7,004.14	7,167.36	7,330.02	7,492.14	7,653.66	7,814.62	7,974.98
01	43.14	06.87	70.08	32.73	94.83	56.35	17.30	77.65
02	45.88	09.60	72.80	35.44	97.52	59.04	19.98	80.32
03	48.61	12.32	75.51	38.15	7,500.22	61.73	22.66	82.99
04	51.34	15.04	78.22	40.86	02.92	64.42	25.34	85.66
05	54.07	17.77	80.94	43.56	05.62	67.11	28.01	88.32
06	56.80	20.50	83.66	46.26	08.32	69.80	30.68	90.98
07	59.53	23.22	86.37	48.97	11.01	72.48	33.36	93.65
08	62.26	25.94	89.08	51.68	13.70	75.16	36.04	96.32
09	65.00	28.66	91.79	54.38	16.40	77.85	38.72	98.99
10	67.74	31.38	94.50	57.08	19.10	80.54	41.40	8,001.66
11	6,870.47	7,034.11	7,197.22	7,359.79	7,521.79	7,683.22	7,844.07	8,004.32
12	73.20	36.84	99.94	62.50	24.48	85.90	46.74	06.98
13	75.93	39.56	7,202.65	65.20	27.18	88.59	49.42	09.65
14	78.66	42.28	05.36	67.90	29.88	91.28	52.10	12.32
15	81.39	45.00	08.08	70.60	32.57	93.96	54.77	14.98
16	84.12	47.72	10.80	73.30	35.26	96.64	57.44	17.64
17	86.85	50.44	13.51	76.01	37.96	99.33	60.12	20.31
18	89.58	53.16	16.22	78.72	40.66	7,702.02	62.80	22.98
19	92.31	55.89	18.93	81.42	43.35	04.70	65.47	25.64
20	95.04	58.62	21.64	84.12	46.04	07.38	68.14	28.30
21	6,897.77	7,061.34	7,224.35	7,386.83	7,548.73	7,710.07	7,870.82	8,030.96
22	6,900.50	64.06	27.06	89.54	51.42	12.76	73.50	33.62
23	03.23	66.78	29.78	92.24	54.12	15.44	76.17	36.29
24	05.96	69.50	32.50	94.94	56.82	18.12	78.84	38.96
25	08.69	72.22	35.21	97.64	59.51	20.80	81.51	41.62
26	11.42	74.94	37.92	7,400.34	62.20	23.48	84.18	44.28
27	14.15	77.66	40.63	03.04	64.89	26.17	86.86	46.94
28	16.88	80.38	43.34	05.74	67.58	28.86	89.54	49.60
29	19.61	83.10	46.05	08.44	70.28	31.54	92.21	52.27
30	22.34	85.82	48.76	11.14	72.98	34.22	94.88	54.94
31	6,925.07	7,088.54	7,251.47	7,413.85	7,575.67	7,736.90	7,897.55	8,057.60
32	27.80	91.26	54.18	16.56	78.36	39.58	7,900.22	60.26
33	30.53	93.98	56.89	19.26	81.05	42.26	02.89	62.92
34	33.26	96.70	59.60	21.96	83.74	44.94	05.56	65.58
35	35.99	99.42	62.31	24.66	86.43	47.63	08.24	68.24
36	38.72	7,102.14	65.02	27.36	89.12	50.32	10.92	70.90
37	41.45	04.86	67.73	30.06	91.81	53.00	13.59	73.59
38	44.18	07.58	70.44	32.76	94.50	55.68	16.26	76.22
39	46.90	10.30	73.15	35.46	97.19	58.36	18.93	78.99
40	49.62	13.02	75.86	38.16	99.88	61.04	21.60	81.56
41	6,952.35	7,115.74	7,278.57	7,440.86	7,602.57	7,763.72	7,924.27	8,084.22
42	55.08	18.46	81.28	43.56	05.26	66.40	26.94	86.88
43	57.81	21.17	83.99	46.26	07.95	69.08	29.61	89.54
44	60.54	23.88	86.70	48.96	10.64	71.76	32.28	92.20
45	63.26	26.60	89.41	51.66	13.33	74.44	34.95	94.86
46	65.98	29.32	92.12	54.36	16.02	77.12	37.62	97.52
47	68.71	32.04	94.83	57.06	18.71	79.80	40.29	8,100.18
48	71.44	34.76	97.54	59.76	21.40	82.48	42.96	02.84
49	74.17	37.48	7,300.25	62.46	24.09	85.16	45.63	05.50
50	76.90	40.20	02.96	65.16	26.78	87.84	48.30	08.16
51	6,979.62	7,142.91	7,305.66	7,467.86	7,629.47	7,790.52	7,950.97	8,110.81
52	82.34	45.62	08.36	70.56	32.16	93.20	53.64	13.46
53	85.07	48.34	11.07	73.25	34.85	95.88	56.31	16.12
54	87.80	51.06	13.78	75.94	37.54	98.56	58.98	18.78
55	90.52	53.78	16.49	78.64	40.23	7,801.24	61.65	21.44
56	93.24	56.50	19.20	81.34	42.92	03.92	64.32	24.10
57	95.97	59.21	21.91	84.04	45.61	06.59	66.98	26.76
58	98.70	61.92	24.62	86.74	48.30	09.26	69.64	29.42
59	7,001.42	64.64	27.32	89.44	50.98	11.94	72.31	32.08

10''=0.45, 20''=0.91, 30''=1.36, first part; 10''=0.45, 20''=0.89, 30''=1.34 second part.

TABLE 22.—*Chords, radius 10,000—Continued*

	48°	49°	50°	51°	52°	53°	54°	55°
00	8, 134. 74	8, 293. 86	8, 452. 36	8, 610. 22	8, 767. 42	8, 923. 96	9, 079. 82	9, 234. 98
01	37. 39	96. 51	55. 00	12. 85	70. 04	26. 56	82. 41	37. 56
02	40. 04	99. 16	57. 64	15. 48	72. 66	29. 16	85. 00	40. 14
03	42. 70	8, 301. 81	60. 28	18. 10	75. 27	31. 76	87. 59	42. 72
04	45. 36	04. 46	62. 92	20. 72	77. 88	34. 36	90. 18	45. 30
05	48. 02	07. 10	65. 55	23. 35	80. 49	36. 97	92. 77	47. 88
06	50. 68	09. 74	68. 18	25. 98	83. 10	39. 58	95. 36	50. 46
07	53. 33	12. 39	70. 82	28. 60	85. 72	42. 18	97. 95	53. 03
08	55. 98	15. 04	73. 46	31. 22	88. 34	44. 78	9, 100. 54	55. 60
09	58. 64	17. 68	76. 09	33. 84	90. 95	47. 38	03. 13	58. 18
10	61. 30	20. 32	78. 72	36. 46	93. 56	49. 98	05. 72	60. 76
11	8, 163. 96	8, 322. 97	8, 481. 35	8, 639. 09	8, 796. 17	8, 952. 58	9, 108. 31	9, 263. 34
12	66. 62	25. 62	83. 98	41. 72	98. 78	55. 18	10. 90	65. 92
13	69. 27	28. 26	86. 62	44. 34	8, 801. 39	57. 78	13. 49	68. 50
14	71. 92	30. 90	89. 26	46. 96	04. 00	60. 38	16. 08	71. 08
15	74. 58	33. 55	91. 89	49. 58	06. 62	62. 98	18. 67	73. 66
16	77. 24	36. 20	94. 52	52. 20	09. 24	65. 58	21. 26	76. 24
17	79. 89	38. 84	97. 16	54. 83	11. 85	68. 18	23. 85	78. 81
18	82. 54	41. 48	99. 80	57. 46	14. 46	70. 78	26. 44	81. 38
19	85. 19	44. 13	8, 502. 43	60. 08	17. 07	73. 38	29. 02	83. 96
20	87. 84	46. 78	05. 06	62. 70	19. 68	75. 98	31. 60	86. 54
21	8, 190. 50	8, 349. 42	8, 507. 69	8, 665. 32	8, 822. 29	8, 978. 58	9, 134. 19	9, 289. 12
22	93. 16	52. 06	10. 32	67. 94	24. 90	81. 15	36. 78	91. 70
23	95. 81	54. 70	12. 95	70. 56	27. 51	83. 78	39. 37	94. 27
24	98. 46	57. 34	15. 58	73. 18	30. 12	86. 38	41. 96	96. 84
25	8, 201. 11	59. 98	18. 22	75. 80	32. 73	88. 98	44. 55	99. 42
26	03. 76	62. 62	20. 86	78. 42	35. 34	91. 58	47. 14	9, 302. 00
27	06. 42	65. 27	23. 49	81. 04	37. 95	94. 18	49. 72	04. 57
28	09. 08	67. 92	26. 12	83. 66	40. 56	96. 78	52. 30	07. 14
29	11. 73	70. 56	28. 75	86. 28	43. 17	99. 37	54. 89	09. 72
30	14. 38	73. 20	31. 38	88. 90	45. 78	9, 001. 96	57. 48	12. 30
31	8, 217. 03	8, 375. 84	8, 534. 01	8, 691. 52	8, 848. 39	9, 004. 56	9, 160. 07	9, 314. 87
32	19. 68	78. 48	36. 64	94. 15	51. 00	07. 16	62. 66	17. 44
33	22. 33	81. 12	39. 27	96. 76	53. 60	09. 76	65. 24	20. 01
34	24. 98	83. 76	41. 90	99. 38	56. 20	12. 36	67. 82	22. 58
35	27. 63	86. 40	44. 53	8, 702. 00	58. 81	14. 96	70. 41	25. 16
36	30. 28	89. 04	47. 16	04. 62	61. 42	17. 56	73. 00	27. 74
37	32. 94	91. 68	49. 79	07. 24	64. 03	20. 15	75. 58	30. 31
38	35. 60	94. 32	52. 42	09. 86	66. 64	22. 74	78. 16	32. 88
39	38. 25	96. 96	55. 05	12. 48	69. 25	25. 34	80. 75	35. 45
40	40. 90	99. 60	57. 68	15. 10	71. 86	27. 94	83. 34	38. 02
41	8, 243. 55	8, 402. 24	8, 560. 31	8, 717. 72	8, 874. 46	9, 030. 53	9, 185. 92	9, 340. 59
42	46. 20	04. 88	62. 94	20. 34	77. 06	33. 12	88. 50	43. 16
43	48. 85	07. 52	65. 57	22. 95	79. 67	35. 72	91. 08	45. 74
44	51. 50	10. 16	68. 20	25. 56	82. 28	38. 32	93. 66	48. 32
45	54. 15	12. 80	70. 82	28. 18	84. 89	40. 91	96. 25	50. 89
46	56. 80	15. 44	73. 44	30. 80	87. 50	43. 50	98. 84	53. 46
47	59. 44	18. 08	76. 07	33. 42	90. 10	46. 10	9, 201. 42	56. 03
48	62. 08	20. 72	78. 70	36. 04	92. 70	48. 70	04. 00	58. 60
49	64. 73	23. 36	81. 33	38. 65	95. 31	51. 29	06. 58	61. 17
50	67. 38	26. 00	83. 96	41. 26	97. 92	53. 88	09. 16	63. 74
51	8, 270. 03	8, 428. 63	8, 586. 59	8, 743. 88	8, 900. 52	9, 056. 48	9, 211. 74	9, 366. 31
52	72. 68	31. 26	89. 22	46. 50	93. 12	59. 08	14. 32	68. 88
53	75. 33	33. 90	91. 84	49. 12	95. 73	61. 67	16. 90	71. 45
54	77. 98	36. 54	94. 46	51. 74	98. 34	64. 26	19. 48	74. 02
55	80. 63	39. 18	97. 09	54. 35	10. 94	66. 85	22. 07	76. 59
56	83. 28	41. 82	99. 72	56. 96	13. 54	69. 44	24. 66	79. 16
57	85. 93	44. 46	8, 602. 35	59. 58	16. 15	72. 03	27. 24	81. 73
58	88. 58	47. 10	04. 98	62. 20	18. 76	74. 62	29. 82	84. 30
59	91. 22	49. 73	07. 60	64. 81	21. 36	77. 22	32. 40	86. 87

10''=0.44, 20''=0.88, 30''=1.32, first part; 10''=0.43, 20''=0.87, 30''=1.30, second part.

TABLE 22.—Chords, radius 10,000—Continued

	56°	57°	58°	59°		56°	57°	58°	59°
00	9, 389.44	9, 543.18	9, 696.20	9, 848.48	30	66.40	19.78	72.42	24.34
01	92.00	45.73	98.74	51.01	31	9, 468.96	9, 622.33	9, 774.96	9, 926.86
02	94.56	48.28	9, 701.28	53.54	32	71.52	24.88	77.50	29.38
03	97.13	50.84	03.82	56.07	33	74.08	27.43	80.04	31.91
04	99.70	53.40	06.36	58.60	34	76.64	29.98	82.58	34.44
05	9, 402.27	55.96	08.91	61.13	35	79.20	32.53	85.12	36.96
06	04.84	58.52	11.46	63.66	36	81.76	35.08	87.66	39.48
07	07.41	61.07	14.00	66.19	37	84.32	37.63	90.19	42.00
08	09.98	63.62	16.54	68.72	38	86.88	40.18	92.72	44.52
09	12.54	66.17	19.08	71.25	39	89.44	42.72	95.26	47.05
10	15.10	68.72	21.62	73.78	40	92.00	45.26	97.80	49.58
11	9, 417.67	9, 571.28	9, 724.16	9, 876.31	41	9, 494.56	9, 647.81	9, 800.33	9, 952.10
12	20.24	73.84	26.70	78.84	42	97.12	50.36	02.86	54.62
13	22.81	76.39	29.25	81.37	43	99.68	52.91	05.40	57.14
14	25.38	78.94	31.80	83.90	44	9, 502.24	55.46	07.94	59.66
15	27.94	81.50	34.34	86.43	45	04.80	58.01	10.47	62.19
16	30.50	84.06	36.88	88.96	46	07.36	60.56	13.00	64.72
17	33.07	86.61	39.42	91.49	47	09.92	63.10	15.54	67.24
18	35.64	89.16	41.96	94.02	48	12.48	65.64	18.08	69.76
19	38.20	91.71	44.50	96.54	49	15.04	68.19	20.61	72.28
20	40.76	94.26	47.04	99.06	50	17.60	70.74	23.14	74.80
21	9, 443.32	9, 596.81	9, 749.58	9, 901.59	51	9, 520.16	9, 673.29	9, 825.68	9, 977.32
22	45.88	99.36	52.12	04.12	52	22.72	75.84	28.22	79.84
23	48.45	9, 601.92	54.66	06.65	53	25.28	78.38	30.75	82.36
24	51.02	04.48	57.20	09.18	54	27.84	80.92	33.28	84.88
25	53.58	07.03	59.74	11.70	55	30.39	83.47	35.81	87.40
26	56.14	09.58	62.28	14.22	56	32.94	86.02	38.34	89.92
27	58.70	12.13	64.81	16.75	57	35.50	88.56	40.87	92.44
28	61.26	14.68	67.34	19.28	58	38.06	91.10	43.40	94.96
29	63.83	17.23	69.88	21.81	59	40.62	93.65	45.94	97.48

10''=0.42, 20''=0.85, 30''=1.27.

TABLE 23.—Mean refraction

Apparent altitude	Refraction	Apparent altitude	Refraction	Apparent altitude	Refraction	Apparent altitude	Refraction
° ' "	' "	° ' "	' "	° ' "	' "	° ' "	' "
5 00	9 52	11 00	4 51	21 00	2 31	30 00	1 41
10	9 36	20	4 43	20	2 28	31 00	1 37
20	9 21	40	4 35	40	2 26	32 00	1 33
30	9 07					33 00	1 29
40	8 53	12 00	4 27	22 00	2 23	34 00	1 26
50	8 40	20	4 20	20	2 21	35 00	1 23
		40	4 14	40	2 19		
6 00	8 28	13 00	4 07			36 00	1 20
10	8 16	20	4 01	23 00	2 16	37 00	1 17
20	8 05	40	3 55	20	2 14	38 00	1 14
30	7 54			40	2 12	39 00	1 12
40	7 44	14 00	3 49			40 00	1 09
50	7 34	20	3 44	24 00	2 10		
		40	3 39	20	2 08	41 00	1 07
7 00	7 24			40	2 06	42 00	1 05
10	7 15	15 00	3 34			43 00	1 02
20	7 06	20	3 29	25 00	2 04	44 00	1 00
30	6 57	40	3 25	20	2 02		
40	6 49			40	2 01	45 00	0 58
50	6 41	16 00	3 21			46 00	0 56
		20	3 16	26 00	1 59	47 00	0 54
8 00	6 33	40	3 12	20	1 57	48 00	0 52
10	6 26			40	1 55	49 00	0 51
20	6 19	17 00	3 08			50 00	0 49
30	6 12	20	3 05	27 00	1 54		
40	6 05	40	3 01	20	1 52	51 00	0 47
50	5 59			40	1 51	52 00	0 45
		18 00	2 57			53 00	0 44
9 00	5 53	20	2 54	28 00	1 49	54 00	0 42
10	5 47	40	2 51	20	1 48		
20	5 41			40	1 46	55 00	0 41
30	5 35	19 00	2 48			60 00	0 34
40	5 30	20	2 45	29 00	1 45	65 00	0 27
50	5 24	40	2 42	20	1 43		
				40	1 42	70 00	0 21
10 00	5 19	20 00	2 39			80 00	0 10
20	5 09	20	2 36			90 00	0 00
40	5 00	40	2 33				

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